# Computer algebra independent integration tests

Summer 2022 edition

## 5-Inverse-trig-functions/5.6-Inverse-cosecant/158-5.6.1-u-a+b-arccsc-c-x-^n

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# Contents

# Chapter 1

# Introduction

## Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [ 178 ]. This is test number [ 158 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 100.00 ( 178 ) | 0.00 ( 0 ) |
| Mathematica | 96.63 ( 172 ) | 3.37 ( 6 ) |
| Maple | 82.02 ( 146 ) | 17.98 ( 32 ) |
| Fricas | 64.04 ( 114 ) | 35.96 ( 64 ) |
| Giac | 52.25 ( 93 ) | 47.75 ( 85 ) |
| Maxima | 48.31 ( 86 ) | 51.69 ( 92 ) |
| Sympy | 36.52 ( 65 ) | 63.48 ( 113 ) |
| Mupad | 32.02 ( 57 ) | 67.98 ( 121 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

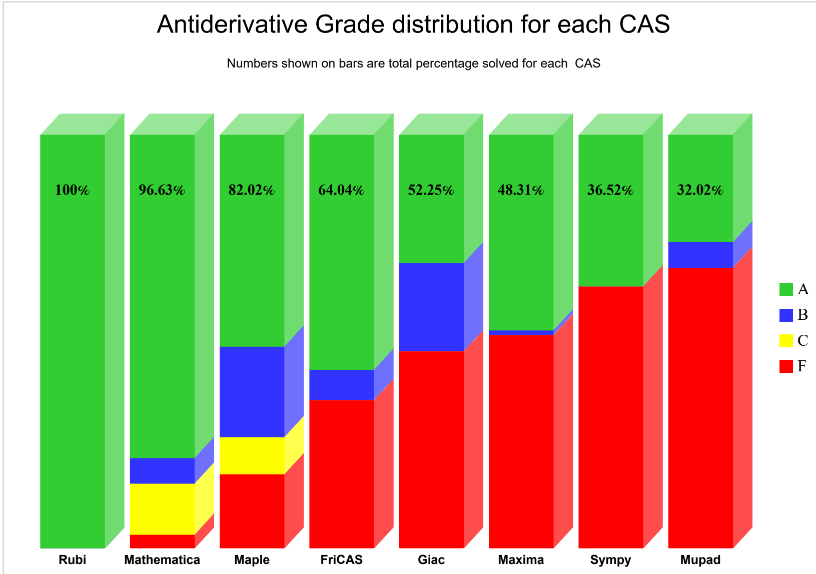
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

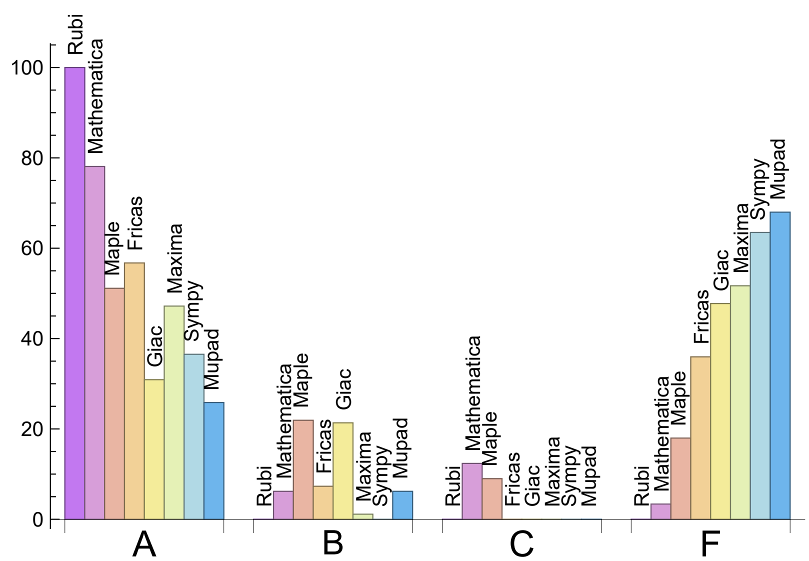
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 100.00 | 0.00 | 0.00 | 0.00 |
| Mathematica | 78.09 | 6.18 | 12.36 | 3.37 |
| Fricas | 56.74 | 7.30 | 0.00 | 35.96 |
| Maple | 51.12 | 21.91 | 8.99 | 17.98 |
| Maxima | 47.19 | 1.12 | 0.00 | 51.69 |
| Sympy | 36.52 | 0.00 | 0.00 | 63.48 |
| Giac | 30.90 | 21.35 | 0.00 | 47.75 |
| Mupad | N/A | 6.18 | 0.00 | 67.98 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the CAS systems for each grade level.




The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 % | 0.00 % | 0.00 % |
| Mathematica | 6 | 83.33 % | 16.67 % | 0.00 % |
| Maple | 32 | 96.88 % | 3.12 % | 0.00 % |
| Fricas | 64 | 73.44 % | 12.50 % | 14.06 % |
| Giac | 85 | 64.71 % | 4.71 % | 30.59 % |
| Maxima | 92 | 81.52 % | 0.00 % | 18.48 % |
| Sympy | 113 | 60.18 % | 36.28 % | 3.54 % |
| Mupad | 121 | 100.00 % | 0.00 % | 0.00 % |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

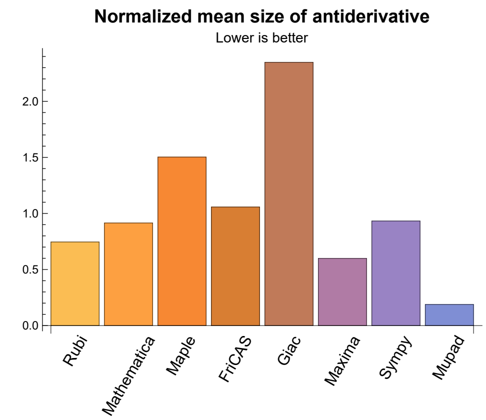
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

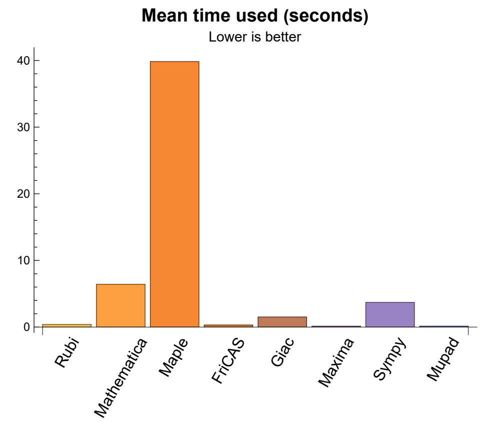
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.39 | 206.92 | 0.75 | 134.50 | 1.00 |
| Mathematica | 6.41 | 279.97 | 0.91 | 126.00 | 0.89 |
| Maple | 39.83 | 431.07 | 1.50 | 185.00 | 1.50 |
| Maxima | 0.14 | 69.87 | 0.60 | 0.00 | 0.00 |
| Fricas | 0.31 | 172.15 | 1.06 | 64.00 | 0.69 |
| Sympy | 3.70 | 122.97 | 0.93 | 37.00 | 1.03 |
| Giac | 1.51 | 314.38 | 2.35 | 54.00 | 1.29 |
| Mupad | 0.14 | 13.42 | 0.19 | -1.00 | -0.04 |

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.5
1.0
1.5
2.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
10
20
30
40
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

## 1.4 list of integrals that has no closed form antiderivative

$\{33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178\}$

## 1.5 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** $\{25, 51, 53, 57, 60, 72, 75, 98, 103, 104, 107, 108, 109, 110, 111, 115, 116, 117\}$

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

# 1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

# 1.9 Important notes about some of the results

## 1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

### 1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

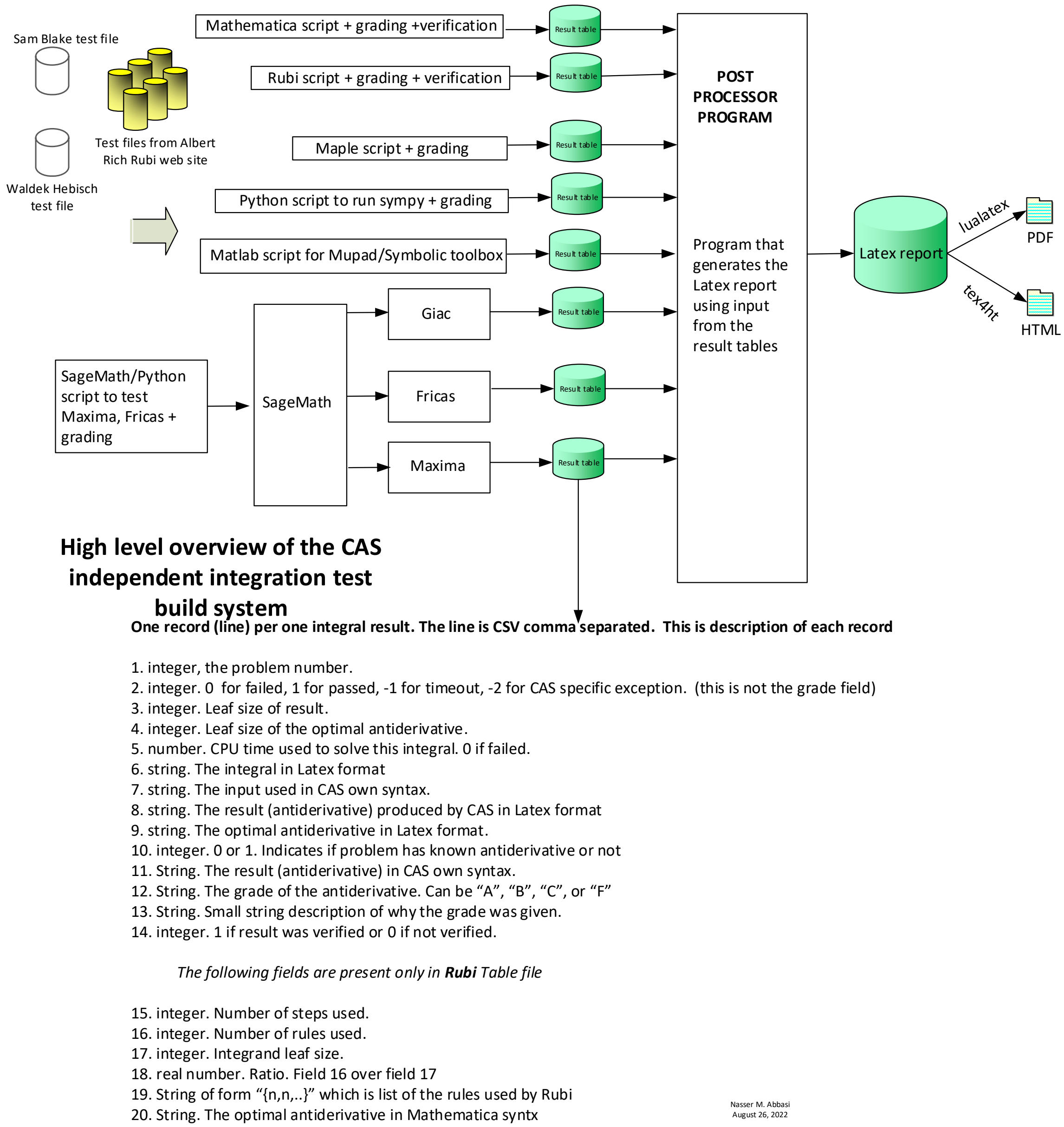
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



# Chapter 2

# detailed summary tables of results

## Local contents

## 2.1 List of integrals sorted by grade for each CAS

### Local contents

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 55, 60, 61, 62, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 101, 108, 109, 110, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { 25, 53, 72, 98, 99, 100, 102, 103, 104, 107, 111 }

C grade: { 51, 52, 56, 57, 58, 59, 63, 64, 65, 69, 70, 71, 105, 112, 113, 126, 127, 136, 137, 146, 155, 164 }

F grade: { 54, 106, 114, 165, 166, 167 }

### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 11, 13, 15, 16, 18, 22, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 90, 91, 92, 93, 96, 97, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 5, 10, 12, 14, 17, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 44, 45, 48, 49, 50, 51, 52, 56, 70, 71, 72, 75, 84, 85, 88, 89, 94, 95, 105, 112, 113 }

C grade: { 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 110, 111, 114, 115, 116, 117 }

F grade: { 41, 109, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 17, 20, 29, 33, 34, 35, 39, 40, 42, 43, 44, 45, 46, 47, 54, 55, 61, 62, 67, 68, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 15, 22 }

C grade: { }

F grade: { 8, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 41, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 35, 39, 40, 42, 43, 44, 45, 46, 54, 55, 61, 62, 67, 68, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 105, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 141, 142, 143, 144, 147, 150, 151, 152, 153, 154, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { 7, 17, 47, 49, 50, 112, 113, 140, 148, 149, 156, 157, 158 }

C grade: { }

F grade: { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 41, 48, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 126, 127, 136, 137, 145, 146, 155, 163, 164, 165, 166, 167 }

### 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 33, 34, 35, 39, 40, 42, 43, 44, 45, 46, 47, 62, 68, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 121, 122, 123, 124, 125, 130, 131, 133, 134, 135, 141, 142, 143, 144, 150, 153, 168, 171, 172, 177, 178 }

B grade: { }

C grade: { }

F grade: { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 132, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 173, 174, 175, 176 }

### 2.1.7 Giac

A grade: { 9, 10, 11, 12, 34, 35, 36, 37, 38, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 80, 81, 92, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 17, 20, 21, 22, 23, 29, 30, 31, 32, 44, 45, 46, 47, 76, 77, 78, 79, 82, 83, 84, 85, 88, 89, 90, 91, 93, 94, 95 }

C grade: { }

F grade: { 8, 16, 18, 19, 24, 25, 26, 27, 28, 33, 41, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

### 2.1.8 Mupad

A grade: { 33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 6, 7, 8, 9, 10, 20, 29, 47, 79, 86, 87 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 107 | 184 | 161 | 115 | 221 | 646 | -1 |
| N.S. | 1 | 1.00 | 0.94 | 1.61 | 1.41 | 1.01 | 1.94 | 5.67 | -0.01 |
| time (sec) | N/A | 0.045 | 0.094 | 0.146 | 0.260 | 0.472 | 11.779 | 0.935 | 0.000 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 72 | 83 | 80 | 62 | 153 | 518 | -1 |
| N.S. | 1 | 1.00 | 0.81 | 0.93 | 0.90 | 0.70 | 1.72 | 5.82 | -0.01 |
| time (sec) | N/A | 0.033 | 0.088 | 0.148 | 0.267 | 0.344 | 3.082 | 0.479 | 0.000 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 97 | 148 | 132 | 106 | 175 | 480 | -1 |
| N.S. | 1 | 1.00 | 1.09 | 1.66 | 1.48 | 1.19 | 1.97 | 5.39 | -0.01 |
| time (sec) | N/A | 0.036 | 0.049 | 0.142 | 0.270 | 0.413 | 4.801 | 0.716 | 0.000 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 62 | 74 | 59 | 52 | 107 | 352 | -1 |
| N.S. | 1 | 1.00 | 0.97 | 1.16 | 0.92 | 0.81 | 1.67 | 5.50 | -0.02 |
| time (sec) | N/A | 0.020 | 0.067 | 0.145 | 0.268 | 0.359 | 1.924 | 0.456 | 0.000 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 85 | 112 | 97 | 94 | 107 | 310 | -1 |
| N.S. | 1 | 1.00 | 1.33 | 1.75 | 1.52 | 1.47 | 1.67 | 4.84 | -0.02 |
| time (sec) | N/A | 0.025 | 0.042 | 0.144 | 0.312 | 0.412 | 2.635 | 0.683 | 0.000 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 50 | 65 | 36 | 39 | 58 | 182 | 40 |
| N.S. | 1 | 1.00 | 1.28 | 1.67 | 0.92 | 1.00 | 1.49 | 4.67 | 1.03 |
| time (sec) | N/A | 0.010 | 0.019 | 0.152 | 0.261 | 0.336 | 1.415 | 0.453 | 0.653 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 58 | 37 | 53 | 64 | 32 | 62 | 33 |
| N.S. | 1 | 1.00 | 1.87 | 1.19 | 1.71 | 2.06 | 1.03 | 2.00 | 1.06 |
| time (sec) | N/A | 0.017 | 0.045 | 0.075 | 0.258 | 0.347 | 1.633 | 0.421 | 0.872 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F(-2) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 53 | 140 | 0 | 0 | 0 | 0 | 65 |
| N.S. | 1 | 1.00 | 0.83 | 2.19 | 0.00 | 0.00 | 0.00 | 0.00 | 1.02 |
| time (sec) | N/A | 0.064 | 0.028 | 0.298 | 0.000 | 0.000 | 0.000 | 0.000 | 0.763 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 41 | 63 | 33 | 26 | 37 | 42 | 37 |
| N.S. | 1 | 1.00 | 1.28 | 1.97 | 1.03 | 0.81 | 1.16 | 1.31 | 1.16 |
| time (sec) | N/A | 0.017 | 0.022 | 0.147 | 0.254 | 0.390 | 0.482 | 0.438 | 0.636 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 66 | 114 | 83 | 40 | 121 | 66 | 50 |
| N.S. | 1 | 1.00 | 1.29 | 2.24 | 1.63 | 0.78 | 2.37 | 1.29 | 0.98 |
| time (sec) | N/A | 0.026 | 0.026 | 0.148 | 0.472 | 0.405 | 2.469 | 0.427 | 0.726 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 59 | 75 | 58 | 39 | 112 | 87 | -1 |
| N.S. | 1 | 1.00 | 0.98 | 1.25 | 0.97 | 0.65 | 1.87 | 1.45 | -0.02 |
| time (sec) | N/A | 0.028 | 0.045 | 0.145 | 0.277 | 0.391 | 2.146 | 0.439 | 0.000 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 78 | 150 | 125 | 53 | 194 | 117 | -1 |
| N.S. | 1 | 1.00 | 1.03 | 1.97 | 1.64 | 0.70 | 2.55 | 1.54 | -0.01 |
| time (sec) | N/A | 0.034 | 0.041 | 0.155 | 0.472 | 0.350 | 4.694 | 0.418 | 0.000 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 69 | 83 | 76 | 50 | 158 | 149 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 1.01 | 0.93 | 0.61 | 1.93 | 1.82 | -0.01 |
| time (sec) | N/A | 0.036 | 0.061 | 0.146 | 0.265 | 0.343 | 5.067 | 0.432 | 0.000 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 88 | 186 | 165 | 63 | 243 | 174 | -1 |
| N.S. | 1 | 1.00 | 0.87 | 1.84 | 1.63 | 0.62 | 2.41 | 1.72 | -0.01 |
| time (sec) | N/A | 0.043 | 0.070 | 0.141 | 0.501 | 0.336 | 10.912 | 0.434 | 0.000 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 124 | 181 | 197 | 146 | 0 | 811 | -1 |
| N.S. | 1 | 1.00 | 1.16 | 1.69 | 1.84 | 1.36 | 0.00 | 7.58 | -0.01 |
| time (sec) | N/A | 0.082 | 0.149 | 0.411 | 0.505 | 0.428 | 0.000 | 0.535 | 0.000 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 210 | 304 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.51 | 2.19 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.096 | 0.977 | 0.779 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 89 | 127 | 84 | 111 | 0 | 427 | -1 |
| N.S. | 1 | 1.00 | 1.62 | 2.31 | 1.53 | 2.02 | 0.00 | 7.76 | -0.02 |
| time (sec) | N/A | 0.055 | 0.085 | 0.407 | 0.291 | 0.425 | 0.000 | 0.498 | 0.000 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 147 | 188 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.75 | 2.24 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.054 | 0.146 | 0.283 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 91 | 137 | 361 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.51 | 3.97 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.095 | 0.089 | 0.363 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 71 | 118 | 79 | 57 | 0 | 104 | 88 |
| N.S. | 1 | 1.00 | 1.42 | 2.36 | 1.58 | 1.14 | 0.00 | 2.08 | 1.76 |
| time (sec) | N/A | 0.050 | 0.092 | 0.268 | 0.267 | 0.397 | 0.000 | 0.444 | 0.812 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 102 | 201 | 0 | 82 | 0 | 163 | -1 |
| N.S. | 1 | 1.00 | 1.16 | 2.28 | 0.00 | 0.93 | 0.00 | 1.85 | -0.01 |
| time (sec) | N/A | 0.057 | 0.071 | 0.266 | 0.000 | 0.394 | 0.000 | 0.443 | 0.000 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 108 | 154 | 197 | 93 | 0 | 224 | -1 |
| N.S. | 1 | 1.00 | 1.06 | 1.51 | 1.93 | 0.91 | 0.00 | 2.20 | -0.01 |
| time (sec) | N/A | 0.078 | 0.149 | 0.406 | 0.519 | 0.403 | 0.000 | 0.449 | 0.000 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 148 | 274 | 0 | 120 | 0 | 304 | -1 |
| N.S. | 1 | 1.00 | 1.10 | 2.04 | 0.00 | 0.90 | 0.00 | 2.27 | -0.01 |
| time (sec) | N/A | 0.086 | 0.110 | 0.385 | 0.000 | 0.353 | 0.000 | 0.445 | 0.000 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 207 | 207 | 285 | 458 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.38 | 2.21 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.157 | 0.597 | 0.819 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 220 | 580 | 603 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.64 | 2.74 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.153 | 7.313 | 1.003 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 182 | 323 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.44 | 2.56 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.111 | 0.327 | 0.747 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 265 | 412 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.84 | 2.86 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.091 | 0.194 | 0.551 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 242 | 666 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.95 | 5.37 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.106 | 0.130 | 0.401 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 135 | 199 | 147 | 98 | 0 | 195 | 155 |
| N.S. | 1 | 1.00 | 1.69 | 2.49 | 1.84 | 1.22 | 0.00 | 2.44 | 1.94 |
| time (sec) | N/A | 0.066 | 0.126 | 0.289 | 0.263 | 0.419 | 0.000 | 0.481 | 0.792 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 186 | 338 | 0 | 150 | 0 | 302 | -1 |
| N.S. | 1 | 1.00 | 1.49 | 2.70 | 0.00 | 1.20 | 0.00 | 2.42 | -0.01 |
| time (sec) | N/A | 0.075 | 0.134 | 0.354 | 0.000 | 0.352 | 0.000 | 0.464 | 0.000 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 204 | 299 | 0 | 173 | 0 | 428 | -1 |
| N.S. | 1 | 1.00 | 1.20 | 1.76 | 0.00 | 1.02 | 0.00 | 2.52 | -0.01 |
| time (sec) | N/A | 0.119 | 0.204 | 0.413 | 0.000 | 0.504 | 0.000 | 0.479 | 0.000 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 208 | 283 | 485 | 0 | 225 | 0 | 576 | -1 |
| N.S. | 1 | 1.00 | 1.36 | 2.33 | 0.00 | 1.08 | 0.00 | 2.77 | -0.00 |
| time (sec) | N/A | 0.125 | 0.225 | 0.519 | 0.000 | 0.420 | 0.000 | 0.447 | 0.000 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.07 |
| time (sec) | N/A | 0.011 | 2.649 | 1.458 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.08 |
| time (sec) | N/A | 0.005 | 2.411 | 0.910 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.06 |
| time (sec) | N/A | 0.017 | 0.202 | 0.406 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 43 | 48 | 0 | 0 | 0 | 54 | -1 |
| N.S. | 1 | 1.00 | 0.91 | 1.02 | 0.00 | 0.00 | 0.00 | 1.15 | -0.02 |
| time (sec) | N/A | 0.081 | 0.057 | 0.243 | 0.000 | 0.000 | 0.000 | 0.560 | 0.000 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 56 | 58 | 0 | 0 | 0 | 95 | -1 |
| N.S. | 1 | 1.00 | 0.89 | 0.92 | 0.00 | 0.00 | 0.00 | 1.51 | -0.02 |
| time (sec) | N/A | 0.114 | 0.056 | 0.200 | 0.000 | 0.000 | 0.000 | 0.422 | 0.000 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 91 | 102 | 0 | 0 | 0 | 200 | -1 |
| N.S. | 1 | 1.00 | 0.78 | 0.87 | 0.00 | 0.00 | 0.00 | 1.71 | -0.01 |
| time (sec) | N/A | 0.194 | 0.132 | 0.207 | 0.000 | 0.000 | 0.000 | 0.423 | 0.000 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.016 | 4.884 | 1.760 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.017 | 3.202 | 1.744 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 83 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 |
| time (sec) | N/A | 0.034 | 0.113 | 1.830 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.019 | 0.614 | 2.691 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 |
| time (sec) | N/A | 0.020 | 1.225 | 1.408 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 167 | 165 | 413 | 267 | 283 | 362 | 1130 | -1 |
| N.S. | 1 | 1.00 | 0.99 | 2.47 | 1.60 | 1.69 | 2.17 | 6.77 | -0.01 |
| time (sec) | N/A | 0.296 | 0.191 | 0.232 | 0.280 | 0.474 | 5.993 | 2.531 | 0.000 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 122 | 315 | 198 | 208 | 228 | 602 | -1 |
| N.S. | 1 | 1.00 | 0.99 | 2.56 | 1.61 | 1.69 | 1.85 | 4.89 | -0.01 |
| time (sec) | N/A | 0.191 | 0.110 | 0.227 | 0.281 | 0.406 | 4.897 | 1.974 | 0.000 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 113 | 139 | 94 | 134 | 104 | 341 | -1 |
| N.S. | 1 | 1.00 | 1.36 | 1.67 | 1.13 | 1.61 | 1.25 | 4.11 | -0.01 |
| time (sec) | N/A | 0.117 | 0.151 | 0.158 | 0.271 | 0.481 | 3.248 | 0.592 | 0.000 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 58 | 37 | 53 | 64 | 32 | 62 | 33 |
| N.S. | 1 | 1.00 | 1.87 | 1.19 | 1.71 | 2.06 | 1.03 | 2.00 | 1.06 |
| time (sec) | N/A | 0.017 | 0.035 | 0.083 | 0.255 | 0.434 | 1.776 | 0.422 | 0.002 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 257 | 257 | 411 | 893 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.60 | 3.47 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.288 | 0.464 | 1.066 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 141 | 216 | 0 | 442 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.38 | 2.12 | 0.00 | 4.33 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.114 | 0.140 | 2.318 | 0.000 | 0.420 | 0.000 | 0.000 | 0.000 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 250 | 971 | 0 | 1062 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.45 | 5.65 | 0.00 | 6.17 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.221 | 0.338 | 4.423 | 0.000 | 0.655 | 0.000 | 0.000 | 0.000 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | F(-1) | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 496 | 693 | 870 | 1204 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.40 | 1.75 | 2.43 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.031 | 40.711 | 0.729 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 404 | 502 | 368 | 827 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.24 | 0.91 | 2.05 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.682 | 10.953 | 0.655 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | F(-2) | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 315 | 315 | 657 | 386 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.09 | 1.23 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.336 | 28.377 | 0.566 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F(-1) | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.049 | 180.001 | 3.827 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.051 | 6.048 | 2.533 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 372 | 372 | 333 | 798 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.90 | 2.15 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.523 | 10.900 | 0.584 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 714 | 714 | 873 | 1233 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.22 | 1.73 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.906 | 30.533 | 0.619 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 530 | 530 | 784 | 850 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.48 | 1.60 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.463 | 30.501 | 0.579 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 344 | 344 | 289 | 410 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 1.19 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.203 | 10.906 | 0.566 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 212 | 212 | 243 | 252 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.15 | 1.19 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.226 | 3.827 | 0.553 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.049 | 4.586 | 1.427 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.055 | 6.972 | 3.257 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 551 | 551 | 814 | 880 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.48 | 1.60 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.859 | 30.757 | 0.582 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | F(-1) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 369 | 369 | 750 | 439 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.03 | 1.19 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.411 | 30.556 | 0.535 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 238 | 238 | 226 | 282 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.95 | 1.18 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.202 | 11.218 | 0.507 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 124 | 215 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.04 | 1.81 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.169 | 0.161 | 0.489 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.058 | 14.464 | 6.179 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.064 | 15.659 | 5.211 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 602 | 602 | 887 | 1067 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.47 | 1.77 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.154 | 30.675 | 0.960 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 440 | 440 | 856 | 1026 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.95 | 2.33 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.727 | 30.592 | 0.933 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F(-2) | F(-1) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 314 | 314 | 345 | 900 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.10 | 2.87 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.544 | 11.081 | 0.884 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 298 | 608 | 875 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.04 | 2.94 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.298 | 18.791 | 0.905 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.062 | 44.758 | 8.010 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.067 | 43.613 | 7.737 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F | F(-2) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 540 | 637 | 1002 | 1620 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.18 | 1.86 | 3.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.538 | 30.852 | 0.931 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 206 | 140 | 341 | 298 | 196 | 408 | 1166 | -1 |
| N.S. | 1 | 1.00 | 0.68 | 1.66 | 1.45 | 0.95 | 1.98 | 5.66 | -0.00 |
| time (sec) | N/A | 0.091 | 0.170 | 0.409 | 0.263 | 0.464 | 14.913 | 1.770 | 0.000 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 121 | 267 | 234 | 177 | 294 | 822 | -1 |
| N.S. | 1 | 1.00 | 0.75 | 1.66 | 1.45 | 1.10 | 1.83 | 5.11 | -0.01 |
| time (sec) | N/A | 0.075 | 0.122 | 0.422 | 0.261 | 0.410 | 6.297 | 1.278 | 0.000 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 149 | 191 | 155 | 147 | 153 | 473 | -1 |
| N.S. | 1 | 1.00 | 1.37 | 1.75 | 1.42 | 1.35 | 1.40 | 4.34 | -0.01 |
| time (sec) | N/A | 0.037 | 0.195 | 0.267 | 0.269 | 0.428 | 4.560 | 1.095 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 104 | 137 | 91 | 130 | 73 | 1052 | 75 |
| N.S. | 1 | 1.00 | 1.20 | 1.57 | 1.05 | 1.49 | 0.84 | 12.09 | 0.86 |
| time (sec) | N/A | 0.046 | 0.091 | 0.273 | 0.263 | 0.400 | 3.431 | 0.623 | 0.833 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 69 | 121 | 96 | 70 | 151 | 136 | -1 |
| N.S. | 1 | 1.00 | 0.66 | 1.15 | 0.91 | 0.67 | 1.44 | 1.30 | -0.01 |
| time (sec) | N/A | 0.055 | 0.079 | 0.270 | 0.265 | 0.360 | 2.889 | 0.464 | 0.000 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 152 | 94 | 140 | 139 | 93 | 280 | 245 | -1 |
| N.S. | 1 | 1.00 | 0.62 | 0.92 | 0.91 | 0.61 | 1.84 | 1.61 | -0.01 |
| time (sec) | N/A | 0.068 | 0.119 | 0.289 | 0.265 | 0.353 | 6.251 | 0.447 | 0.000 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 197 | 110 | 158 | 174 | 113 | 372 | 367 | -1 |
| N.S. | 1 | 1.00 | 0.56 | 0.80 | 0.88 | 0.57 | 1.89 | 1.86 | -0.01 |
| time (sec) | N/A | 0.087 | 0.122 | 0.305 | 0.265 | 0.357 | 34.486 | 0.438 | 0.000 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 196 | 115 | 152 | 185 | 130 | 364 | 1244 | -1 |
| N.S. | 1 | 1.00 | 0.59 | 0.78 | 0.94 | 0.66 | 1.86 | 6.35 | -0.01 |
| time (sec) | N/A | 0.110 | 0.172 | 0.415 | 0.267 | 0.382 | 5.362 | 0.542 | 0.000 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 153 | 97 | 307 | 144 | 111 | 272 | 900 | -1 |
| N.S. | 1 | 1.00 | 0.63 | 2.01 | 0.94 | 0.73 | 1.78 | 5.88 | -0.01 |
| time (sec) | N/A | 0.088 | 0.155 | 0.522 | 0.277 | 0.382 | 3.796 | 0.470 | 0.000 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 78 | 238 | 100 | 89 | 177 | 556 | -1 |
| N.S. | 1 | 1.00 | 0.57 | 1.72 | 0.72 | 0.64 | 1.28 | 4.03 | -0.01 |
| time (sec) | N/A | 0.068 | 0.076 | 0.503 | 0.259 | 0.365 | 2.362 | 0.452 | 0.000 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F(-2) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 108 | 198 | 0 | 0 | 0 | 0 | 111 |
| N.S. | 1 | 1.00 | 0.87 | 1.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.90 |
| time (sec) | N/A | 0.214 | 0.065 | 4.875 | 0.000 | 0.000 | 0.000 | 0.000 | 0.925 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 125 | 204 | 0 | 0 | 0 | 0 | 124 |
| N.S. | 1 | 1.00 | 0.91 | 1.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.91 |
| time (sec) | N/A | 0.214 | 0.075 | 1.111 | 0.000 | 0.000 | 0.000 | 0.000 | 0.991 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 252 | 252 | 184 | 475 | 404 | 272 | 542 | 1579 | -1 |
| N.S. | 1 | 1.00 | 0.73 | 1.88 | 1.60 | 1.08 | 2.15 | 6.27 | -0.00 |
| time (sec) | N/A | 0.167 | 0.225 | 0.520 | 0.272 | 0.568 | 15.162 | 5.304 | 0.000 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 191 | 151 | 358 | 296 | 236 | 355 | 1033 | -1 |
| N.S. | 1 | 1.00 | 0.79 | 1.87 | 1.55 | 1.24 | 1.86 | 5.41 | -0.01 |
| time (sec) | N/A | 0.084 | 0.155 | 0.369 | 0.264 | 0.489 | 7.349 | 3.451 | 0.000 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 163 | 134 | 273 | 197 | 234 | 207 | 2502 | -1 |
| N.S. | 1 | 1.00 | 0.82 | 1.67 | 1.21 | 1.44 | 1.27 | 15.35 | -0.01 |
| time (sec) | N/A | 0.092 | 0.122 | 0.367 | 0.273 | 0.456 | 5.552 | 2.560 | 0.000 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 157 | 125 | 251 | 158 | 233 | 211 | 4288 | -1 |
| N.S. | 1 | 1.00 | 0.80 | 1.60 | 1.01 | 1.48 | 1.34 | 27.31 | -0.01 |
| time (sec) | N/A | 0.096 | 0.180 | 0.388 | 0.259 | 0.414 | 5.531 | 101.449 | 0.000 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 183 | 127 | 191 | 181 | 131 | 335 | 314 | -1 |
| N.S. | 1 | 1.00 | 0.69 | 1.04 | 0.99 | 0.72 | 1.83 | 1.72 | -0.01 |
| time (sec) | N/A | 0.116 | 0.162 | 0.384 | 0.257 | 0.390 | 6.897 | 0.443 | 0.000 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 241 | 241 | 153 | 223 | 241 | 165 | 510 | 491 | -1 |
| N.S. | 1 | 1.00 | 0.63 | 0.93 | 1.00 | 0.68 | 2.12 | 2.04 | -0.00 |
| time (sec) | N/A | 0.140 | 0.161 | 0.384 | 0.277 | 0.351 | 35.888 | 0.475 | 0.000 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 242 | 159 | 499 | 253 | 184 | 493 | 1706 | -1 |
| N.S. | 1 | 1.00 | 0.66 | 2.06 | 1.05 | 0.76 | 2.04 | 7.05 | -0.00 |
| time (sec) | N/A | 0.160 | 0.196 | 0.614 | 0.264 | 0.388 | 6.470 | 0.579 | 0.000 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 124 | 376 | 189 | 150 | 352 | 1160 | -1 |
| N.S. | 1 | 1.00 | 0.64 | 1.93 | 0.97 | 0.77 | 1.81 | 5.95 | -0.01 |
| time (sec) | N/A | 0.105 | 0.183 | 0.627 | 0.263 | 0.367 | 4.093 | 0.523 | 0.000 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 157 | 301 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 1.62 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.300 | 0.245 | 8.671 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 194 | 309 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.03 | 1.63 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.300 | 0.493 | 6.217 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F | F | F(-2) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 565 | 565 | 1260 | 407 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.23 | 0.72 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.096 | 1.246 | 3.691 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 507 | 507 | 1123 | 420 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.21 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.975 | 0.321 | 2.313 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 529 | 529 | 1068 | 272 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.02 | 0.51 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.777 | 0.311 | 1.954 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 479 | 479 | 407 | 2875 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.85 | 6.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.764 | 0.616 | 3.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 572 | 572 | 1241 | 332 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.17 | 0.58 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.016 | 1.162 | 2.224 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 628 | 628 | 1480 | 764 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.36 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.145 | 4.087 | 2.608 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 593 | 590 | 1442 | 563 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.99 | 2.43 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.063 | 1.535 | 2.033 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | A | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 131 | 286 | 350 | 0 | 395 | 0 | 0 | -1 |
| N.S. | 1 | 0.98 | 2.13 | 2.61 | 0.00 | 2.95 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.088 | 0.534 | 7.335 | 0.000 | 0.415 | 0.000 | 0.000 | 0.000 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | C | F | F | F(-1) | F(-2) | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 566 | 566 | 0 | 3036 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 5.36 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.027 | 40.444 | 3.961 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F | F | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 803 | 803 | 1634 | 1888 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.03 | 2.35 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.267 | 6.043 | 12.786 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F | F | F(-2) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 765 | 765 | 1482 | 1722 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.94 | 2.25 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.130 | 1.825 | 4.824 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F(-1) | F | F | F | F(-2) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 762 | 759 | 1477 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.090 | 1.905 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F | F | F(-2) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 806 | 806 | 1525 | 1784 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.89 | 2.21 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.212 | 1.761 | 17.079 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | C | F | F | F(-1) | F(-1) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 727 | 727 | 2053 | 1520 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.82 | 2.09 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.224 | 7.203 | 4.236 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F(-1) | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 157 | 390 | 1831 | 0 | 1021 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 2.48 | 11.66 | 0.00 | 6.50 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.128 | 0.745 | 1.483 | 0.000 | 0.596 | 0.000 | 0.000 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F(-1) | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 193 | 385 | 1796 | 0 | 888 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.99 | 9.31 | 0.00 | 4.60 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.147 | 0.653 | 1.465 | 0.000 | 0.635 | 0.000 | 0.000 | 0.000 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | C | F | F | F(-1) | F(-2) | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 704 | 704 | 0 | 5294 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.00 | 7.52 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.140 | 60.538 | 17.462 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F | F(-1) | F(-2) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1144 | 1144 | 2067 | 3188 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.81 | 2.79 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.473 | 6.047 | 11.318 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F | F(-1) | F(-2) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1144 | 1144 | 2075 | 2342 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.81 | 2.05 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.862 | 6.041 | 13.876 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | F | F(-1) | F(-2) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1134 | 1134 | 2060 | 3181 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.82 | 2.81 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.540 | 6.033 | 13.699 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 403 | 403 | 321 | 0 | 0 | 868 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.80 | 0.00 | 0.00 | 2.15 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.844 | 0.330 | 180.000 | 0.000 | 2.429 | 0.000 | 0.000 | 0.000 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 294 | 257 | 0 | 0 | 721 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 2.45 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.276 | 0.250 | 180.000 | 0.000 | 1.211 | 0.000 | 0.000 | 0.000 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 196 | 0 | 0 | 588 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.01 | 0.00 | 0.00 | 3.02 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.132 | 0.160 | 180.000 | 0.000 | 0.661 | 0.000 | 0.000 | 0.000 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.063 | 3.113 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.066 | 2.914 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.068 | 8.063 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.030 | 34.704 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.055 | 1.112 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 328 | 328 | 247 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.286 | 7.248 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 453 | 453 | 325 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.412 | 9.779 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 374 | 374 | 295 | 0 | 0 | 865 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 2.31 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.358 | 0.333 | 180.000 | 0.000 | 2.368 | 0.000 | 0.000 | 0.000 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 262 | 262 | 235 | 0 | 0 | 716 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.90 | 0.00 | 0.00 | 2.73 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.194 | 2.776 | 180.000 | 0.000 | 1.156 | 0.000 | 0.000 | 0.000 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.078 | 3.751 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.088 | 3.043 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.077 | 8.382 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.028 | 35.294 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.065 | 68.855 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.066 | 4.582 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 416 | 416 | 303 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.381 | 10.197 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 554 | 554 | 383 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.535 | 10.574 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 321 | 321 | 259 | 0 | 0 | 724 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.81 | 0.00 | 0.00 | 2.26 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.719 | 0.268 | 180.000 | 0.000 | 1.338 | 0.000 | 0.000 | 0.000 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 225 | 200 | 0 | 0 | 593 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 2.64 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.213 | 0.193 | 180.000 | 0.000 | 0.622 | 0.000 | 0.000 | 0.000 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 136 | 0 | 0 | 466 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.03 | 0.00 | 0.00 | 3.53 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.102 | 0.083 | 180.000 | 0.000 | 0.442 | 0.000 | 0.000 | 0.000 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.062 | 1.137 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.072 | 9.874 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.064 | 69.469 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.020 | 0.722 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 247 | 140 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.175 | 0.129 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 362 | 362 | 249 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.319 | 5.528 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 252 | 252 | 231 | 0 | 0 | 774 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.92 | 0.00 | 0.00 | 3.07 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.723 | 0.232 | 180.000 | 0.000 | 0.598 | 0.000 | 0.000 | 0.000 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 146 | 0 | 0 | 572 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 3.67 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.170 | 0.136 | 180.000 | 0.000 | 0.446 | 0.000 | 0.000 | 0.000 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 96 | 0 | 0 | 304 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.22 | 0.00 | 0.00 | 3.85 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.067 | 0.089 | 180.000 | 0.000 | 0.401 | 0.000 | 0.000 | 0.000 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.076 | 25.420 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.092 | 40.812 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.068 | 7.504 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.067 | 2.973 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 112 | 0 | 0 | 79 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.04 | 0.00 | 0.00 | 0.73 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.061 | 0.134 | 180.000 | 0.000 | 0.124 | 0.000 | 0.000 | 0.000 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 275 | 213 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.220 | 3.956 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 243 | 243 | 237 | 0 | 0 | 1110 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 4.57 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.743 | 1.227 | 180.000 | 0.000 | 0.586 | 0.000 | 0.000 | 0.000 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 163 | 173 | 0 | 0 | 690 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.06 | 0.00 | 0.00 | 4.23 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.164 | 0.161 | 180.000 | 0.000 | 0.484 | 0.000 | 0.000 | 0.000 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | B | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 157 | 0 | 0 | 596 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 1.14 | 0.00 | 0.00 | 4.32 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.095 | 0.130 | 180.000 | 0.000 | 0.467 | 0.000 | 0.000 | 0.000 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.086 | 49.550 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.095 | 58.060 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.079 | 10.585 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.073 | 9.389 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 276 | 276 | 185 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.189 | 0.192 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 296 | 296 | 249 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.164 | 0.351 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 585 | 566 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.566 | 0.092 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 371 | 352 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.319 | 0.070 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 215 | 202 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.144 | 0.057 | 9.309 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.049 | 1.290 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.048 | 4.242 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.072 | 0.675 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.063 | 0.069 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.067 | 0.866 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.04 |
| time (sec) | N/A | 0.071 | 1.030 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-2) | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 401 | 401 | 194 | 0 | 0 | 238 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.48 | 0.00 | 0.00 | 0.59 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.746 | 0.188 | 180.000 | 0.000 | 0.431 | 0.000 | 0.000 | 0.000 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 268 | 268 | 159 | 0 | 0 | 181 | 0 | 0 | -1 |
| N.S. | 1 | 1.00 | 0.59 | 0.00 | 0.00 | 0.68 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.593 | 0.258 | 180.000 | 0.000 | 0.364 | 0.000 | 0.000 | 0.000 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 135 | 138 | 0 | 0 | 124 | 0 | 0 | -1 |
| N.S. | 1 | 1.07 | 1.10 | 0.00 | 0.00 | 0.98 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.143 | 0.114 | 180.000 | 0.000 | 0.391 | 0.000 | 0.000 | 0.000 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.060 | 0.291 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | A |
| verified | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| N.S. | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.03 |
| time (sec) | N/A | 0.069 | 2.439 | 180.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [174] had the largest ratio of [26]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 7 | 5 | 1.00 | 12 | 0.417 |
| 2 | A | 4 | 3 | 1.00 | 12 | 0.250 |
| 3 | A | 6 | 5 | 1.00 | 12 | 0.417 |
| 4 | A | 3 | 3 | 1.00 | 12 | 0.250 |
| 5 | A | 5 | 5 | 1.00 | 12 | 0.417 |
| 6 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 7 | A | 5 | 4 | 1.00 | 8 | 0.500 |
| 8 | A | 6 | 6 | 1.00 | 12 | 0.500 |
| 9 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 10 | A | 4 | 4 | 1.00 | 12 | 0.333 |
| 11 | A | 4 | 3 | 1.00 | 12 | 0.250 |
| 12 | A | 5 | 4 | 1.00 | 12 | 0.333 |
| 13 | A | 4 | 3 | 1.00 | 12 | 0.250 |
| 14 | A | 6 | 4 | 1.00 | 12 | 0.333 |
| 15 | A | 5 | 5 | 1.00 | 14 | 0.357 |
| 16 | A | 8 | 6 | 1.00 | 14 | 0.429 |
| 17 | A | 4 | 4 | 1.00 | 12 | 0.333 |
| 18 | A | 7 | 5 | 1.00 | 10 | 0.500 |
| 19 | A | 6 | 6 | 1.00 | 14 | 0.429 |
| 20 | A | 4 | 3 | 1.00 | 14 | 0.214 |
| 21 | A | 4 | 3 | 1.00 | 14 | 0.214 |
| 22 | A | 5 | 5 | 1.00 | 14 | 0.357 |
| 23 | A | 5 | 3 | 1.00 | 14 | 0.214 |
| 24 | A | 10 | 10 | 1.00 | 14 | 0.714 |
| 25 | A | 11 | 8 | 1.00 | 14 | 0.571 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 26 | A | 7 | 7 | 1.00 | 12 | 0.583 |
| 27 | A | 9 | 6 | 1.00 | 10 | 0.600 |
| 28 | A | 7 | 7 | 1.00 | 14 | 0.500 |
| 29 | A | 5 | 3 | 1.00 | 14 | 0.214 |
| 30 | A | 6 | 6 | 1.00 | 14 | 0.429 |
| 31 | A | 8 | 6 | 1.00 | 14 | 0.429 |
| 32 | A | 10 | 6 | 1.00 | 14 | 0.429 |
| 33 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 34 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 35 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 36 | A | 4 | 4 | 1.00 | 14 | 0.286 |
| 37 | A | 6 | 6 | 1.00 | 14 | 0.429 |
| 38 | A | 9 | 5 | 1.00 | 14 | 0.357 |
| 39 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 40 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 41 | A | 3 | 3 | 1.00 | 14 | 0.214 |
| 42 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 43 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 44 | A | 11 | 9 | 1.00 | 16 | 0.562 |
| 45 | A | 10 | 9 | 1.00 | 16 | 0.562 |
| 46 | A | 9 | 9 | 1.00 | 14 | 0.643 |
| 47 | A | 5 | 4 | 1.00 | 8 | 0.500 |
| 48 | A | 4 | 2 | 1.00 | 16 | 0.125 |
| 49 | A | 7 | 7 | 1.00 | 16 | 0.438 |
| 50 | A | 8 | 8 | 1.00 | 16 | 0.500 |
| 51 | A | 31 | 16 | 1.40 | 21 | 0.762 |
| 52 | A | 24 | 15 | 1.24 | 19 | 0.790 |
| 53 | A | 15 | 11 | 1.00 | 18 | 0.611 |
| 54 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 55 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 56 | A | 22 | 13 | 1.00 | 18 | 0.722 |
| 57 | A | 27 | 17 | 1.00 | 21 | 0.810 |
| 58 | A | 20 | 15 | 1.00 | 21 | 0.714 |
| 59 | A | 14 | 13 | 1.00 | 19 | 0.684 |
| 60 | A | 9 | 9 | 1.00 | 18 | 0.500 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 61 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 62 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 63 | A | 23 | 15 | 1.00 | 21 | 0.714 |
| 64 | A | 16 | 13 | 1.00 | 21 | 0.619 |
| 65 | A | 11 | 11 | 1.00 | 19 | 0.579 |
| 66 | A | 6 | 6 | 1.00 | 18 | 0.333 |
| 67 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 68 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 69 | A | 31 | 18 | 1.00 | 21 | 0.857 |
| 70 | A | 25 | 17 | 1.00 | 21 | 0.810 |
| 71 | A | 19 | 14 | 1.00 | 19 | 0.737 |
| 72 | A | 12 | 11 | 1.00 | 18 | 0.611 |
| 73 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 74 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 75 | A | 19 | 14 | 1.18 | 18 | 0.778 |
| 76 | A | 7 | 7 | 1.00 | 19 | 0.368 |
| 77 | A | 6 | 7 | 1.00 | 19 | 0.368 |
| 78 | A | 5 | 5 | 1.00 | 16 | 0.312 |
| 79 | A | 4 | 5 | 1.00 | 19 | 0.263 |
| 80 | A | 4 | 5 | 1.00 | 19 | 0.263 |
| 81 | A | 5 | 6 | 1.00 | 19 | 0.316 |
| 82 | A | 6 | 6 | 1.00 | 19 | 0.316 |
| 83 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 84 | A | 5 | 5 | 1.00 | 19 | 0.263 |
| 85 | A | 6 | 5 | 1.00 | 17 | 0.294 |
| 86 | A | 11 | 11 | 1.00 | 19 | 0.579 |
| 87 | A | 13 | 13 | 1.00 | 19 | 0.684 |
| 88 | A | 7 | 8 | 1.00 | 21 | 0.381 |
| 89 | A | 6 | 7 | 1.00 | 18 | 0.389 |
| 90 | A | 6 | 7 | 1.00 | 21 | 0.333 |
| 91 | A | 6 | 7 | 1.00 | 21 | 0.333 |
| 92 | A | 5 | 6 | 1.00 | 21 | 0.286 |
| 93 | A | 6 | 7 | 1.00 | 21 | 0.333 |
| 94 | A | 5 | 6 | 1.00 | 21 | 0.286 |
| 95 | A | 6 | 5 | 1.00 | 19 | 0.263 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 96 | A | 12 | 13 | 1.00 | 21 | 0.619 |
| 97 | A | 14 | 15 | 1.00 | 21 | 0.714 |
| 98 | A | 25 | 12 | 1.00 | 21 | 0.571 |
| 99 | A | 26 | 9 | 1.00 | 19 | 0.474 |
| 100 | A | 19 | 7 | 1.00 | 18 | 0.389 |
| 101 | A | 19 | 7 | 1.00 | 21 | 0.333 |
| 102 | A | 24 | 10 | 1.00 | 21 | 0.476 |
| 103 | A | 31 | 14 | 1.00 | 21 | 0.667 |
| 104 | A | 29 | 12 | 0.99 | 21 | 0.571 |
| 105 | A | 7 | 5 | 0.98 | 19 | 0.263 |
| 106 | A | 24 | 10 | 1.00 | 21 | 0.476 |
| 107 | A | 51 | 15 | 1.00 | 21 | 0.714 |
| 108 | A | 27 | 10 | 1.00 | 21 | 0.476 |
| 109 | A | 47 | 11 | 1.00 | 18 | 0.611 |
| 110 | A | 50 | 13 | 1.00 | 21 | 0.619 |
| 111 | A | 33 | 13 | 1.00 | 21 | 0.619 |
| 112 | A | 6 | 7 | 1.00 | 21 | 0.333 |
| 113 | A | 8 | 6 | 1.00 | 19 | 0.316 |
| 114 | A | 28 | 11 | 1.00 | 21 | 0.524 |
| 115 | A | 35 | 11 | 1.00 | 21 | 0.524 |
| 116 | A | 63 | 12 | 1.00 | 21 | 0.571 |
| 117 | A | 81 | 12 | 1.00 | 18 | 0.667 |
| 118 | A | 12 | 12 | 1.00 | 23 | 0.522 |
| 119 | A | 11 | 12 | 1.00 | 23 | 0.522 |
| 120 | A | 9 | 9 | 1.00 | 21 | 0.429 |
| 121 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 122 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 123 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 124 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 125 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 126 | A | 11 | 11 | 1.00 | 23 | 0.478 |
| 127 | A | 12 | 12 | 1.00 | 23 | 0.522 |
| 128 | A | 12 | 12 | 1.00 | 23 | 0.522 |
| 129 | A | 10 | 10 | 1.00 | 21 | 0.476 |
| 130 | A | 0 | 0 | 0.00 | 0 | 0.000 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 131 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 132 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 133 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 134 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 135 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 136 | A | 12 | 12 | 1.00 | 23 | 0.522 |
| 137 | A | 13 | 12 | 1.00 | 23 | 0.522 |
| 138 | A | 11 | 12 | 1.00 | 23 | 0.522 |
| 139 | A | 10 | 12 | 1.00 | 23 | 0.522 |
| 140 | A | 9 | 9 | 1.00 | 21 | 0.429 |
| 141 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 142 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 143 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 144 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 145 | A | 11 | 11 | 1.00 | 23 | 0.478 |
| 146 | A | 11 | 12 | 1.00 | 23 | 0.522 |
| 147 | A | 10 | 11 | 1.00 | 23 | 0.478 |
| 148 | A | 9 | 11 | 1.00 | 23 | 0.478 |
| 149 | A | 4 | 4 | 1.00 | 21 | 0.190 |
| 150 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 151 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 152 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 153 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 154 | A | 5 | 5 | 1.00 | 20 | 0.250 |
| 155 | A | 10 | 11 | 1.00 | 23 | 0.478 |
| 156 | A | 10 | 11 | 1.00 | 23 | 0.478 |
| 157 | A | 7 | 8 | 1.00 | 23 | 0.348 |
| 158 | A | 5 | 5 | 1.00 | 21 | 0.238 |
| 159 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 160 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 161 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 162 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 163 | A | 10 | 10 | 1.00 | 23 | 0.435 |
| 164 | A | 10 | 11 | 1.00 | 20 | 0.550 |
| 165 | A | 6 | 7 | 0.97 | 23 | 0.304 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 166 | A | 6 | 7 | 0.95 | 23 | 0.304 |
| 167 | A | 5 | 6 | 0.94 | 21 | 0.286 |
| 168 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 169 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 170 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 171 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 172 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 173 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 174 | A | 16 | 11 | 1.00 | 26 | 0.423 |
| 175 | A | 13 | 11 | 1.00 | 26 | 0.423 |
| 176 | A | 8 | 9 | 1.07 | 26 | 0.346 |
| 177 | A | 0 | 0 | 0.00 | 0 | 0.000 |
| 178 | A | 0 | 0 | 0.00 | 0 | 0.000 |

# Chapter 3

# Listing of integrals

## Local contents

## 3.1 $\int x^6(a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=114

$$\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{112c^5}+\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)+\frac{5b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}$$

```
[Out] 1/7*x^7*(a+b*arccsc(c*x))+5/112*b*arctanh((1-1/c^2/x^2)^(1/2))/c^7+5/112*b*
x^2*(1-1/c^2/x^2)^(1/2)/c^5+5/168*b*x^4*(1-1/c^2/x^2)^(1/2)/c^3+1/42*b*x^6*
(1-1/c^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5329, 272, 44, 65, 214}

$$\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)+\frac{bx^6\sqrt{1-\frac{1}{c^2x^2}}}{42c}+\frac{5b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}+\frac{5bx^2\sqrt{1-\frac{1}{c^2x^2}}}{112c^5}+\frac{5bx^4\sqrt{1-\frac{1}{c^2x^2}}}{168c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (5*b*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(112*c^5) + (5*b*Sqrt[1 - 1/(c^2*x^2)]*x^4)
/(168*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^6)/(42*c) + (x^7*(a + b*ArcCsc[c*x]
))/7 + (5*b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(112*c^7)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^6\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x^5}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{7c}$$

$$=\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)-\frac{b\text{Subst}\left(\int\frac{1}{x^4\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{14c}$$

$$=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)-\frac{(5b)\text{Subst}\left(\int\frac{1}{x^3\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right.}{84c^3}$$

$$=\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx)\right)-\frac{(5b)\text{Subst}\left(\int\right.}{}$$

$$=\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{112c^5}+\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx\right.$$

$$=\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{112c^5}+\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx\right.$$

$$=\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{112c^5}+\frac{5b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\csc^{-1}(cx\right.$$

**Mathematica [A]**
time = 0.09, size = 107, normalized size = 0.94

$$\frac{ax^7}{7}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(\frac{5x^2}{112c^5}+\frac{5x^4}{168c^3}+\frac{x^6}{42c}\right)+\frac{1}{7}bx^7\csc^{-1}(cx)+\frac{5b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{112c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*ArcCsc[c*x]),x]
```

[Out] (a*x^7)/7 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((5*x^2)/(112*c^5) + (5*x^4)/(168*c^3) + x^6/(42*c)) + (b*x^7*ArcCsc[c*x])/7 + (5*b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])])/(112*c^7)

**Maple [A]**
time = 0.15, size = 184, normalized size = 1.61

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c^7x^7a}{7}+\frac{b\,c^7x^7\,\mathrm{arccsc}(cx)}{7}+\frac{b(c^2x^2-1)c^4x^4}{42\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{5b(c^2x^2-1)c^2x^2}{168\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{5b(c^2x^2-1)}{112\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{5b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{112\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^7}$ |
| default | $\frac{\frac{c^7x^7a}{7}+\frac{b\,c^7x^7\,\mathrm{arccsc}(cx)}{7}+\frac{b(c^2x^2-1)c^4x^4}{42\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{5b(c^2x^2-1)c^2x^2}{168\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{5b(c^2x^2-1)}{112\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{5b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{112\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^7}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/c^7*(1/7*c^7*x^7*a+1/7*b*c^7*x^7*arccsc(c*x)+1/42*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*c^4*x^4+5/168*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*c^2*x^2+5/112*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)+5/112*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*ln(c*x+(c^2*x^2-1)^(1/2)))

**Maxima [A]**
time = 0.26, size = 161, normalized size = 1.41

$$\frac{1}{7}ax^7+\frac{1}{672}\left(96\,x^7\,\mathrm{arccsc}\,(cx)+\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3c^6\left(\frac{1}{c^2x^2}-1\right)^2+3c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="maxima")

[Out] 1/7*a*x^7 + 1/672*(96*x^7*arccsc(c*x) + (2*(15*(-1/(c^2*x^2) + 1)^(5/2) - 40*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^6)/c)*b

**Fricas [A]**
time = 0.47, size = 115, normalized size = 1.01

$$\frac{48\,ac^7x^7-96\,bc^7\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+48\,(bc^7x^7-bc^7)\,\mathrm{arccsc}\,(cx)-15\,b\log\left(-cx+\sqrt{c^2x^2-1}\right)+(8\,bc^5x^5+10\,bc^3x^3+15\,bcx)\sqrt{c^2x^2-1}}{336\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/336*(48*a*c^7*x^7 - 96*b*c^7*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 48*(b*c^7
*x^7 - b*c^7)*arccsc(c*x) - 15*b*log(-c*x + sqrt(c^2*x^2 - 1)) + (8*b*c^5*x
^5 + 10*b*c^3*x^3 + 15*b*c*x)*sqrt(c^2*x^2 - 1))/c^7
```

**Sympy [A]**
time = 11.78, size = 221, normalized size = 1.94

$$\frac{ax^7}{7} + \frac{bx^7 \operatorname{acsc}(cx)}{7} + \frac{b\left(\begin{cases} \frac{cx^7}{6\sqrt{c^2x^2-1}} + \frac{x^5}{24c\sqrt{c^2x^2-1}} + \frac{5x^3}{48c^3\sqrt{c^2x^2-1}} - \frac{5x}{16c^5\sqrt{c^2x^2-1}} + \frac{5\operatorname{acosh}(cx)}{16c^6} & \text{for } |c^2x^2| > 1 \\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}} - \frac{ix^5}{24c\sqrt{-c^2x^2+1}} - \frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}} + \frac{5ix}{16c^5\sqrt{-c^2x^2+1}} - \frac{5i\operatorname{asin}(cx)}{16c^6} & \text{otherwise} \end{cases}\right)}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**7/7 + b*x**7*acsc(c*x)/7 + b*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1))
 + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1)) -
 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2)
 > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 +
 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*
x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(96) = 192.
time = 0.94, size = 646, normalized size = 5.67

$$\frac{1}{2688}\left(\frac{3bx^7\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^7\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{3ax^7\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^7}{c} + \frac{bx^6\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^6}{c^2} + \frac{21bx^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5\arcsin\left(\frac{1}{cx}\right)}{c^3} + \frac{21ax^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5}{c^3} + \frac{9bx^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c^4} + \frac{63bx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)}{c^5} + \frac{63ax^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c^5} + \frac{45bx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c^6} + \frac{105bx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{cx}\right)}{c^7} + \frac{105ax\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^7} + \frac{120b\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^8} - \frac{120b\log\left(\frac{1}{|c||x|}\right)}{c^8} + \frac{105b\arcsin\left(\frac{1}{cx}\right)}{c^9x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} + \frac{105a}{c^9x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} - \frac{45b}{c^{10}x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2} + \frac{63b\arcsin\left(\frac{1}{cx}\right)}{c^{11}x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3} + \frac{63a}{c^{11}x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3} - \frac{9b}{c^{12}x^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4} + \frac{21b\arcsin\left(\frac{1}{cx}\right)}{c^{13}x^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5} + \frac{21a}{c^{13}x^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5} - \frac{b}{c^{14}x^6\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^6} + \frac{3b\arcsin\left(\frac{1}{cx}\right)}{c^{15}x^7\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^7} + \frac{3a}{c^{15}x^7\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^7}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/2688*(3*b*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*arcsin(1/(c*x))/c + 3*a*x^7*
(sqrt(-1/(c^2*x^2) + 1) + 1)^7/c + b*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c^2
 + 21*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c^3 + 21*a*x^5*(
sqrt(-1/(c^2*x^2) + 1) + 1)^5/c^3 + 9*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/
c^4 + 63*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^5 + 63*a*x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^5 + 45*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1
)^2/c^6 + 105*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^7 + 105*a*
x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^7 + 120*b*log(sqrt(-1/(c^2*x^2) + 1) + 1)/
c^8 - 120*b*log(1/(abs(c)*abs(x)))/c^8 + 105*b*arcsin(1/(c*x))/(c^9*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)) + 105*a/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 45*
b/(c^10*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 63*b*arcsin(1/(c*x))/(c^11*x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 63*a/(c^11*x^3*(sqrt(-1/(c^2*x^2) + 1)
```

```
+ 1)^3) - 9*b/(c^12*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 21*b*arcsin(1/(c*
x))/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 21*a/(c^13*x^5*(sqrt(-1/(c^
2*x^2) + 1) + 1)^5) - b/(c^14*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 3*b*arc
sin(1/(c*x))/(c^15*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7) + 3*a/(c^15*x^7*(sqr
t(-1/(c^2*x^2) + 1) + 1)^7))*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*asin(1/(c*x))),x)
[Out] int(x^6*(a + b*asin(1/(c*x))), x)
```

# 3.2 $\int x^5(a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=89

$$\frac{4b\sqrt{1-\frac{1}{c^2x^2}}\,x}{45c^5}+\frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{45c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)$$

```
[Out] 1/6*x^6*(a+b*arccsc(c*x))+4/45*b*x*(1-1/c^2/x^2)^(1/2)/c^5+2/45*b*x^3*(1-1/
c^2/x^2)^(1/2)/c^3+1/30*b*x^5*(1-1/c^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {5329, 277, 197}

$$\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{bx^5\sqrt{1-\frac{1}{c^2x^2}}}{30c}+\frac{4bx\sqrt{1-\frac{1}{c^2x^2}}}{45c^5}+\frac{2bx^3\sqrt{1-\frac{1}{c^2x^2}}}{45c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (4*b*Sqrt[1 - 1/(c^2*x^2)]*x)/(45*c^5) + (2*b*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(4
5*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^5)/(30*c) + (x^6*(a + b*ArcCsc[c*x]))/6
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^5\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{6c}$$
$$=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{(2b)\int\frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{15c^3}$$
$$=\frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{45c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)+\frac{(4b)\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}}{45c^5}$$
$$=\frac{4b\sqrt{1-\frac{1}{c^2x^2}}\,x}{45c^5}+\frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{45c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)$$

**Mathematica** [A]
time = 0.09, size = 72, normalized size = 0.81

$$\frac{ax^6}{6}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(\frac{4x}{45c^5}+\frac{2x^3}{45c^3}+\frac{x^5}{30c}\right)+\frac{1}{6}bx^6\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (a*x^6)/6 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((4*x)/(45*c^5) + (2*x^3)/(45*
c^3) + x^5/(30*c)) + (b*x^6*ArcCsc[c*x])/6
```

**Maple** [A]
time = 0.15, size = 83, normalized size = 0.93

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{c^6x^6a}{6}+b\left(\frac{c^6x^6\operatorname{arccsc}(cx)}{6}+\frac{(c^2x^2-1)(3c^4x^4+4c^2x^2+8)}{90\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^6}$ | 83 |
| default | $\frac{\frac{c^6x^6a}{6}+b\left(\frac{c^6x^6\operatorname{arccsc}(cx)}{6}+\frac{(c^2x^2-1)(3c^4x^4+4c^2x^2+8)}{90\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^6}$ | 83 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(1/6*c^6*x^6*a+b*(1/6*c^6*x^6*arccsc(c*x)+1/90*(c^2*x^2-1)*(3*c^4*x^4
+4*c^2*x^2+8)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))
```

**Maxima [A]**
time = 0.27, size = 80, normalized size = 0.90

$$\frac{1}{6}\,ax^6+\frac{1}{90}\left(15\,x^6\,\mathrm{arccsc}\left(cx\right)+\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2)
+ 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b
```

**Fricas [A]**
time = 0.34, size = 62, normalized size = 0.70

$$\frac{15\,bc^6x^6\,\mathrm{arccsc}\left(cx\right)+15\,ac^6x^6+\left(3\,bc^4x^4+4\,bc^2x^2+8\,b\right)\sqrt{c^2x^2-1}}{90\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*b*c^6*x^6*arccsc(c*x) + 15*a*c^6*x^6 + (3*b*c^4*x^4 + 4*b*c^2*x^2
+ 8*b)*sqrt(c^2*x^2 - 1))/c^6
```

**Sympy [A]**
time = 3.08, size = 153, normalized size = 1.72

$$\frac{ax^6}{6}+\frac{bx^6\,\mathrm{acsc}\left(cx\right)}{6}+\frac{b\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for}\ \left|c^2x^2\right|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**6/6 + b*x**6*acsc(c*x)/6 + b*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c)
 + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5),
Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c*
*2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6*c)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(75) = 150$.
time = 0.48, size = 518, normalized size = 5.82

$$\frac{1}{5760}\left(\frac{15\,bx^{6}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{6}\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{15\,ax^{6}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{6}}{c}+\frac{6\,bx^{5}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{5}}{c^{2}}+\frac{90\,bx^{4}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{4}\arcsin\left(\frac{1}{cx}\right)}{c^{3}}+\frac{90\,ax^{4}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{4}}{c^{3}}+\frac{50\,bx^{3}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{3}}{c^{4}}+\frac{225\,bx^{2}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{2}\arcsin\left(\frac{1}{cx}\right)}{c^{5}}+\frac{225\,ax^{2}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{2}}{c^{5}}+\frac{300\,bx\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)}{c^{6}}+\frac{300\,b\arcsin\left(\frac{1}{cx}\right)}{c^{7}}+\frac{300\,a}{c^{7}}-\frac{300\,b}{c^{8}x\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)}+\frac{225\,b\arcsin\left(\frac{1}{cx}\right)}{c^{9}x^{2}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{2}}+\frac{225\,a}{c^{9}x^{2}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{2}}-\frac{50\,b}{c^{10}x^{3}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{3}}+\frac{90\,b\arcsin\left(\frac{1}{cx}\right)}{c^{11}x^{4}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{4}}+\frac{90\,a}{c^{11}x^{4}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{4}}-\frac{6\,b}{c^{12}x^{5}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{5}}+\frac{15\,b\arcsin\left(\frac{1}{cx}\right)}{c^{13}x^{6}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{6}}+\frac{15\,a}{c^{13}x^{6}\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)^{6}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/5760*(15*b*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c + 15*a*x^
6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 6*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5
/c^2 + 90*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 90*a*x
^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^3 + 50*b*x^3*(sqrt(-1/(c^2*x^2) + 1) +
1)^3/c^4 + 225*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^5 + 2
25*a*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 300*b*x*(sqrt(-1/(c^2*x^2) +
1) + 1)/c^6 + 300*b*arcsin(1/(c*x))/c^7 + 300*a/c^7 - 300*b/(c^8*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)) + 225*b*arcsin(1/(c*x))/(c^9*x^2*(sqrt(-1/(c^2*x^2) +
 1) + 1)^2) + 225*a/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 50*b/(c^10*x
^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 90*b*arcsin(1/(c*x))/(c^11*x^4*(sqrt(-
1/(c^2*x^2) + 1) + 1)^4) + 90*a/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) -
 6*b/(c^12*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 15*b*arcsin(1/(c*x))/(c^13
*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 15*a/(c^13*x^6*(sqrt(-1/(c^2*x^2) +
1) + 1)^6))*c
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^5*(a + b*asin(1/(c*x))), x)
```

# 3.3 $\int x^4(a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=89

$$\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{40c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)+\frac{3b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}$$

```
[Out] 1/5*x^5*(a+b*arccsc(c*x))+3/40*b*arctanh((1-1/c^2/x^2)^(1/2))/c^5+3/40*b*x^
2*(1-1/c^2/x^2)^(1/2)/c^3+1/20*b*x^4*(1-1/c^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5329, 272, 44, 65, 214}

$$\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)+\frac{bx^4\sqrt{1-\frac{1}{c^2x^2}}}{20c}+\frac{3b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}+\frac{3bx^2\sqrt{1-\frac{1}{c^2x^2}}}{40c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (3*b*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(40*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^4)/(2
0*c) + (x^5*(a + b*ArcCsc[c*x]))/5 + (3*b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(
40*c^5)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^4\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x^3}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{5c}$$

$$=\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)-\frac{b\text{Subst}\left(\int\frac{1}{x^3\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{10c}$$

$$=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)-\frac{(3b)\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{40c^3}$$

$$=\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{40c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)-\frac{(3b)\text{Subst}\left(\int\cdots\right)}{\cdots}$$

$$=\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{40c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)+\frac{(3b)\text{Subst}\left(\int\cdots\right)}{\cdots}$$

$$=\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{40c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)+\frac{3b\tanh^{-1}\left(\sqrt{1\cdots}\right)}{40c^5\cdots}$$

**Mathematica [A]**
time = 0.05, size = 97, normalized size = 1.09

$$\frac{ax^5}{5}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(\frac{3x^2}{40c^3}+\frac{x^4}{20c}\right)+\frac{1}{5}bx^5\csc^{-1}(cx)+\frac{3b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{40c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (a*x^5)/5 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((3*x^2)/(40*c^3) + x^4/(20*c)
) + (b*x^5*ArcCsc[c*x])/5 + (3*b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])
])/(40*c^5)
```

**Maple** [A]
time = 0.14, size = 148, normalized size = 1.66

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{c^{5}x^{5}a}{5}+\frac{b\,c^{5}x^{5}\operatorname{arccsc}(cx)}{5}+\frac{b\left(c^{2}x^{2}-1\right)c^{2}x^{2}}{20\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{3b\left(c^{2}x^{2}-1\right)}{40\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{3b\sqrt{c^{2}x^{2}-1}\,\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)}{40\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}}{c^{5}}$ | 148 |
| default | $\frac{\frac{c^{5}x^{5}a}{5}+\frac{b\,c^{5}x^{5}\operatorname{arccsc}(cx)}{5}+\frac{b\left(c^{2}x^{2}-1\right)c^{2}x^{2}}{20\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{3b\left(c^{2}x^{2}-1\right)}{40\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{3b\sqrt{c^{2}x^{2}-1}\,\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)}{40\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}}{c^{5}}$ | 148 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/5*c^5*x^5*a+1/5*b*c^5*x^5*arccsc(c*x)+1/20*b*(c^2*x^2-1)/((c^2*x^2
-1)/c^2/x^2)^(1/2)*c^2*x^2+3/40*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)+3
/40*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*ln(c*x+(c^2*x^2-1)^
(1/2)))
```

**Maxima** [A]
time = 0.27, size = 132, normalized size = 1.48

$$\frac{1}{5}\,ax^{5}+\frac{1}{80}\left(16\,x^{5}\operatorname{arccsc}\left(cx\right)-\frac{\frac{2\left(3\left(-\frac{1}{c^{2}x^{2}}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^{2}x^{2}}+1}\right)}{c^{4}\left(\frac{1}{c^{2}x^{2}}-1\right)^{2}+2\,c^{4}\left(\frac{1}{c^{2}x^{2}}-1\right)+c^{4}}-\frac{3\,\log\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)}{c^{4}}+\frac{3\,\log\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}-1\right)}{c^{4}}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/80*(16*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*s
qrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) +
 c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1
) - 1)/c^4)/c)*b
```

**Fricas** [A]
time = 0.41, size = 106, normalized size = 1.19

$$\frac{8\,ac^{5}x^{5}-16\,bc^{5}\arctan\left(-cx+\sqrt{c^{2}x^{2}-1}\right)+8\,(bc^{5}x^{5}-bc^{5})\operatorname{arccsc}\left(cx\right)-3\,b\log\left(-cx+\sqrt{c^{2}x^{2}-1}\right)+(2\,bc^{3}x^{3}+3\,bcx)\sqrt{c^{2}x^{2}-1}}{40\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*c^5*x^5 - 16*b*c^5*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 8*(b*c^5*x^
5 - b*c^5)*arccsc(c*x) - 3*b*log(-c*x + sqrt(c^2*x^2 - 1)) + (2*b*c^3*x^3 +
 3*b*c*x)*sqrt(c^2*x^2 - 1))/c^5
```

**Sympy** [A]
time = 4.80, size = 175, normalized size = 1.97

$$\frac{ax^5}{5} + \frac{bx^5\operatorname{acsc}(cx)}{5} + \frac{b\left(\begin{cases} \frac{cx^5}{4\sqrt{c^2x^2-1}} + \frac{x^3}{8c\sqrt{c^2x^2-1}} - \frac{3x}{8c^3\sqrt{c^2x^2-1}} + \frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2| > 1 \\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}} - \frac{ix^3}{8c\sqrt{-c^2x^2+1}} + \frac{3ix}{8c^3\sqrt{-c^2x^2+1}} - \frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise} \end{cases}\right)}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**5/5 + b*x**5*acsc(c*x)/5 + b*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1))
 + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*ac
osh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1))
 - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1))
- 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(75) = 150.
time = 0.72, size = 480, normalized size = 5.39

$$\frac{1}{320}\left(\frac{2bx^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{2ax^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5}{c} + \frac{bx^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c^2} + \frac{10bx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)}{c^3} + \frac{10ax^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c^3} + \frac{8bx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c^4} + \frac{20bx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{cx}\right)}{c^5} + \frac{20ax\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^5} + \frac{24b\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6} - \frac{24b\log\left(\frac{1}{|c||x|}\right)}{c^6} + \frac{20b\arcsin\left(\frac{1}{cx}\right)}{c^7x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} + \frac{20a}{c^7x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} - \frac{8b}{c^8x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2} + \frac{10b\arcsin\left(\frac{1}{cx}\right)}{c^9x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3} + \frac{10a}{c^9x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3} - \frac{b}{c^{10}x^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4} + \frac{2b\arcsin\left(\frac{1}{cx}\right)}{c^{11}x^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5} + \frac{2a}{c^{11}x^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/320*(2*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c + 2*a*x^5*(
sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^2
+ 10*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^3 + 10*a*x^3*(s
qrt(-1/(c^2*x^2) + 1) + 1)^3/c^3 + 8*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c
^4 + 20*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 20*a*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)/c^5 + 24*b*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 24
*b*log(1/(abs(c)*abs(x)))/c^6 + 20*b*arcsin(1/(c*x))/(c^7*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)) + 20*a/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 8*b/(c^8*x^2*(
sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 10*b*arcsin(1/(c*x))/(c^9*x^3*(sqrt(-1/(c^
2*x^2) + 1) + 1)^3) + 10*a/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - b/(c^
10*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 2*b*arcsin(1/(c*x))/(c^11*x^5*(sqr
t(-1/(c^2*x^2) + 1) + 1)^5) + 2*a/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)
)*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asin(1/(c*x))),x)
[Out] int(x^4*(a + b*asin(1/(c*x))), x)
```

# 3.4 $\int x^3(a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=64

$$\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)$$

```
[Out] 1/4*x^4*(a+b*arccsc(c*x))+1/6*b*x*(1-1/c^2/x^2)^(1/2)/c^3+1/12*b*x^3*(1-1/c
^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5329, 277, 197}

$$\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)+\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}}{12c}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x)/(6*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(12*c)
+ (x^4*(a + b*ArcCsc[c*x]))/4
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{4c}$$
$$=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{6c^3}$$
$$=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)$$

**Mathematica** [**A**]
time = 0.07, size = 62, normalized size = 0.97

$$\frac{ax^4}{4}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(\frac{x}{6c^3}+\frac{x^3}{12c}\right)+\frac{1}{4}bx^4\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (a*x^4)/4 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*(x/(6*c^3) + x^3/(12*c)) + (b*
x^4*ArcCsc[c*x])/4
```

**Maple** [**A**]
time = 0.14, size = 74, normalized size = 1.16

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{c^4x^4a}{4}+b\left(\frac{c^4x^4\operatorname{arccsc}(cx)}{4}+\frac{(c^2x^2-1)(c^2x^2+2)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$ | 74 |
| default | $\frac{\frac{c^4x^4a}{4}+b\left(\frac{c^4x^4\operatorname{arccsc}(cx)}{4}+\frac{(c^2x^2-1)(c^2x^2+2)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$ | 74 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*c^4*x^4*a+b*(1/4*c^4*x^4*arccsc(c*x)+1/12*(c^2*x^2-1)*(c^2*x^2+2
)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))
```

**Maxima** [A]
time = 0.27, size = 59, normalized size = 0.92

$$\frac{1}{4}\,ax^4+\frac{1}{12}\,\left(3\,x^4\operatorname{arccsc}\left(cx\right)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3
*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b
```

**Fricas** [A]
time = 0.36, size = 52, normalized size = 0.81

$$\frac{3\,bc^4x^4\operatorname{arccsc}\left(cx\right)+3\,ac^4x^4+\left(bc^2x^2+2\,b\right)\sqrt{c^2x^2-1}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c^4*x^4*arccsc(c*x) + 3*a*c^4*x^4 + (b*c^2*x^2 + 2*b)*sqrt(c^2*x^
2 - 1))/c^4
```

**Sympy** [A]
time = 1.92, size = 107, normalized size = 1.67

$$\frac{ax^4}{4}+\frac{bx^4\operatorname{acsc}\left(cx\right)}{4}+\frac{b\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for}\ \left|c^2x^2\right|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**4/4 + b*x**4*acsc(c*x)/4 + b*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c)
 + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*
x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(54) = 108.
time = 0.46, size = 352, normalized size = 5.50

$$\frac{1}{192}\left(\frac{3\,bx^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{3\,ax^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c}+\frac{2\,bx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c^2}+\frac{12\,bx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)}{c^3}+\frac{12\,ax^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c^3}+\frac{18\,bx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{18\,b\arcsin\left(\frac{1}{cx}\right)}{c^5}+\frac{18\,a}{c^5}-\frac{18\,b}{c^6x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}+\frac{12\,b\arcsin\left(\frac{1}{cx}\right)}{c^7x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}+\frac{12\,a}{c^7x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}-\frac{2\,b}{c^8x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}+\frac{3\,b\arcsin\left(\frac{1}{cx}\right)}{c^9x^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}+\frac{3\,a}{c^9x^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a*x^4*(
sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 2*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^
2 + 12*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 12*a*x^2*
(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 18*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^
4 + 18*b*arcsin(1/(c*x))/c^5 + 18*a/c^5 - 18*b/(c^6*x*(sqrt(-1/(c^2*x^2) +
1) + 1)) + 12*b*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) +
12*a/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 2*b/(c^8*x^3*(sqrt(-1/(c^2*
x^2) + 1) + 1)^3) + 3*b*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) +
1)^4) + 3*a/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^3*(a + b*asin(1/(c*x))), x)
```

# 3.5 $\int x^2(a + b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=64

$$\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

```
[Out] 1/3*x^3*(a+b*arccsc(c*x))+1/6*b*arctanh((1-1/c^2/x^2)^(1/2))/c^3+1/6*b*x^2*
(1-1/c^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5329, 272, 44, 65, 214}

$$\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{bx^2\sqrt{1-\frac{1}{c^2x^2}}}{6c}+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(6*c) + (x^3*(a + b*ArcCsc[c*x]))/3 + (b*ArcT
anh[Sqrt[1 - 1/(c^2*x^2)]])/(6*c^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\csc^{-1}(cx)\right)\,dx &= \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{3c}\\
&= \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)-\frac{b\mathrm{Subst}\left(\int\frac{1}{x^2\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{6c}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)-\frac{b\mathrm{Subst}\left(\int\frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{12c^3}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\mathrm{Subst}\left(\int\frac{1}{c^2-c^2x^2}\,dx,x,\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}\\
&= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}
\end{aligned}$$

**Mathematica [A]**
time = 0.04, size = 85, normalized size = 1.33

$$\frac{ax^3}{3}+\frac{bx^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{6c}+\frac{1}{3}bx^3\csc^{-1}(cx)+\frac{b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*ArcCsc[c*x]),x]

[Out] (a*x^3)/3 + (b*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + (b*x^3*ArcCsc[c*x])/3 + (b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(54) = 108$.
time = 0.14, size = 112, normalized size = 1.75

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{c^3x^3a}{3}+\frac{b\,c^3x^3\operatorname{arccsc}(cx)}{3}+\frac{b\left(c^2x^2-1\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^3}$ | 112 |
| default | $\frac{\frac{c^3x^3a}{3}+\frac{b\,c^3x^3\operatorname{arccsc}(cx)}{3}+\frac{b\left(c^2x^2-1\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\,\ln\left(cx+\sqrt{c^2x^2-1}\right)}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^3}$ | 112 |

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/c^3*(1/3*c^3*x^3*a+1/3*b*c^3*x^3*arccsc(c*x)+1/6*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)+1/6*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*ln(c*x+(c^2*x^2-1)^(1/2)))

**Maxima** [A]
time = 0.31, size = 97, normalized size = 1.52

$$\frac{1}{3}\,ax^3+\frac{1}{12}\left(4\,x^3\operatorname{arccsc}(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arccsc(c*x)),x, algorithm="maxima")

[Out] 1/3*a*x^3 + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b

**Fricas [A]**
time = 0.41, size = 94, normalized size = 1.47

$$\frac{2\,ac^3x^3 - 4\,bc^3\arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) + \sqrt{c^2x^2 - 1}\,bcx + 2\,(bc^3x^3 - bc^3)\,\mathrm{arccsc}\,(cx) - b\log\left(-cx + \sqrt{c^2x^2 - 1}\right)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3 - 4*b*c^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^2
- 1)*b*c*x + 2*(b*c^3*x^3 - b*c^3)*arccsc(c*x) - b*log(-c*x + sqrt(c^2*x^2
- 1)))/c^3
```

**Sympy [A]**
time = 2.64, size = 107, normalized size = 1.67

$$\frac{ax^3}{3} + \frac{bx^3\,\mathrm{acsc}\,(cx)}{3} + \frac{b\left(\begin{cases} \frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for } |c^2x^2| > 1 \\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise} \end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**3/3 + b*x**3*acsc(c*x)/3 + b*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) +
acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1
)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(54) = 108.
time = 0.68, size = 310, normalized size = 4.84

$$\frac{1}{24}\left(\frac{bx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{ax^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c} + \frac{bx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c^2} + \frac{3bx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{cx}\right)}{c^3} + \frac{3ax\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^3} + \frac{4b\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4} - \frac{4b\log\left(\frac{1}{|c||x|}\right)}{c^4} + \frac{3b\arcsin\left(\frac{1}{cx}\right)}{c^5x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} + \frac{3a}{c^5x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} - \frac{b}{c^6x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2} + \frac{b\arcsin\left(\frac{1}{cx}\right)}{c^7x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3} + \frac{a}{c^7x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/24*(b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + a*x^3*(sqrt(
-1/(c^2*x^2) + 1) + 1)^3/c + b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 3*b
*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 3*a*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)/c^3 + 4*b*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 4*b*log(1/(ab
s(c)*abs(x)))/c^4 + 3*b*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)
) + 3*a/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - b/(c^6*x^2*(sqrt(-1/(c^2*x^2
) + 1) + 1)^2) + b*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 + a/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))*c
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asin(1/(c*x))),x)
[Out] int(x^2*(a + b*asin(1/(c*x))), x)
```

# 3.6 $\int x(a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=39

$$\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)$$

```
[Out] 1/2*x^2*(a+b*arccsc(c*x))+1/2*b*x*(1-1/c^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5329, 197}

$$\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c) + (x^2*(a + b*ArcCsc[c*x]))/2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\csc^{-1}(cx)\right)\,dx&=\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)+\frac{b\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{2c}\\&=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)\end{aligned}$$

**Mathematica [A]**
time = 0.02, size = 50, normalized size = 1.28

$$\frac{ax^2}{2}+\frac{bx\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c}+\frac{1}{2}bx^2\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCsc[c*x]),x]
[Out] (a*x^2)/2 + (b*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*x^2*ArcCsc[c*x]
)/2
```

**Maple [A]**
time = 0.15, size = 65, normalized size = 1.67

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{a\,c^2x^2}{2}+b\left(\frac{c^2x^2\,\mathrm{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$ | 65 |
| default | $\frac{\frac{a\,c^2x^2}{2}+b\left(\frac{c^2x^2\,\mathrm{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$ | 65 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/c^2*(1/2*a*c^2*x^2+b*(1/2*c^2*x^2*arccsc(c*x)+1/2/((c^2*x^2-1)/c^2/x^2)^(
1/2)/c/x*(c^2*x^2-1)))
```

**Maxima [A]**
time = 0.26, size = 36, normalized size = 0.92

$$\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\,\mathrm{arccsc}\left(cx\right)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/2*(x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)*b
```

**Fricas** [A]
time = 0.34, size = 39, normalized size = 1.00

$$\frac{bc^2x^2 \operatorname{arccsc}(cx) + ac^2x^2 + \sqrt{c^2x^2-1}\, b}{2\, c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*c^2*x^2*arccsc(c*x) + a*c^2*x^2 + sqrt(c^2*x^2 - 1)*b)/c^2
```

**Sympy** [A]
time = 1.41, size = 58, normalized size = 1.49

$$\frac{ax^2}{2} + \frac{bx^2 \operatorname{acsc}(cx)}{2} + \frac{b\left(\begin{cases} \frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2| > 1 \\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise} \end{cases}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x)),x)
```

```
[Out] a*x**2/2 + b*x**2*acsc(c*x)/2 + b*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**
2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(33) = 66.
time = 0.45, size = 182, normalized size = 4.67

$$\frac{1}{8}\left(\frac{bx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2 \arcsin\left(\frac{1}{cx}\right)}{c} + \frac{ax^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c} + \frac{2\,bx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{2\,b\arcsin\left(\frac{1}{cx}\right)}{c^3} + \frac{2\,a}{c^3} - \frac{2\,b}{c^4x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} + \frac{b\arcsin\left(\frac{1}{cx}\right)}{c^5x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2} + \frac{a}{c^5x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + a*x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2/c + 2*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 2*b*ar
csin(1/(c*x))/c^3 + 2*a/c^3 - 2*b/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + b*
arcsin(1/(c*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + a/(c^5*x^2*(sqrt
(-1/(c^2*x^2) + 1) + 1)^2))*c
```

**Mupad** [B]
time = 0.65, size = 40, normalized size = 1.03

$$\frac{a\,x^2}{2} + \frac{b\,x^2\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{2} + \frac{b\,x\,\sqrt{1-\frac{1}{c^2\,x^2}}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asin(1/(c*x))),x)
[Out] (a*x^2)/2 + (b*x^2*asin(1/(c*x)))/2 + (b*x*(1 - 1/(c^2*x^2))^(1/2))/(2*c)
```

## 3.7 $\int (a + b \csc^{-1}(cx))\, dx$

**Optimal**. Leaf size=31

$$ax + bx\csc^{-1}(cx) + \frac{b\tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] a*x+b*x*arccsc(c*x)+b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

**Rubi** [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5323, 272, 65, 214}

$$ax + \frac{b\tanh^{-1}\left(\sqrt{1 - \frac{1}{c^2x^2}}\right)}{c} + bx\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCsc[c*x],x]
```

```
[Out] a*x + b*x*ArcCsc[c*x] + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5323

```
Int[ArcCsc[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsc[c*x], x] + Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx = ax+b\int \csc^{-1}(cx)\,dx$$

$$= ax+bx\csc^{-1}(cx)+\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x}\,dx}{c}$$

$$= ax+bx\csc^{-1}(cx)-\frac{b\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{2c}$$

$$= ax+bx\csc^{-1}(cx)+(bc)\mathrm{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx,x,\sqrt{1-\frac{1}{c^2x^2}}\right)$$

$$= ax+bx\csc^{-1}(cx)+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

**Mathematica** [A]
time = 0.05, size = 58, normalized size = 1.87

$$ax+bx\csc^{-1}(cx)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCsc[c*x],x]
[Out] a*x + b*x*ArcCsc[c*x] + (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

**Maple** [A]
time = 0.08, size = 37, normalized size = 1.19

| method | result | size |
|---|---|---|

| | | |
|---|---|---|
| default | $ax+bx\,\mathrm{arccsc}(cx)+\frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)}{c}$ | 37 |
| derivativedivides | $\frac{acx+\mathrm{arccsc}(cx)bcx+\ln\left(cx+cx\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)b}{c}$ | 40 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccsc(c*x),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arccsc(c*x)+b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

**Maxima** [A]
time = 0.26, size = 53, normalized size = 1.71

$$ax+\frac{\left(2\,cx\,\mathrm{arccsc}\left(cx\right)+\log\left(\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^{2}x^{2}}+1}+1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(
-1/(c^2*x^2) + 1) + 1))*b/c
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(29) = 58$.
time = 0.35, size = 64, normalized size = 2.06

$$\frac{acx-2\,bc\arctan\left(-cx+\sqrt{c^{2}x^{2}-1}\right)+\left(bcx-bc\right)\mathrm{arccsc}\left(cx\right)-b\log\left(-cx+\sqrt{c^{2}x^{2}-1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="fricas")
```

```
[Out] (a*c*x - 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arccsc(c*x)
 - b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

**Sympy** [A]
time = 1.63, size = 32, normalized size = 1.03

$$ax+b\left(x\,\mathrm{acsc}\left(cx\right)+\frac{\begin{cases}\mathrm{acosh}\left(cx\right) & \text{for}\ \left|c^{2}x^{2}\right|>1\\-i\,\mathrm{asin}\left(cx\right) & \text{otherwise}\end{cases}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acsc(c*x),x)
```

```
[Out] a*x + b*(x*acsc(c*x) + Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin
(c*x), True))/c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(29) = 58$.
time = 0.42, size = 62, normalized size = 2.00

$$\frac{1}{2}\,bc\left(\frac{2\,x\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="giac")
```

```
[Out] 1/2*b*c*(2*x*arcsin(1/(c*x))/c + (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sq
rt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

**Mupad** [B]
time = 0.87, size = 33, normalized size = 1.06

$$a\,x+b\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right)+\frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asin(1/(c*x)),x)
```

```
[Out] a*x + b*x*asin(1/(c*x)) + (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

## 3.8 $\int \frac{a+b\csc^{-1}(cx)}{x}\,dx$

**Optimal**. Leaf size=64

$$\frac{i(a+b\csc^{-1}(cx))^2}{2b} - (a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right) + \frac{1}{2}ib\text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)$$

```
[Out] 1/2*I*(a+b*arccsc(c*x))^2/b-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/
2))^2)+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

**Rubi** [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5327, 4721, 3798, 2221, 2317, 2438}

$$\frac{i(a+b\csc^{-1}(cx))^2}{2b} - \log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx)) + \frac{1}{2}ib\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x,x]
```

```
[Out] ((I/2)*(a + b*ArcCsc[c*x])^2)/b - (a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcC
sc[c*x])] + (I/2)*b*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
```

```
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5327

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcSin[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\csc^{-1}(cx)}{x}\,dx &= -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i(a+b\csc^{-1}(cx))^2}{2b} + 2i\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i(a+b\csc^{-1}(cx))^2}{2b} - \left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + b\text{Subst}\left(\int \log\left(1-\right.\right.\\
&= \frac{i(a+b\csc^{-1}(cx))^2}{2b} - \left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right) - \frac{1}{2}(ib)\text{Subst}\left(\int \frac{\log(}{}\right.\\
&= \frac{i(a+b\csc^{-1}(cx))^2}{2b} - \left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + \frac{1}{2}ib\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)
\end{aligned}
$$

**Mathematica [A]**
time = 0.03, size = 53, normalized size = 0.83

$$
-b\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + a\log(x) + \frac{1}{2}ib\left(\csc^{-1}(cx)^2 + \text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x,x]
```

```
[Out] -(b*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])]) + a*Log[x] + (I/2)*b*(ArcCs
c[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])])
```

**Maple [A]**
time = 0.30, size = 140, normalized size = 2.19

| method | result |
|---|---|
| derivativedivides | $a\ln(cx)+\frac{ib\operatorname{arccsc}(cx)^2}{2}-b\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)-b\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}\right.$ |
| default | $a\ln(cx)+\frac{ib\operatorname{arccsc}(cx)^2}{2}-b\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)-b\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(c*x)+1/2*I*b*arccsc(c*x)^2-b*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2
))-b*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I*b*polylog(2,-I/c/x-(1-1/
c^2/x^2)^(1/2))+I*b*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

```
[Out] (c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) + a
rctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b + a*log(x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/x, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\operatorname{acsc}(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))/x, x)
```

**Giac** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]Undef/
Unsigned
```

**Mupad** [B]
time = 0.76, size = 65, normalized size = 1.02

$$\frac{b\,\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2\mathrm{i}}\right)\,1\mathrm{i}}{2}+\frac{b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)^2\,1\mathrm{i}}{2}+a\,\ln\left(x\right)-b\,\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{c\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x,x)
```

```
[Out] (b*polylog(2, exp(asin(1/(c*x))*2i))*1i)/2 + (b*asin(1/(c*x))^2*1i)/2 + a*l
og(x) - b*log(1 - exp(asin(1/(c*x))*2i))*asin(1/(c*x))
```

## 3.9 $\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx$

**Optimal**. Leaf size=32

$$-bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{x}$$

```
[Out] (-a-b*arccsc(c*x))/x-b*c*(1-1/c^2/x^2)^(1/2)
```

**Rubi** [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5329, 267}

$$-\frac{a+b\csc^{-1}(cx)}{x}-bc\sqrt{1-\frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x^2,x]
```

```
[Out] -(b*c*Sqrt[1 - 1/(c^2*x^2)]) - (a + b*ArcCsc[c*x])/x
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx &= -\frac{a+b\csc^{-1}(cx)}{x}-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^3}\,dx}{c}\\ &= -bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{x}\end{aligned}$$

**Mathematica** [A]
time = 0.02, size = 41, normalized size = 1.28

$$-\frac{a}{x}-bc\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\csc^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^2,x]
```

```
[Out] -(a/x) - b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*ArcCsc[c*x])/x
```

**Maple** [A]
time = 0.15, size = 63, normalized size = 1.97

| method | result | size |
|---|---|---|
| derivativedivides | $c\left(-\frac{a}{cx}+b\left(-\frac{\mathrm{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$ | 63 |
| default | $c\left(-\frac{a}{cx}+b\left(-\frac{\mathrm{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$ | 63 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a/c/x+b*(-1/c/x*arccsc(c*x)-1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*
x^2-1)))
```

**Maxima** [A]
time = 0.25, size = 33, normalized size = 1.03

$$-\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\mathrm{arccsc}\left(cx\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b - a/x
```

**Fricas** [A]
time = 0.39, size = 26, normalized size = 0.81

$$-\frac{b\,\mathrm{arccsc}\left(cx\right)+\sqrt{c^2x^2-1}\,b+a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*arccsc(c*x) + sqrt(c^2*x^2 - 1)*b + a)/x
```

**Sympy [A]**
time = 0.48, size = 37, normalized size = 1.16

$$\begin{cases} -\frac{a}{x} - bc\sqrt{1 - \frac{1}{c^2x^2}} - \frac{b\operatorname{acsc}(cx)}{x} & \text{for } c \neq 0 \\ -\frac{a + \tilde{\infty}b}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2,x)
```

```
[Out] Piecewise((-a/x - b*c*sqrt(1 - 1/(c**2*x**2)) - b*acsc(c*x)/x, Ne(c, 0)), (
-(a + zoo*b)/x, True))
```

**Giac [A]**
time = 0.44, size = 42, normalized size = 1.31

$$-\left(b\sqrt{-\frac{1}{c^2x^2} + 1} + \frac{b\arcsin\left(\frac{1}{cx}\right)}{cx} + \frac{a}{cx}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

```
[Out] -(b*sqrt(-1/(c^2*x^2) + 1) + b*arcsin(1/(c*x))/(c*x) + a/(c*x))*c
```

**Mupad [B]**
time = 0.64, size = 37, normalized size = 1.16

$$-\frac{a}{x} - b\,c\,\sqrt{1 - \frac{1}{c^2\,x^2}} - \frac{b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x^2,x)
```

```
[Out] - a/x - b*c*(1 - 1/(c^2*x^2))^(1/2) - (b*asin(1/(c*x)))/x
```

# 3.10 $\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx$

**Optimal**. Leaf size=51

$$-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2\csc^{-1}(cx)-\frac{a+b\csc^{-1}(cx)}{2x^2}$$

```
[Out] 1/4*b*c^2*arccsc(c*x)+1/2*(-a-b*arccsc(c*x))/x^2-1/4*b*c*(1-1/c^2/x^2)^(1/2
)/x
```

**Rubi** [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5329, 342, 327, 222}

$$-\frac{a+b\csc^{-1}(cx)}{2x^2}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x^3,x]
```

```
[Out] -1/4*(b*c*Sqrt[1 - 1/(c^2*x^2)])/x + (b*c^2*ArcCsc[c*x])/4 - (a + b*ArcCsc[
c*x])/(2*x^2)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx &= -\frac{a+b\csc^{-1}(cx)}{2x^2}-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^4}\,dx}{2c}\\ &= -\frac{a+b\csc^{-1}(cx)}{2x^2}+\frac{b\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)}{2c}\\ &= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}-\frac{a+b\csc^{-1}(cx)}{2x^2}+\frac{1}{4}(bc)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx,x,\frac{1}{x}\right)\\ &= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2\csc^{-1}(cx)-\frac{a+b\csc^{-1}(cx)}{2x^2}\end{aligned}$$

**Mathematica** [A]
time = 0.03, size = 66, normalized size = 1.29

$$-\frac{a}{2x^2}-\frac{bc\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{4x}-\frac{b\csc^{-1}(cx)}{2x^2}+\frac{1}{4}bc^2\mathrm{ArcSin}\left(\frac{1}{cx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^3,x]
```

```
[Out] -1/2*a/x^2 - (b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*ArcCsc[c*x])/(
2*x^2) + (b*c^2*ArcSin[1/(c*x)])/4
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(46) = 92$.
time = 0.15, size = 114, normalized size = 2.24

| method | result | size |
|---|---|---|
| derivativedivides | $c^2\left(-\frac{a}{2c^2x^2}-\frac{b\,\mathrm{arccsc}(cx)}{2c^2x^2}+\frac{b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}-\frac{b(c^2x^2-1)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}\right)$ | 114 |
| default | $c^2\left(-\frac{a}{2c^2x^2}-\frac{b\,\mathrm{arccsc}(cx)}{2c^2x^2}+\frac{b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}-\frac{b(c^2x^2-1)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}\right)$ | 114 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a/c^2/x^2-1/2*b/c^2/x^2*arccsc(c*x)+1/4*b*(c^2*x^2-1)^(1/2)/((c^2
*x^2-1)/c^2/x^2)^(1/2)/c/x*arctan(1/(c^2*x^2-1)^(1/2))-1/4*b*(c^2*x^2-1)/((
c^2*x^2-1)/c^2/x^2)^(1/2)/c^3/x^3)
```

**Maxima [A]**
time = 0.47, size = 83, normalized size = 1.63

$$\frac{1}{4}\,b\left(\frac{\frac{c^4x\sqrt{-\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}-c^3\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c}-\frac{2\,\mathrm{arccsc}\,(cx)}{x^2}\right)-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*
arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) - 1/2*a/x^2
```

**Fricas [A]**
time = 0.40, size = 40, normalized size = 0.78

$$\frac{(bc^2x^2-2\,b)\,\mathrm{arccsc}\,(cx)-\sqrt{c^2x^2-1}\,b-2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*((b*c^2*x^2 - 2*b)*arccsc(c*x) - sqrt(c^2*x^2 - 1)*b - 2*a)/x^2
```

**Sympy** [A]
time = 2.47, size = 121, normalized size = 2.37

$$-\frac{a}{2x^2}-\frac{b\operatorname{acsc}(cx)}{2x^2}-\frac{b\left(\begin{cases}\frac{ic^3\operatorname{acosh}\left(\frac{1}{cx}\right)}{2}-\frac{ic^2}{2x\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{i}{2x^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|}>1\\ -\frac{c^3\operatorname{asin}\left(\frac{1}{cx}\right)}{2}+\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**3,x)
```

```
[Out] -a/(2*x**2) - b*acsc(c*x)/(2*x**2) - b*Piecewise((I*c**3*acosh(1/(c*x))/2 -
 I*c**2/(2*x*sqrt(-1 + 1/(c**2*x**2))) + I/(2*x**3*sqrt(-1 + 1/(c**2*x**2))
), 1/Abs(c**2*x**2) > 1), (-c**3*asin(1/(c*x))/2 + c**2*sqrt(1 - 1/(c**2*x*
*2))/(2*x), True))/(2*c)
```

**Giac** [A]
time = 0.43, size = 66, normalized size = 1.29

$$-\frac{1}{4}\left(2\,bc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)+2\,ac\left(\frac{1}{c^2x^2}-1\right)+bc\arcsin\left(\frac{1}{cx}\right)+\frac{b\sqrt{-\frac{1}{c^2x^2}+1}}{x}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 2*a*c*(1/(c^2*x^2) - 1) + b
*c*arcsin(1/(c*x)) + b*sqrt(-1/(c^2*x^2) + 1)/x)*c
```

**Mupad** [B]
time = 0.73, size = 50, normalized size = 0.98

$$-\frac{a}{2\,x^2}-\frac{b\,c^2\operatorname{asin}\left(\frac{1}{c\,x}\right)\,\left(\frac{2}{c^2\,x^2}-1\right)}{4}-\frac{b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x^3,x)
```

```
[Out] - a/(2*x^2) - (b*c^2*asin(1/(c*x))*(2/(c^2*x^2) - 1))/4 - (b*c*(1 - 1/(c^2*
x^2))^(1/2))/(4*x)
```

# 3.11 $\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx$

**Optimal**. Leaf size=60

$$-\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}+\frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{a+b\csc^{-1}(cx)}{3x^3}$$

```
[Out] 1/9*b*c^3*(1-1/c^2/x^2)^(3/2)+1/3*(-a-b*arccsc(c*x))/x^3-1/3*b*c^3*(1-1/c^2
/x^2)^(1/2)
```

**Rubi** [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5329, 272, 45}

$$-\frac{a+b\csc^{-1}(cx)}{3x^3}+\frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x^4,x]
```

```
[Out] -1/3*(b*c^3*Sqrt[1 - 1/(c^2*x^2)]) + (b*c^3*(1 - 1/(c^2*x^2))^(3/2))/9 - (a
 + b*ArcCsc[c*x])/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx = -\frac{a+b\csc^{-1}(cx)}{3x^3} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^5}dx}{3c}$$

$$= -\frac{a+b\csc^{-1}(cx)}{3x^3} + \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x}{c^2}}}dx, x, \frac{1}{x^2}\right)}{6c}$$

$$= -\frac{a+b\csc^{-1}(cx)}{3x^3} + \frac{b\text{Subst}\left(\int \left(\frac{c^2}{\sqrt{1-\frac{x}{c^2}}} - c^2\sqrt{1-\frac{x}{c^2}}\right)dx, x, \frac{1}{x^2}\right)}{6c}$$

$$= -\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}} + \frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2} - \frac{a+b\csc^{-1}(cx)}{3x^3}$$

**Mathematica** [A]
time = 0.04, size = 59, normalized size = 0.98

$$-\frac{a}{3x^3} + b\left(-\frac{2c^3}{9} - \frac{c}{9x^2}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^4,x]
```

```
[Out] -1/3*a/x^3 + b*((-2*c^3)/9 - c/(9*x^2))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b
*ArcCsc[c*x])/(3*x^3)
```

**Maple** [A]
time = 0.14, size = 75, normalized size = 1.25

| method | result | size |
|---|---|---|
| derivativedivides | $c^3\left(-\frac{a}{3c^3x^3} + b\left(-\frac{\text{arccsc}(cx)}{3c^3x^3} - \frac{(c^2x^2-1)(2c^2x^2+1)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)\right)$ | 75 |
| default | $c^3\left(-\frac{a}{3c^3x^3} + b\left(-\frac{\text{arccsc}(cx)}{3c^3x^3} - \frac{(c^2x^2-1)(2c^2x^2+1)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)\right)$ | 75 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a/c^3/x^3+b*(-1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+
1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4))
```

**Maxima [A]**
time = 0.28, size = 58, normalized size = 0.97

$$\frac{1}{9}\,b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\operatorname{arccsc}\left(cx\right)}{x^3}\right)-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*
arccsc(c*x)/x^3) - 1/3*a/x^3
```

**Fricas [A]**
time = 0.39, size = 39, normalized size = 0.65

$$-\frac{3\,b\operatorname{arccsc}\left(cx\right)+\left(2\,bc^2x^2+b\right)\sqrt{c^2x^2-1}+3\,a}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b*arccsc(c*x) + (2*b*c^2*x^2 + b)*sqrt(c^2*x^2 - 1) + 3*a)/x^3
```

**Sympy [A]**
time = 2.15, size = 112, normalized size = 1.87

$$-\frac{a}{3x^3}-\frac{b\operatorname{acsc}\left(cx\right)}{3x^3}-\frac{b\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for}\ \left|c^2x^2\right|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**4,x)
```

```
[Out] -a/(3*x**3) - b*acsc(c*x)/(3*x**3) - b*Piecewise((2*c**3*sqrt(c**2*x**2 - 1
)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sq
rt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

**Giac [A]**
time = 0.44, size = 87, normalized size = 1.45

$$\frac{1}{9}\left(bc^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,bc^2\sqrt{-\frac{1}{c^2x^2}+1}-\frac{3\,bc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}-\frac{3\,bc\arcsin\left(\frac{1}{cx}\right)}{x}-\frac{3\,a}{cx^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/9*(b*c^2*(-1/(c^2*x^2) + 1)^(3/2) - 3*b*c^2*sqrt(-1/(c^2*x^2) + 1) - 3*b*
c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 3*b*c*arcsin(1/(c*x))/x - 3*a/(c*x^
3))*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x^4,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/x^4, x)
```

# 3.12 $\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx$

**Optimal**. Leaf size=76

$$-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}-\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x}+\frac{3}{32}bc^4\csc^{-1}(cx)-\frac{a+b\csc^{-1}(cx)}{4x^4}$$

```
[Out] 3/32*b*c^4*arccsc(c*x)+1/4*(-a-b*arccsc(c*x))/x^4-1/16*b*c*(1-1/c^2/x^2)^(1
/2)/x^3-3/32*b*c^3*(1-1/c^2/x^2)^(1/2)/x
```

**Rubi** [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5329, 342, 327, 222}

$$-\frac{a+b\csc^{-1}(cx)}{4x^4}+\frac{3}{32}bc^4\csc^{-1}(cx)-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3}-\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x^5,x]
```

```
[Out] -1/16*(b*c*Sqrt[1 - 1/(c^2*x^2)])/x^3 - (3*b*c^3*Sqrt[1 - 1/(c^2*x^2)])/(32
*x) + (3*b*c^4*ArcCsc[c*x])/32 - (a + b*ArcCsc[c*x])/(4*x^4)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx = -\frac{a+b\csc^{-1}(cx)}{4x^4} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^6}\,dx}{4c}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4x^4} + \frac{b\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4c}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} - \frac{a+b\csc^{-1}(cx)}{4x^4} + \frac{1}{16}(3bc)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x} - \frac{a+b\csc^{-1}(cx)}{4x^4} + \frac{1}{32}\left(3bc^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\cdots}}\right.$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x} + \frac{3}{32}bc^4\csc^{-1}(cx) - \frac{a+b\csc^{-1}(cx)}{4x^4}$$

**Mathematica** [A]
time = 0.04, size = 78, normalized size = 1.03

$$-\frac{a}{4x^4} + b\left(-\frac{c}{16x^3} - \frac{3c^3}{32x}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{4x^4} + \frac{3}{32}bc^4\mathrm{ArcSin}\left(\frac{1}{cx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^5,x]
```

```
[Out] -1/4*a/x^4 + b*(-1/16*c/x^3 - (3*c^3)/(32*x))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)
] - (b*ArcCsc[c*x])/(4*x^4) + (3*b*c^4*ArcSin[1/(c*x)])/32
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(67) = 134.
time = 0.16, size = 150, normalized size = 1.97

| method | result |
|---|---|
| derivativedivides | $c^4\left(-\frac{a}{4c^4x^4}-\frac{b\,\mathrm{arccsc}(cx)}{4c^4x^4}+\frac{3b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}-\frac{3b(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}-\frac{b(c^2\cdots}{16\sqrt{\frac{c^2\cdots}{c^2\cdots}}}\right.$ |
| default | $c^4\left(-\frac{a}{4c^4x^4}-\frac{b\,\mathrm{arccsc}(cx)}{4c^4x^4}+\frac{3b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}-\frac{3b(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}-\frac{b(c^2\cdots}{16\sqrt{\frac{c^2\cdots}{c^2\cdots}}}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(-1/4*a/c^4/x^4-1/4*b/c^4/x^4*arccsc(c*x)+3/32*b*(c^2*x^2-1)^(1/2)/((c^
2*x^2-1)/c^2/x^2)^(1/2)/c/x*arctan(1/(c^2*x^2-1)^(1/2))-3/32*b*(c^2*x^2-1)/
((c^2*x^2-1)/c^2/x^2)^(1/2)/c^3/x^3-1/16*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2
)^(1/2)/c^5/x^5)
```

**Maxima [A]**
time = 0.47, size = 125, normalized size = 1.64

$$-\frac{1}{32}\,b\left(\frac{3\,c^5\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)+\frac{3\,c^8x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+5\,c^6x\sqrt{-\frac{1}{c^2x^2}+1}}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c}+\frac{8\,\mathrm{arccsc}\left(cx\right)}{x^4}\right)-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] -1/32*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*x^
2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) - 1)^
2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c + 8*arccsc(c*x)/x^4) - 1/4*a/x^4
```

**Fricas [A]**
time = 0.35, size = 53, normalized size = 0.70

$$\frac{(3\,bc^4x^4-8\,b)\,\mathrm{arccsc}\left(cx\right)-(3\,bc^2x^2+2\,b)\sqrt{c^2x^2-1}-8\,a}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^5,x, algorithm="fricas")
```

[Out] 1/32*((3*b*c^4*x^4 - 8*b)*arccsc(c*x) - (3*b*c^2*x^2 + 2*b)*sqrt(c^2*x^2 - 1) - 8*a)/x^4

**Sympy** [A]
time = 4.69, size = 194, normalized size = 2.55

$$-\frac{a}{4x^4}-\frac{b\operatorname{acsc}(cx)}{4x^4}-\frac{b\left(\begin{cases}\frac{3ic^5\operatorname{acosh}\left(\frac{1}{cx}\right)}{8}-\frac{3ic^4}{8x\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{ic^2}{8x^3\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{i}{4x^5\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|}>1\\ -\frac{3c^5\operatorname{asin}\left(\frac{1}{cx}\right)}{8}+\frac{3c^4}{8x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{c^2}{8x^3\sqrt{1-\frac{1}{c^2x^2}}}-\frac{1}{4x^5\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise}\end{cases}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*acsc(c*x))/x**5,x)

[Out] -a/(4*x**4) - b*acsc(c*x)/(4*x**4) - b*Piecewise((3*I*c**5*acosh(1/(c*x))/8 - 3*I*c**4/(8*x*sqrt(-1 + 1/(c**2*x**2))) + I*c**2/(8*x**3*sqrt(-1 + 1/(c**2*x**2))) + I/(4*x**5*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (-3*c**5*asin(1/(c*x))/8 + 3*c**4/(8*x*sqrt(1 - 1/(c**2*x**2))) - c**2/(8*x**3*sqrt(1 - 1/(c**2*x**2))) - 1/(4*x**5*sqrt(1 - 1/(c**2*x**2))), True))/(4*c)

**Giac** [A]
time = 0.42, size = 117, normalized size = 1.54

$$-\frac{1}{32}\left(8\,bc^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)+16\,bc^3\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)+5\,bc^3\arcsin\left(\frac{1}{cx}\right)-\frac{2\,bc^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}{x}+\frac{5\,bc^2\sqrt{-\frac{1}{c^2x^2}+1}}{x}+\frac{8\,a}{cx^4}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccsc(c*x))/x^5,x, algorithm="giac")

[Out] -1/32*(8*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x)) + 16*b*c^3*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 5*b*c^3*arcsin(1/(c*x)) - 2*b*c^2*(-1/(c^2*x^2) + 1)^(3/2)/x + 5*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 8*a/(c*x^4))*c

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*asin(1/(c*x)))/x^5,x)

[Out] int((a + b*asin(1/(c*x)))/x^5, x)

# 3.13 $\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx$

**Optimal**. Leaf size=82

$$-\frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}}+\frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2}-\frac{a+b\csc^{-1}(cx)}{5x^5}$$

```
[Out] 2/15*b*c^5*(1-1/c^2/x^2)^(3/2)-1/25*b*c^5*(1-1/c^2/x^2)^(5/2)+1/5*(-a-b*arc
csc(c*x))/x^5-1/5*b*c^5*(1-1/c^2/x^2)^(1/2)
```

**Rubi** [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5329, 272, 45}

$$-\frac{a+b\csc^{-1}(cx)}{5x^5}-\frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2}+\frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x^6,x]
```

```
[Out] -1/5*(b*c^5*Sqrt[1 - 1/(c^2*x^2)]) + (2*b*c^5*(1 - 1/(c^2*x^2))^(3/2))/15 -
 (b*c^5*(1 - 1/(c^2*x^2))^(5/2))/25 - (a + b*ArcCsc[c*x])/(5*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = -\frac{a+b\csc^{-1}(cx)}{5x^5} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^7}\,dx}{5c}$$

$$= -\frac{a+b\csc^{-1}(cx)}{5x^5} + \frac{b\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{10c}$$

$$= -\frac{a+b\csc^{-1}(cx)}{5x^5} + \frac{b\text{Subst}\left(\int \left(\frac{c^4}{\sqrt{1-\frac{x}{c^2}}} - 2c^4\sqrt{1-\frac{x}{c^2}} + c^4\left(1-\frac{x}{c^2}\right)^{3/2}\right) dx, x,\right.}{10c}$$

$$= -\frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}} + \frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2} - \frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2} - \frac{a+b\csc^{-1}(cx)}{5x^5}$$

**Mathematica [A]**
time = 0.06, size = 69, normalized size = 0.84

$$-\frac{a}{5x^5} + b\left(-\frac{8c^5}{75} - \frac{c}{25x^4} - \frac{4c^3}{75x^2}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^6,x]
```

```
[Out] -1/5*a/x^5 + b*((-8*c^5)/75 - c/(25*x^4) - (4*c^3)/(75*x^2))*Sqrt[(-1 + c^2
*x^2)/(c^2*x^2)] - (b*ArcCsc[c*x])/(5*x^5)
```

**Maple [A]**
time = 0.15, size = 83, normalized size = 1.01

| method | result | size |
|---|---|---|
| derivativedivides | $c^5\left(-\frac{a}{5c^5x^5} + b\left(-\frac{\operatorname{arccsc}(cx)}{5c^5x^5} - \frac{(c^2x^2-1)(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$ | 83 |
| default | $c^5\left(-\frac{a}{5c^5x^5} + b\left(-\frac{\operatorname{arccsc}(cx)}{5c^5x^5} - \frac{(c^2x^2-1)(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$ | 83 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(-1/5*a/c^5/x^5+b*(-1/5/c^5/x^5*arccsc(c*x)-1/75*(c^2*x^2-1)*(8*c^4*x^4
+4*c^2*x^2+3)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6))
```

**Maxima** [A]
time = 0.26, size = 76, normalized size = 0.93

$$-\frac{1}{75}\,b\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\mathrm{arccsc}\left(cx\right)}{x^5}\right)-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/75*b*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2)
+ 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) - 1/5*a/x^5
```

**Fricas** [A]
time = 0.34, size = 50, normalized size = 0.61

$$-\frac{15\,b\,\mathrm{arccsc}\left(cx\right)+\left(8\,bc^4x^4+4\,bc^2x^2+3\,b\right)\sqrt{c^2x^2-1}+15\,a}{75\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/75*(15*b*arccsc(c*x) + (8*b*c^4*x^4 + 4*b*c^2*x^2 + 3*b)*sqrt(c^2*x^2 -
1) + 15*a)/x^5
```

**Sympy** [A]
time = 5.07, size = 158, normalized size = 1.93

$$-\frac{a}{5x^5}-\frac{b\,\mathrm{acsc}\left(cx\right)}{5x^5}-\frac{b\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for}\ \left|c^2x^2\right|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**6,x)
```

```
[Out] -a/(5*x**5) - b*acsc(c*x)/(5*x**5) - b*Piecewise((8*c**5*sqrt(c**2*x**2 - 1
)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*
x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**
3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True)
)/(5*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(68) = 136.
time = 0.43, size = 149, normalized size = 1.82

$$-\frac{1}{75}\left(3\,bc^4\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}-10\,bc^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{15\,bc^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x}+15\,bc^4\sqrt{-\frac{1}{c^2x^2}+1}+\frac{30\,bc^3\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{15\,bc^3\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{15\,a}{cx^5}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/75*(3*b*c^4*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) - 10*b*c^4*(-1/(c
^2*x^2) + 1)^(3/2) + 15*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 15*b*
c^4*sqrt(-1/(c^2*x^2) + 1) + 30*b*c^3*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x +
 15*b*c^3*arcsin(1/(c*x))/x + 15*a/(c*x^5))*c
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x^6,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/x^6, x)
```

# 3.14 $\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx$

**Optimal**. Leaf size=101

$$-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5}-\frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3}-\frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x}+\frac{5}{96}bc^6\csc^{-1}(cx)-\frac{a+b\csc^{-1}(cx)}{6x^6}$$

```
[Out] 5/96*b*c^6*arccsc(c*x)+1/6*(-a-b*arccsc(c*x))/x^6-1/36*b*c*(1-1/c^2/x^2)^(1
/2)/x^5-5/144*b*c^3*(1-1/c^2/x^2)^(1/2)/x^3-5/96*b*c^5*(1-1/c^2/x^2)^(1/2)/
x
```

**Rubi** [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5329, 342, 327, 222}

$$-\frac{a+b\csc^{-1}(cx)}{6x^6}+\frac{5}{96}bc^6\csc^{-1}(cx)-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5}-\frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x}-\frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/x^7,x]
```

```
[Out] -1/36*(b*c*Sqrt[1 - 1/(c^2*x^2)])/x^5 - (5*b*c^3*Sqrt[1 - 1/(c^2*x^2)])/(14
4*x^3) - (5*b*c^5*Sqrt[1 - 1/(c^2*x^2)])/(96*x) + (5*b*c^6*ArcCsc[c*x])/96
- (a + b*ArcCsc[c*x])/(6*x^6)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx = -\frac{a+b\csc^{-1}(cx)}{6x^6} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^8}\,dx}{6c}$$

$$= -\frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{b\text{Subst}\left(\int \frac{x^6}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6c}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{1}{36}(5bc)\text{Subst}\left(\int \frac{x^4}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} - \frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{1}{48}\left(5bc^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\cdots}}\right.$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} - \frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} - \frac{a+b\csc^{-1}(cx)}{6x^6} + \frac{1}{96}(5b\cdots$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3} - \frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x} + \frac{5}{96}bc^6\csc^{-1}(cx) - \frac{a+b\,\mathrm{c}\cdots}{6\cdots}$$

**Mathematica [A]**
time = 0.07, size = 88, normalized size = 0.87

$$-\frac{a}{6x^6} + b\left(-\frac{c}{36x^5} - \frac{5c^3}{144x^3} - \frac{5c^5}{96x}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{6x^6} + \frac{5}{96}bc^6\text{ArcSin}\left(\frac{1}{cx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/x^7,x]
```

```
[Out] -1/6*a/x^6 + b*(-1/36*c/x^5 - (5*c^3)/(144*x^3) - (5*c^5)/(96*x))*Sqrt[(-1
+ c^2*x^2)/(c^2*x^2)] - (b*ArcCsc[c*x])/(6*x^6) + (5*b*c^6*ArcSin[1/(c*x)])
/96
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(88) = 176$.
time = 0.14, size = 186, normalized size = 1.84

| method | result |
|---|---|
| derivativedivides | $c^{6}\left(-\frac{a}{6c^{6}x^{6}}-\frac{b\,\mathrm{arccsc}(cx)}{6c^{6}x^{6}}+\frac{5b\sqrt{c^{2}x^{2}-1}\,\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)}{96\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}-\frac{5b(c^{2}x^{2}-1)}{96\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{3}x^{3}}-\frac{5b(c}{144\sqrt{\frac{c^2}{}}}\right.$ |
| default | $c^{6}\left(-\frac{a}{6c^{6}x^{6}}-\frac{b\,\mathrm{arccsc}(cx)}{6c^{6}x^{6}}+\frac{5b\sqrt{c^{2}x^{2}-1}\,\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)}{96\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}-\frac{5b(c^{2}x^{2}-1)}{96\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c^{3}x^{3}}-\frac{5b(c}{144\sqrt{\frac{c^2}{}}}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] c^6*(-1/6*a/c^6/x^6-1/6*b/c^6/x^6*arccsc(c*x)+5/96*b*(c^2*x^2-1)^(1/2)/((c^
2*x^2-1)/c^2/x^2)^(1/2)/c/x*arctan(1/(c^2*x^2-1)^(1/2))-5/96*b*(c^2*x^2-1)/
((c^2*x^2-1)/c^2/x^2)^(1/2)/c^3/x^3-5/144*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^
2)^(1/2)/c^5/x^5-1/36*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^7/x^7)
```

**Maxima** [A]
time = 0.50, size = 165, normalized size = 1.63

$$-\frac{1}{288}\,b\left(\frac{15\,c^{7}\arctan\left(cx\sqrt{-\frac{1}{c^{2}x^{2}}+1}\right)-\frac{15\,c^{12}x^{5}\left(-\frac{1}{c^{2}x^{2}}+1\right)^{\frac{5}{2}}+40\,c^{10}x^{3}\left(-\frac{1}{c^{2}x^{2}}+1\right)^{\frac{3}{2}}+33\,c^{8}x\sqrt{-\frac{1}{c^{2}x^{2}}+1}}{c^{6}x^{6}\left(\frac{1}{c^{2}x^{2}}-1\right)^{3}-3\,c^{4}x^{4}\left(\frac{1}{c^{2}x^{2}}-1\right)^{2}+3\,c^{2}x^{2}\left(\frac{1}{c^{2}x^{2}}-1\right)-1}}{c}+\frac{48\,\mathrm{arccsc}\left(cx\right)}{x^{6}}\right)-\frac{a}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/288*b*((15*c^7*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) - (15*c^12*x^5*(-1/(c^
2*x^2) + 1)^(5/2) + 40*c^10*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 33*c^8*x*sqrt(-1
/(c^2*x^2) + 1))/(c^6*x^6*(1/(c^2*x^2) - 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) - 1)
^2 + 3*c^2*x^2*(1/(c^2*x^2) - 1) - 1))/c + 48*arccsc(c*x)/x^6) - 1/6*a/x^6
```

**Fricas** [A]
time = 0.34, size = 63, normalized size = 0.62

$$\frac{3\,(5\,bc^6x^6 - 16\,b)\,\mathrm{arccsc}\,(cx) - (15\,bc^4x^4 + 10\,bc^2x^2 + 8\,b)\sqrt{c^2x^2 - 1} - 48\,a}{288\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] 1/288*(3*(5*b*c^6*x^6 - 16*b)*arccsc(c*x) - (15*b*c^4*x^4 + 10*b*c^2*x^2 +
8*b)*sqrt(c^2*x^2 - 1) - 48*a)/x^6
```

**Sympy** [A]
time = 10.91, size = 243, normalized size = 2.41

$$-\frac{a}{6x^6} - \frac{b\,\mathrm{acsc}\,(cx)}{6x^6} - \frac{b\left(\begin{cases} \frac{5ic^7\,\mathrm{acosh}\left(\frac{1}{cx}\right)}{16} - \frac{5ic^6}{16x\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{5ic^4}{48x^3\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{ic^2}{24x^5\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{i}{6x^7\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|} > 1 \\ -\frac{5c^7\,\mathrm{asin}\left(\frac{1}{cx}\right)}{16} + \frac{5c^6}{16x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{5c^4}{48x^3\sqrt{1-\frac{1}{c^2x^2}}} - \frac{c^2}{24x^5\sqrt{1-\frac{1}{c^2x^2}}} - \frac{1}{6x^7\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise} \end{cases}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**7,x)
```

```
[Out] -a/(6*x**6) - b*acsc(c*x)/(6*x**6) - b*Piecewise((5*I*c**7*acosh(1/(c*x))/1
6 - 5*I*c**6/(16*x*sqrt(-1 + 1/(c**2*x**2))) + 5*I*c**4/(48*x**3*sqrt(-1 +
1/(c**2*x**2))) + I*c**2/(24*x**5*sqrt(-1 + 1/(c**2*x**2))) + I/(6*x**7*sqr
t(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (-5*c**7*asin(1/(c*x))/16 +
5*c**6/(16*x*sqrt(1 - 1/(c**2*x**2))) - 5*c**4/(48*x**3*sqrt(1 - 1/(c**2*x*
*2))) - c**2/(24*x**5*sqrt(1 - 1/(c**2*x**2))) - 1/(6*x**7*sqrt(1 - 1/(c**2
*x**2))), True))/(6*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(85) = 170.
time = 0.43, size = 174, normalized size = 1.72

$$-\frac{1}{288}\left(48\,bc^5\left(\frac{1}{c^2x^2}-1\right)^3\arcsin\left(\frac{1}{cx}\right) + 144\,bc^5\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right) + 144\,bc^5\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right) + 33\,bc^5\arcsin\left(\frac{1}{cx}\right) + \frac{8\,bc^4\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}}{x} - \frac{26\,bc^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}{x} + \frac{33\,bc^4\sqrt{-\frac{1}{c^2x^2}+1}}{x} + \frac{48\,a}{cx^6}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^7,x, algorithm="giac")
```

```
[Out] -1/288*(48*b*c^5*(1/(c^2*x^2) - 1)^3*arcsin(1/(c*x)) + 144*b*c^5*(1/(c^2*x^
2) - 1)^2*arcsin(1/(c*x)) + 144*b*c^5*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 3
3*b*c^5*arcsin(1/(c*x)) + 8*b*c^4*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1
)/x - 26*b*c^4*(-1/(c^2*x^2) + 1)^(3/2)/x + 33*b*c^4*sqrt(-1/(c^2*x^2) + 1)
/x + 48*a/(c*x^6))*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asin}\left(\frac{1}{c x}\right)}{x^7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/x^7,x)
[Out] int((a + b*asin(1/(c*x)))/x^7, x)
```

# 3.15 $\int x^3(a+b\csc^{-1}(cx))^2\,dx$

**Optimal**. Leaf size=107

$$\frac{b^2x^2}{12c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))}{3c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3(a+b\csc^{-1}(cx))}{6c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{3c^4}$$

```
[Out] 1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arccsc(c*x))^2+1/3*b^2*ln(x)/c^4+1/3*b*x*(a+b
*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/c^3+1/6*b*x^3*(a+b*arccsc(c*x))*(1-1/c^2/
x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.08, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5331, 4495, 4270, 4269, 3556}

$$\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{6c}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{3c^3}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{3c^4}+\frac{b^2x^2}{12c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (b^2*x^2)/(12*c^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*ArcCsc[c*x]))/(3*c^3
) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^3*(a + b*ArcCsc[c*x]))/(6*c) + (x^4*(a + b*A
rcCsc[c*x])^2)/4 + (b^2*Log[x])/(3*c^4)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^2\,dx=-\frac{\mathrm{Subst}\left(\int(a+bx)^2\cot(x)\csc^4(x)\,dx,x,\csc^{-1}(cx)\right)}{c^4}$$

$$=\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2-\frac{b\mathrm{Subst}\left(\int(a+bx)\csc^4(x)\,dx,x,\csc^{-1}(cx)\right)}{2c^4}$$

$$=\frac{b^2x^2}{12c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3(a+b\csc^{-1}(cx))}{6c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2-\frac{b\mathrm{Subst}\left(\int\right.}{}$$

$$=\frac{b^2x^2}{12c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))}{3c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3(a+b\csc^{-1}(cx))}{6c}+$$

$$=\frac{b^2x^2}{12c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))}{3c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3(a+b\csc^{-1}(cx))}{6c}+$$

**Mathematica** [A]
time = 0.15, size = 124, normalized size = 1.16

$$\frac{cx\left(b^2cx+3a^2c^3x^3+2ab\sqrt{1-\frac{1}{c^2x^2}}\,(2+c^2x^2)\right)+2bcx\left(3ac^3x^3+b\sqrt{1-\frac{1}{c^2x^2}}\,(2+c^2x^2)\right)\csc^{-1}(cx)+3b^2c^4x^4\csc^{-1}(cx)^2+4b^2\log(x)}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (c*x*(b^2*c*x + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))
+ 2*b*c*x*(3*a*c^3*x^3 + b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))*ArcCsc[c*x]
 + 3*b^2*c^4*x^4*ArcCsc[c*x]^2 + 4*b^2*Log[x])/(12*c^4)
```

**Maple [A]**
time = 0.41, size = 181, normalized size = 1.69

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a^2 c^4 x^4}{4}+\frac{b^2 \operatorname{arccsc}(cx)^2 c^4 x^4}{4}+\frac{b^2 \operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, c^3x^3}{6}+\frac{b^2c^2x^2}{12}+\frac{b^2 \operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}-\frac{b^2\ln\left(\frac{1}{cx}\right)}{3}+2ab\left(\frac{c^4x^4\mathrm{ar}}{}\right.}{c^4}$ |
| default | $\frac{\frac{a^2 c^4 x^4}{4}+\frac{b^2 \operatorname{arccsc}(cx)^2 c^4 x^4}{4}+\frac{b^2 \operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, c^3x^3}{6}+\frac{b^2c^2x^2}{12}+\frac{b^2 \operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}-\frac{b^2\ln\left(\frac{1}{cx}\right)}{3}+2ab\left(\frac{c^4x^4\mathrm{ar}}{}\right.}{c^4}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*a^2*c^4*x^4+1/4*b^2*arccsc(c*x)^2*c^4*x^4+1/6*b^2*arccsc(c*x)*((
c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*x^3+1/12*b^2*c^2*x^2+1/3*b^2*arccsc(c*x)*c*x*
((c^2*x^2-1)/c^2/x^2)^(1/2)-1/3*b^2*ln(1/c/x)+2*a*b*(1/4*c^4*x^4*arccsc(c*x
)+1/12*(c^2*x^2-1)*(c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(93) = 186.
time = 0.51, size = 197, normalized size = 1.84

$$\frac{1}{4}\,b^2x^4\operatorname{arccsc}(cx)^2+\frac{1}{4}\,a^2x^4+\frac{1}{6}\left(3\,x^4\operatorname{arccsc}(cx)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)ab+\frac{\left(2\,c^4x^4\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)+2\,c^2x^2\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)+\left(c^2x^2+2\,\log\left(x^2\right)\right)\sqrt{cx+1}\,\sqrt{cx-1}-4\,\arctan\left(1,\sqrt{cx+1}\,\sqrt{cx-1}\right)\right)b^2}{12\,\sqrt{cx+1}\,\sqrt{cx-1}\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arccsc(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arccsc(c*x) + (c^2*x^3
*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a*b + 1/12*(2*
c^4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*c^2*x^2*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1)) + (c^2*x^2 + 2*log(x^2))*sqrt(c*x + 1)*sqrt(c*x -
1) - 4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x
 - 1)*c^4)
```

**Fricas [A]**
time = 0.43, size = 146, normalized size = 1.36

$$\frac{3\,b^2c^4x^4\operatorname{arccsc}(cx)^2+3\,a^2c^4x^4-12\,abc^4\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+b^2c^2x^2+4\,b^2\log(x)+6\left(abc^4x^4-abc^4\right)\operatorname{arccsc}(cx)+2\left(abc^2x^2+2\,ab+\left(b^2c^2x^2+2\,b^2\right)\operatorname{arccsc}(cx)\right)\sqrt{c^2x^2-1}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4*arccsc(c*x)^2 + 3*a^2*c^4*x^4 - 12*a*b*c^4*arctan(-c*x
+ sqrt(c^2*x^2 - 1)) + b^2*c^2*x^2 + 4*b^2*log(x) + 6*(a*b*c^4*x^4 - a*b*c^
4)*arccsc(c*x) + 2*(a*b*c^2*x^2 + 2*a*b + (b^2*c^2*x^2 + 2*b^2)*arccsc(c*x)
)*sqrt(c^2*x^2 - 1))/c^4
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a + b\,\mathrm{acsc}\,(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))**2,x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))**2, x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 811 vs. 2(93) = 186.
time = 0.53, size = 811, normalized size = 7.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*b^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))^2/c + 6*a*b
*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a^2*x^4*(sqrt(-1/
(c^2*x^2) + 1) + 1)^4/c + 4*b^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1
/(c*x))/c^2 + 4*a*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 12*b^2*x^2*(sq
rt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))^2/c^3 + 24*a*b*x^2*(sqrt(-1/(c^
2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 12*a^2*x^2*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^2/c^3 + 4*b^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 36*b^2*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^4 + 36*a*b*x*(sqrt(-1/(c^2*x^2) +
 1) + 1)/c^4 + 18*b^2*arcsin(1/(c*x))^2/c^5 + 36*a*b*arcsin(1/(c*x))/c^5 -
128*b^2*log(2)/c^5 + 64*b^2*log(2*sqrt(-1/(c^2*x^2) + 1) + 2)/c^5 - 64*b^2*
log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^5 - 64*b^2*log(1/(abs(c)*abs(x)))/c^5 + 1
8*a^2/c^5 + 8*b^2/c^5 - 36*b^2*arcsin(1/(c*x))/(c^6*x*(sqrt(-1/(c^2*x^2) +
1) + 1)) - 36*a*b/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 12*b^2*arcsin(1/(c
*x))^2/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 24*a*b*arcsin(1/(c*x))/(c
^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 12*a^2/(c^7*x^2*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^2) + 4*b^2/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 4*b^2*arcs
in(1/(c*x))/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 4*a*b/(c^8*x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3) + 3*b^2*arcsin(1/(c*x))^2/(c^9*x^4*(sqrt(-1/(c^2
*x^2) + 1) + 1)^4) + 6*a*b*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^4) + 3*a^2/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(a+b \operatorname{asin}\left(\frac{1}{c x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int(x^3*(a + b*asin(1/(c*x)))^2, x)
```

# 3.16 $\int x^2(a+b\csc^{-1}(cx))^2\,dx$

**Optimal**. Leaf size=139

$$\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\csc^{-1}(cx))}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2b(a+b\csc^{-1}(cx))\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{3c^3}-\cdots ib$$

```
[Out] 1/3*b^2*x/c^2+1/3*x^3*(a+b*arccsc(c*x))^2+2/3*b*(a+b*arccsc(c*x))*arctanh(I
/c/x+(1-1/c^2/x^2)^(1/2))/c^3-1/3*I*b^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2
))/c^3+1/3*I*b^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+1/3*b*x^2*(a+b*ar
ccsc(c*x))*(1-1/c^2/x^2)^(1/2)/c
```

**Rubi** [A]

time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5331, 4495, 4270, 4268, 2317, 2438}

$$\frac{2b\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{3c^3}+\frac{bx^2\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2-\frac{ib^2\text{Li}_2\left(-e^{i\csc^{-1}(cx)}\right)}{3c^3}+\frac{ib^2\text{Li}_2\left(e^{i\csc^{-1}(cx)}\right)}{3c^3}+\frac{b^2x}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcCsc[c*x]))/(3*c) +
 (x^3*(a + b*ArcCsc[c*x])^2)/3 + (2*b*(a + b*ArcCsc[c*x])*ArcTanh[E^(I*ArcC
sc[c*x])])/(3*c^3) - ((I/3)*b^2*PolyLog[2, -E^(I*ArcCsc[c*x])])/c^3 + ((I/3
)*b^2*PolyLog[2, E^(I*ArcCsc[c*x])])/c^3
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
```

```
[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^2\,dx=-\frac{\text{Subst}\left(\int(a+bx)^2\cot(x)\csc^3(x)\,dx,x,\csc^{-1}(cx)\right)}{c^3}$$

$$=\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2-\frac{(2b)\text{Subst}\left(\int(a+bx)\csc^3(x)\,dx,x,\csc^{-1}(cx)\right)}{3c^3}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\csc^{-1}(cx))}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2-\frac{b\text{Subst}\left(\int(a\right.}{}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\csc^{-1}(cx))}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2b(a+b\csc}{}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\csc^{-1}(cx))}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2b(a+b\csc}{}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\csc^{-1}(cx))}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2b(a+b\csc}{}$$

**Mathematica [A]**
time = 0.98, size = 210, normalized size = 1.51

$$\frac{1}{3}\left(a^2x^3+2abx^3\csc^{-1}(cx)+\frac{ab\left(-cx+c^3x^3+\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)\right)}{c^4\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{ib^2\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c^3}+\frac{b^2\left(cx+c^3x^3\csc^{-1}(cx)^2+\csc^{-1}(cx)\left(c^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2-\log\left(1-e^{i\csc^{-1}(cx)}\right)+\log\left(1+e^{i\csc^{-1}(cx)}\right)\right)+i\text{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)\right)}{c^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (a^2*x^3 + 2*a*b*x^3*ArcCsc[c*x] + (a*b*(-(c*x) + c^3*x^3 + Sqrt[-1 + c^2*x
^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]]))/(c^4*Sqrt[1 - 1/(c^2*x^2)]*x) - (I*
b^2*PolyLog[2, -E^(I*ArcCsc[c*x])])/c^3 + (b^2*(c*x + c^3*x^3*ArcCsc[c*x]^2
 + ArcCsc[c*x]*(c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2 - Log[1 - E^(I*ArcCsc[c*x])]
+ Log[1 + E^(I*ArcCsc[c*x])]) + I*PolyLog[2, E^(I*ArcCsc[c*x])]))/c^3)/3
```

**Maple [A]**
time = 0.78, size = 304, normalized size = 2.19

| method | result |
|---|---|
| derivativedivides | $\dfrac{\frac{c^3x^3a^2}{3}+\frac{b^2\operatorname{arccsc}(cx)^2c^3x^3}{3}+\frac{b^2\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}{3}+\frac{b^2cx}{3}-\frac{b^2\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{3}+\frac{ib^2\operatorname{polylog}\left(\cdots\right.}{\cdots}}{}$ |
| default | $\dfrac{\frac{c^3x^3a^2}{3}+\frac{b^2\operatorname{arccsc}(cx)^2c^3x^3}{3}+\frac{b^2\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}{3}+\frac{b^2cx}{3}-\frac{b^2\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{3}+\frac{ib^2\operatorname{polylog}\left(\cdots\right.}{\cdots}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/3*c^3*x^3*a^2+1/3*b^2*arccsc(c*x)^2*c^3*x^3+1/3*b^2*arccsc(c*x)*((
c^2*x^2-1)/c^2/x^2)^(1/2)*c^2*x^2+1/3*b^2*c*x-1/3*b^2*arccsc(c*x)*ln(1-I/c/
x-(1-1/c^2/x^2)^(1/2))+1/3*I*b^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+1/3*b
^2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/3*I*b^2*polylog(2,-I/c/x-(
1-1/c^2/x^2)^(1/2))+2/3*a*b*c^3*x^3*arccsc(c*x)+1/3*a*b*(c^2*x^2-1)/((c^2*x
^2-1)/c^2/x^2)^(1/2)+1/3*a*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/
c/x*ln(c*x+(c^2*x^2-1)^(1/2)))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/6*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c
^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^
2*x^2) + 1) - 1)/c^2)/c)*a*b + 1/12*(4*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*
x - 1))^2 - x^3*log(c^2*x^2)^2 - 2*c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x +
 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 + 36*c^2*integrate(1/3*x^4*log(c^2*x
^2)/(c^2*x^2 - 1), x)*log(c) - 72*c^2*integrate(1/3*x^4*log(x)/(c^2*x^2 - 1
), x)*log(c) + 36*c^2*integrate(1/3*x^4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1),
x) - 36*c^2*integrate(1/3*x^4*log(x)^2/(c^2*x^2 - 1), x) + 12*c^2*integrate
(1/3*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x) + 6*(2*x/c^2 - log(c*x + 1)/c^3 + l
og(c*x - 1)/c^3)*log(c)^2 - 36*integrate(1/3*x^2*log(c^2*x^2)/(c^2*x^2 - 1)
, x)*log(c) + 72*integrate(1/3*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) + 24*int
egrate(1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x
 - 1)))/(c^2*x^2 - 1), x) - 36*integrate(1/3*x^2*log(c^2*x^2)*log(x)/(c^2*x
^2 - 1), x) + 36*integrate(1/3*x^2*log(x)^2/(c^2*x^2 - 1), x) - 12*integrat
e(1/3*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arccsc(c*x)^2 + 2*a*b*x^2*arccsc(c*x) + a^2*x^2, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\operatorname{acsc}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^2*x^2, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asin(1/(c*x)))^2,x)
[Out] int(x^2*(a + b*asin(1/(c*x)))^2, x)
```

# 3.17 $\int x(a+b\csc^{-1}(cx))^2\,dx$

**Optimal**. Leaf size=55

$$\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))}{c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

```
[Out] 1/2*x^2*(a+b*arccsc(c*x))^2+b^2*ln(x)/c^2+b*x*(a+b*arccsc(c*x))*(1-1/c^2/x^
2)^(1/2)/c
```

**Rubi** [A]
time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5331, 4495, 4269, 3556}

$$\frac{bx\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*ArcCsc[c*x]))/c + (x^2*(a + b*ArcCsc[c*x]
)^2)/2 + (b^2*Log[x])/c^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
```

```
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int (a+bx)^2\cot(x)\csc^2(x)\,dx, x, \csc^{-1}(cx)\right)}{c^2}\\ &= \frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2-\frac{b\text{Subst}\left(\int (a+bx)\csc^2(x)\,dx, x, \csc^{-1}(cx)\right)}{c^2}\\ &= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))}{c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2-\frac{b^2\text{Subst}\left(\int \cot(x)\,d\right.}{c^2}\\ &= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))}{c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}\end{aligned}$$

**Mathematica** [A]
time = 0.09, size = 89, normalized size = 1.62

$$\frac{acx\left(2b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)+2bcx\left(b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)\csc^{-1}(cx)+b^2c^2x^2\csc^{-1}(cx)^2+2b^2\log(cx)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (a*c*x*(2*b*Sqrt[1 - 1/(c^2*x^2)] + a*c*x) + 2*b*c*x*(b*Sqrt[1 - 1/(c^2*x^2
)] + a*c*x)*ArcCsc[c*x] + b^2*c^2*x^2*ArcCsc[c*x]^2 + 2*b^2*Log[c*x])/(2*c^
2)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(51) = 102$.
time = 0.41, size = 127, normalized size = 2.31

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{c^2x^2a^2}{2}+\frac{b^2\text{arccsc}(cx)^2c^2x^2}{2}+b^2\text{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}-b^2\ln\left(\frac{1}{cx}\right)+2ab\left(\frac{c^2x^2\text{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$ | 127 |

| | | |
|---|---|---|
| default | $\frac{\frac{c^2x^2a^2}{2}+\frac{b^2\operatorname{arccsc}(cx)^2c^2x^2}{2}+b^2\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}-b^2\ln\left(\frac{1}{cx}\right)+2ab\left(\frac{c^2x^2\operatorname{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$ | 127 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*c^2*x^2*a^2+1/2*b^2*arccsc(c*x)^2*c^2*x^2+b^2*arccsc(c*x)*c*x*((
c^2*x^2-1)/c^2/x^2)^(1/2)-b^2*ln(1/c/x)+2*a*b*(1/2*c^2*x^2*arccsc(c*x)+1/2/
((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*(c^2*x^2-1)))
```

**Maxima** [A]
time = 0.29, size = 84, normalized size = 1.53

$$\frac{1}{2}b^2x^2\operatorname{arccsc}(cx)^2+\frac{1}{2}a^2x^2+\left(x^2\operatorname{arccsc}(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)ab+\left(\frac{x\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arccsc}(cx)}{c}+\frac{\log(x)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arccsc(c*x)^2 + 1/2*a^2*x^2 + (x^2*arccsc(c*x) + x*sqrt(-1/(c^2
*x^2) + 1)/c)*a*b + (x*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x)/c + log(x)/c^2)*b
^2
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(51) = 102$.
time = 0.43, size = 111, normalized size = 2.02

$$\frac{b^2c^2x^2\operatorname{arccsc}(cx)^2+a^2c^2x^2-4abc^2\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2b^2\log(x)+2(abc^2x^2-abc^2)\operatorname{arccsc}(cx)+2\sqrt{c^2x^2-1}\,(b^2\operatorname{arccsc}(cx)+ab)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2*arccsc(c*x)^2 + a^2*c^2*x^2 - 4*a*b*c^2*arctan(-c*x + sqrt
(c^2*x^2 - 1)) + 2*b^2*log(x) + 2*(a*b*c^2*x^2 - a*b*c^2)*arccsc(c*x) + 2*s
qrt(c^2*x^2 - 1)*(b^2*arccsc(c*x) + a*b))/c^2
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\operatorname{acsc}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))**2,x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))**2, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(51) = 102.
time = 0.50, size = 427, normalized size = 7.76

$$\frac{1}{8}\left(\frac{b^2x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)^2}{c}+\frac{2\,abx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{a^2x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c}+\frac{4\,b^2x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{cx}\right)}{c^2}+\frac{4\,abx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{2\,b^2\arcsin\left(\frac{1}{cx}\right)^2}{c^3}+\frac{4\,ab\arcsin\left(\frac{1}{cx}\right)}{c^3}-\frac{16\,b^2\log\left(2\right)}{c^3}+\frac{8\,b^2\log\left(2\sqrt{-\frac{1}{c^2x^2}+1}+2\right)}{c^3}-\frac{8\,b^2\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^3}-\frac{8\,b^2\log\left(\frac{1}{|c||x|}\right)}{c^3}+\frac{2\,a^2}{c^3}-\frac{4\,b^2\arcsin\left(\frac{1}{cx}\right)}{c^4x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}-\frac{4\,ab}{c^4x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}+\frac{b^2\arcsin\left(\frac{1}{cx}\right)^2}{c^5x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}+\frac{2\,ab\arcsin\left(\frac{1}{cx}\right)}{c^5x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}+\frac{a^2}{c^5x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/8*(b^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))^2/c + 2*a*b*x^2
*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + a^2*x^2*(sqrt(-1/(c^2*x
^2) + 1) + 1)^2/c + 4*b^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^
2 + 4*a*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 2*b^2*arcsin(1/(c*x))^2/c^3
+ 4*a*b*arcsin(1/(c*x))/c^3 - 16*b^2*log(2)/c^3 + 8*b^2*log(2*sqrt(-1/(c^2*
x^2) + 1) + 2)/c^3 - 8*b^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 - 8*b^2*log(
1/(abs(c)*abs(x)))/c^3 + 2*a^2/c^3 - 4*b^2*arcsin(1/(c*x))/(c^4*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) - 4*a*b/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + b^2*arc
sin(1/(c*x))^2/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 2*a*b*arcsin(1/(c
*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + a^2/(c^5*x^2*(sqrt(-1/(c^2*
x^2) + 1) + 1)^2))*c
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int(x*(a + b*asin(1/(c*x)))^2, x)
```

# 3.18 $\int (a+b\csc^{-1}(cx))^2\, dx$

**Optimal**. Leaf size=84

$$x\left(a+b\csc^{-1}(cx)\right)^2+\frac{4b(a+b\csc^{-1}(cx))\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c}-\frac{2ib^2\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c}+\frac{2ib^2\text{PolyLog}\left(2\right.}{c}$$

```
[Out] x*(a+b*arccsc(c*x))^2+4*b*(a+b*arccsc(c*x))*arctanh(I/c/x+(1-1/c^2/x^2)^(1/
2))/c-2*I*b^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c+2*I*b^2*polylog(2,I/c
/x+(1-1/c^2/x^2)^(1/2))/c
```

**Rubi** [A]
time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5325, 4495, 4268, 2317, 2438}

$$x\left(a+b\csc^{-1}(cx)\right)^2+\frac{4b\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{c}-\frac{2ib^2\text{Li}_2\left(-e^{i\csc^{-1}(cx)}\right)}{c}+\frac{2ib^2\text{Li}_2\left(e^{i\csc^{-1}(cx)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] x*(a + b*ArcCsc[c*x])^2 + (4*b*(a + b*ArcCsc[c*x])*ArcTanh[E^(I*ArcCsc[c*x]
)])/c - ((2*I)*b^2*PolyLog[2, -E^(I*ArcCsc[c*x])])/c + ((2*I)*b^2*PolyLog[2
, E^(I*ArcCsc[c*x])])/c
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5325

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Sub
st[Int[(a + b*x)^n*Csc[x]*Cot[x], x], x, ArcCsc[c*x]], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\csc^{-1}(cx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int (a+bx)^2\cot(x)\csc(x)\,dx,x,\csc^{-1}(cx)\right)}{c}\\ &= x\left(a+b\csc^{-1}(cx)\right)^2-\frac{(2b)\text{Subst}\left(\int (a+bx)\csc(x)\,dx,x,\csc^{-1}(cx)\right)}{c}\\ &= x\left(a+b\csc^{-1}(cx)\right)^2+\frac{4b(a+b\csc^{-1}(cx))\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c}+\frac{(2b^2)\,\text{Subst}(\int \text{l}}{}\\ &= x\left(a+b\csc^{-1}(cx)\right)^2+\frac{4b(a+b\csc^{-1}(cx))\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c}-\frac{(2ib^2)\,\text{Subst}\left(\int\right.}{}\\ &= x\left(a+b\csc^{-1}(cx)\right)^2+\frac{4b(a+b\csc^{-1}(cx))\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c}-\frac{2ib^2\text{Li}_2\left(-e^{i\csc}\right.}{c}\end{aligned}$$

**Mathematica [A]**
time = 0.15, size = 147, normalized size = 1.75

$$\frac{a^2cx+2abcx\csc^{-1}(cx)+b^2cx\csc^{-1}(cx)^2-2b^2\csc^{-1}(cx)\log\left(1-e^{i\csc^{-1}(cx)}\right)+2b^2\csc^{-1}(cx)\log\left(1+e^{i\csc^{-1}(cx)}\right)+2ab\log\left(\cos\left(\frac{1}{2}\csc^{-1}(cx)\right)\right)-2ab\log\left(\sin\left(\frac{1}{2}\csc^{-1}(cx)\right)\right)-2ib^2\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)+2ib^2\text{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] (a^2*c*x + 2*a*b*c*x*ArcCsc[c*x] + b^2*c*x*ArcCsc[c*x]^2 - 2*b^2*ArcCsc[c*x
]*Log[1 - E^(I*ArcCsc[c*x])] + 2*b^2*ArcCsc[c*x]*Log[1 + E^(I*ArcCsc[c*x])]
 + 2*a*b*Log[Cos[ArcCsc[c*x]/2]] - 2*a*b*Log[Sin[ArcCsc[c*x]/2]] - (2*I)*b^
2*PolyLog[2, -E^(I*ArcCsc[c*x])] + (2*I)*b^2*PolyLog[2, E^(I*ArcCsc[c*x])])
/c
```

**Maple [A]**
time = 0.28, size = 188, normalized size = 2.24

| method | result |
|---|---|
| derivativedivides | $\frac{cx\,a^2+\operatorname{arccsc}(cx)^2b^2cx+2b^2\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-2b^2\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+2i\operatorname{dilog}}{c}$ |
| default | $\frac{cx\,a^2+\operatorname{arccsc}(cx)^2b^2cx+2b^2\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-2b^2\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+2i\operatorname{dilog}}{c}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*x*a^2+arccsc(c*x)^2*b^2*c*x+2*b^2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^
2)^(1/2))-2*b^2*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+2*I*dilog(1-I/c
/x-(1-1/c^2/x^2)^(1/2))*b^2-2*I*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))*b^2+2*ar
ccsc(c*x)*a*b*c*x+2*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))*a*b)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 4*c^
2*integrate(x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 8*c^2*integrate(x^2
*log(x)/(c^2*x^2 - 1), x)*log(c) - 4*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1))^2 - 4*c^2*integrate(x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 4*c^2*i
ntegrate(x^2*log(x)^2/(c^2*x^2 - 1), x) - 4*c^2*integrate(x^2*log(c^2*x^2)/
(c^2*x^2 - 1), x) + x*log(c^2*x^2)^2 + 2*(log(c*x + 1)/c - log(c*x - 1)/c)*
log(c)^2 + 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 8*integrate(
log(x)/(c^2*x^2 - 1), x)*log(c) - 8*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*a
rctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x) + 4*integrate(log(
c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 4*integrate(log(x)^2/(c^2*x^2 - 1), x)
+ 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2 + a^2*x + (2*c*x*arccsc(c
*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*a
*b/c
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acsc}\,(cx))^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^2, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^2, x)
```

## 3.19 $\int \frac{\left(a+b \csc^{-1}(cx)\right)^2}{x} dx$

**Optimal**. Leaf size=91

$$\frac{i(a+b\csc^{-1}(cx))^3}{3b}-\left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)+ib\left(a+b\csc^{-1}(cx)\right)\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)-\frac{1}{2}$$

```
[Out] 1/3*I*(a+b*arccsc(c*x))^3/b-(a+b*arccsc(c*x))^2*ln(1-(I/c/x+(1-1/c^2/x^2)^(
1/2))^2)+I*b*(a+b*arccsc(c*x))*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-1/2
*b^2*polylog(3,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5331, 3798, 2221, 2611, 2320, 6724}

$$ib\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)+\frac{i(a+b\csc^{-1}(cx))^3}{3b}-\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2-\frac{1}{2}b^2\text{Li}_3\left(e^{2i\csc^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^2/x,x]
```

```
[Out] ((I/3)*(a + b*ArcCsc[c*x])^3)/b - (a + b*ArcCsc[c*x])^2*Log[1 - E^((2*I)*Ar
cCsc[c*x])] + I*b*(a + b*ArcCsc[c*x])*PolyLog[2, E^((2*I)*ArcCsc[c*x])] - (
b^2*PolyLog[3, E^((2*I)*ArcCsc[c*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}\,dx &= -\text{Subst}\left(\int (a+bx)^2\cot(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} + 2i\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^2}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} - \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right) + (2b)\text{Subst}\left(\int (a\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} - \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right) + ib\left(a+b\csc^{-1}(c\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} - \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right) + ib\left(a+b\csc^{-1}(c\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b} - \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right) + ib\left(a+b\csc^{-1}(c
\end{aligned}$$

**Mathematica** [A]
time = 0.09, size = 137, normalized size = 1.51

$$-2ab\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + a^2\log(cx) + iab\left(\csc^{-1}(cx)^2 + \text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\right) + \frac{1}{24}ib^2\left(\pi^3 - 8\csc^{-1}(cx)^3 + 24i\csc^{-1}(cx)^2\log\left(1-e^{-2i\csc^{-1}(cx)}\right) - 24\csc^{-1}(cx)\text{PolyLog}\left(2, e^{-2i\csc^{-1}(cx)}\right) + 12i\text{PolyLog}\left(3, e^{-2i\csc^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcCsc[c*x])^2/x,x]

[Out] -2*a*b*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + a^2*Log[c*x] + I*a*b*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]) + (I/24)*b^2*(Pi^3 - 8*ArcCsc[c*x]^3 + (24*I)*ArcCsc[c*x]^2*Log[1 - E^((-2*I)*ArcCsc[c*x])] - 24*ArcCsc[c*x]*PolyLog[2, E^((-2*I)*ArcCsc[c*x])] + (12*I)*PolyLog[3, E^((-2*I)*ArcCsc[c*x])])

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(127) = 254$.
time = 0.36, size = 361, normalized size = 3.97

| method | result |
|---|---|
| derivativedivides | $a^2 \ln(cx) + \frac{i b^2 \operatorname{arccsc}(cx)^3}{3} - b^2 \operatorname{arccsc}(cx)^2 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 2ib^2 \operatorname{arccsc}(cx) \operatorname{polyl}$ |
| default | $a^2 \ln(cx) + \frac{i b^2 \operatorname{arccsc}(cx)^3}{3} - b^2 \operatorname{arccsc}(cx)^2 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 2ib^2 \operatorname{arccsc}(cx) \operatorname{polyl}$ |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccsc(c*x))^2/x,x,method=_RETURNVERBOSE)

[Out] a^2*ln(c*x)+1/3*I*b^2*arccsc(c*x)^3-b^2*arccsc(c*x)^2*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+2*I*b^2*arccsc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-2*b^2*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))-b^2*arccsc(c*x)^2*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+2*I*b^2*arccsc(c*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-2*b^2*polylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2))+I*a*b*arccsc(c*x)^2-2*a*b*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-2*a*b*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+2*I*a*b*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+2*I*a*b*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccsc(c*x))^2/x,x, algorithm="maxima")

[Out] -1/2*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 + b^2*c^2*integrate(x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 2*b^2*c^2*integrate(x^2*log(x)/(c^2*x^3 - x), x)*log(c) + 2*b^2*c^2*integrate(x^2*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) - b^2*c^2*integrate(x^2*log(x)^2/(c^2*x^3 - x), x) + 2*a*b*c^2*integrate(x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^3 - x), x) + 1/2*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 + b^2*arcta

```
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2*log(x) - 1/4*b^2*log(c^2*x^2)^2*log(x)
 - b^2*integrate(log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 2*b^2*integrate(lo
g(x)/(c^2*x^3 - x), x)*log(c) + 2*b^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)
*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^3 - x), x) - 2*b^2*i
ntegrate(log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + b^2*integrate(log(x)^2/(c^
2*x^3 - x), x) - 2*a*b*integrate(arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c
^2*x^3 - x), x) + a^2*log(x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2)/x, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsc}\,(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x, x)
```

**Giac [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]ln of
unsigned
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^2/x,x)
[Out] int((a + b*asin(1/(c*x)))^2/x, x)
```

# 3.20 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2} dx$

**Optimal**. Leaf size=50

$$\frac{2b^2}{x} - 2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right) - \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}$$

```
[Out] 2*b^2/x-(a+b*arccsc(c*x))^2/x-2*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5331, 3377, 2717}

$$-2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right) - \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x} + \frac{2b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^2/x^2,x]
```

```
[Out] (2*b^2)/x - 2*b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]) - (a + b*ArcCsc
[c*x])^2/x
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^2}\,dx = -\left(c\text{Subst}\left(\int (a+bx)^2\cos(x)\,dx, x, \csc^{-1}(cx)\right)\right)$$
$$= -\frac{(a+b\csc^{-1}(cx))^2}{x} + (2bc)\text{Subst}\left(\int (a+bx)\sin(x)\,dx, x, \csc^{-1}(cx)\right)$$
$$= -2bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx)) - \frac{(a+b\csc^{-1}(cx))^2}{x} + (2b^2c)\,\text{Subst}\left(\int \cos(x$$
$$= \frac{2b^2}{x} - 2bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx)) - \frac{(a+b\csc^{-1}(cx))^2}{x}$$

**Mathematica** [**A**]
time = 0.09, size = 71, normalized size = 1.42

$$-\frac{a^2-2b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}\,x+2b\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}\,x\right)\csc^{-1}(cx)+b^2\csc^{-1}(cx)^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x^2,x]
```

```
[Out] -((a^2 - 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 2*b*(a + b*c*Sqrt[1 - 1/
(c^2*x^2)]*x)*ArcCsc[c*x] + b^2*ArcCsc[c*x]^2)/x)
```

**Maple** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(48) = 96$.
time = 0.27, size = 118, normalized size = 2.36

| method | result |
|---|---|
| derivativedivides | $c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{cx}+\frac{2}{cx}-2\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)+2ab\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c\right.\right.$ |
| default | $c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{cx}+\frac{2}{cx}-2\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)+2ab\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c\right.\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a^2/c/x+b^2*(-arccsc(c*x)^2/c/x+2/c/x-2*arccsc(c*x)*((c^2*x^2-1)/c^2/x^
2)^(1/2))+2*a*b*(-1/c/x*arccsc(c*x)-1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(
c^2*x^2-1)))
```

**Maxima** [A]
time = 0.27, size = 79, normalized size = 1.58

$$-2\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)ab-2\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arccsc}(cx)-\frac{1}{x}\right)b^2-\frac{b^2\operatorname{arccsc}(cx)^2}{x}-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*a*b - 2*(c*sqrt(-1/(c^2*x^2)
+ 1)*arccsc(c*x) - 1/x)*b^2 - b^2*arccsc(c*x)^2/x - a^2/x
```

**Fricas** [A]
time = 0.40, size = 57, normalized size = 1.14

$$-\frac{b^2\operatorname{arccsc}(cx)^2+2\,ab\operatorname{arccsc}(cx)+a^2-2\,b^2+2\,\sqrt{c^2x^2-1}\,(b^2\operatorname{arccsc}(cx)+ab)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] -(b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2 - 2*b^2 + 2*sqrt(c^2*x^2 - 1)
*(b^2*arccsc(c*x) + a*b))/x
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\operatorname{acsc}(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x**2, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(48) = 96.
time = 0.44, size = 104, normalized size = 2.08

$$-\left(2\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)+2\,ab\sqrt{-\frac{1}{c^2x^2}+1}+\frac{b^2\arcsin\left(\frac{1}{cx}\right)^2}{cx}+\frac{2\,ab\arcsin\left(\frac{1}{cx}\right)}{cx}+\frac{a^2}{cx}-\frac{2\,b^2}{cx}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] -(2*b^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x)) + 2*a*b*sqrt(-1/(c^2*x^2) +
1) + b^2*arcsin(1/(c*x))^2/(c*x) + 2*a*b*arcsin(1/(c*x))/(c*x) + a^2/(c*x)
- 2*b^2/(c*x))*c
```

**Mupad** [B]
time = 0.81, size = 88, normalized size = 1.76

$$-\frac{a^2}{x}-\frac{b^2\left(\operatorname{asin}\left(\frac{1}{c\,x}\right)^2-2\right)}{x}-2\,b^2\,c\operatorname{asin}\left(\frac{1}{c\,x}\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}-2\,a\,b\,c\left(\sqrt{1-\frac{1}{c^2\,x^2}}+\frac{\operatorname{asin}\left(\frac{1}{c\,x}\right)}{c\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^2/x^2,x)
```

```
[Out] - a^2/x - (b^2*(asin(1/(c*x))^2 - 2))/x - 2*b^2*c*asin(1/(c*x))*(1 - 1/(c^2
*x^2))^(1/2) - 2*a*b*c*((1 - 1/(c^2*x^2))^(1/2) + asin(1/(c*x))/(c*x))
```

# 3.21 $\int \frac{(a+b\csc^{-1}(cx))^2}{x^3}\,dx$

**Optimal**. Leaf size=88

$$\frac{b^2}{4x^2}+\frac{1}{2}abc^2\csc^{-1}(cx)+\frac{1}{4}b^2c^2\csc^{-1}(cx)^2-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{2x}-\frac{(a+b\csc^{-1}(cx))^2}{2x^2}$$

```
[Out] 1/4*b^2/x^2+1/2*a*b*c^2*arccsc(c*x)+1/4*b^2*c^2*arccsc(c*x)^2-1/2*(a+b*arcc
sc(c*x))^2/x^2-1/2*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/x
```

**Rubi** [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5331, 4489, 3391}

$$-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{2x}+\frac{1}{2}abc^2\csc^{-1}(cx)-\frac{(a+b\csc^{-1}(cx))^2}{2x^2}+\frac{1}{4}b^2c^2\csc^{-1}(cx)^2+\frac{b^2}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^2/x^3,x]
```

```
[Out] b^2/(4*x^2) + (a*b*c^2*ArcCsc[c*x])/2 + (b^2*c^2*ArcCsc[c*x]^2)/4 - (b*c*Sq
rt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(2*x) - (a + b*ArcCsc[c*x])^2/(2*x
^2)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
```

```
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^3}\,dx = -\left(c^2\text{Subst}\left(\int (a+bx)^2\cos(x)\sin(x)\,dx, x, \csc^{-1}(cx)\right)\right)$$

$$= -\frac{(a+b\csc^{-1}(cx))^2}{2x^2} + (bc^2)\,\text{Subst}\left(\int (a+bx)\sin^2(x)\,dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{b^2}{4x^2} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{2x} - \frac{(a+b\csc^{-1}(cx))^2}{2x^2} + \frac{1}{2}(bc^2)\,\text{Subst}\left(\int (a$$

$$= \frac{b^2}{4x^2} + \frac{1}{2}abc^2\csc^{-1}(cx) + \frac{1}{4}b^2c^2\csc^{-1}(cx)^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{2x} - \frac{(a$$

**Mathematica** [A]
time = 0.07, size = 102, normalized size = 1.16

$$\frac{-2a^2+b^2-2abc\sqrt{1-\frac{1}{c^2x^2}}\,x-2b\left(2a+bc\sqrt{1-\frac{1}{c^2x^2}}\,x\right)\csc^{-1}(cx)+b^2(-2+c^2x^2)\csc^{-1}(cx)^2+2abc^2x^2\text{ArcSin}\left(\frac{1}{cx}\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x^3,x]
```

```
[Out] (-2*a^2 + b^2 - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 2*b*(2*a + b*c*Sqrt[1 - 1
/(c^2*x^2)]*x)*ArcCsc[c*x] + b^2*(-2 + c^2*x^2)*ArcCsc[c*x]^2 + 2*a*b*c^2*x
^2*ArcSin[1/(c*x)])/(4*x^2)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(76) = 152$.
time = 0.27, size = 201, normalized size = 2.28

| method | result |
|---|---|
| derivativedivides | $c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(\frac{\text{arccsc}(cx)^2(c^2x^2-1)}{2c^2x^2}-\frac{\text{arccsc}(cx)\left(\text{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx}+\frac{\text{arccsc}(cx)^2}{4}+\frac{1}{4c^2x^2}\right)\right.$ |

| | |
|---|---|
| default | $c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(\frac{\operatorname{arccsc}(cx)^2(c^2x^2-1)}{2c^2x^2}-\frac{\operatorname{arccsc}(cx)\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx}+\frac{\operatorname{arccsc}(cx)^2}{4}+\frac{1}{4c^2x^2}\right.\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a^2/c^2/x^2+b^2*(1/2*arccsc(c*x)^2*(c^2*x^2-1)/c^2/x^2-1/2*arccsc
(c*x)*(arccsc(c*x)*c*x+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x+1/4*arccsc(c*x)^2+1
/4/c^2/x^2)-a*b/c^2/x^2*arccsc(c*x)+1/2*a*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)/c/x*arctan(1/(c^2*x^2-1)^(1/2))-1/2*a*b*(c^2*x^2-1)/((c^2*x^
2-1)/c^2/x^2)^(1/2)/c^3/x^3)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^
3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) - 1/8*(4*(c^2*
(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 - 4*c^2*integrate(1/2*x^2
*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 8*c^2*integrate(1/2*x^2*log(x)/(
c^2*x^5 - x^3), x)*log(c) - 4*c^2*integrate(1/2*x^2*log(c^2*x^2)*log(x)/(c^
2*x^5 - x^3), x) + 4*c^2*integrate(1/2*x^2*log(x)^2/(c^2*x^5 - x^3), x) + 2
*c^2*integrate(1/2*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x) - (c^2*log(c*x + 1)
 + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*log(c)^2 + 4*integrate(1/2*log(
c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 8*integrate(1/2*log(x)/(c^2*x^5 - x^3
), x)*log(c) + 4*integrate(1/2*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(sqrt(c
*x + 1)*sqrt(c*x - 1)))/(c^2*x^5 - x^3), x) + 4*integrate(1/2*log(c^2*x^2)*
log(x)/(c^2*x^5 - x^3), x) - 4*integrate(1/2*log(x)^2/(c^2*x^5 - x^3), x) -
 2*integrate(1/2*log(c^2*x^2)/(c^2*x^5 - x^3), x))*x^2 + 4*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1))^2 - log(c^2*x^2)^2)*b^2/x^2 - 1/2*a^2/x^2
```

**Fricas [A]**
time = 0.39, size = 82, normalized size = 0.93

$$\frac{(b^2c^2x^2-2\,b^2)\operatorname{arccsc}(cx)^2-2\,a^2+b^2+2\,(abc^2x^2-2\,ab)\operatorname{arccsc}(cx)-2\,\sqrt{c^2x^2-1}\,(b^2\operatorname{arccsc}(cx)+ab)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*c^2*x^2 - 2*b^2)*arccsc(c*x)^2 - 2*a^2 + b^2 + 2*(a*b*c^2*x^2 - 2
*a*b)*arccsc(c*x) - 2*sqrt(c^2*x^2 - 1)*(b^2*arccsc(c*x) + a*b))/x^2
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsc}(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x**3, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(76) = 152.
time = 0.44, size = 163, normalized size = 1.85

$$-\frac{1}{8}\left(4\,b^2c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^2+8\,abc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)+2\,b^2c\arcsin\left(\frac{1}{cx}\right)^2+4\,a^2c\left(\frac{1}{c^2x^2}-1\right)-2\,b^2c\left(\frac{1}{c^2x^2}-1\right)+4\,abc\arcsin\left(\frac{1}{cx}\right)-b^2c+\frac{4\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{4\,ab\sqrt{-\frac{1}{c^2x^2}+1}}{x}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^2*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^2 + 8*a*b*c*(1/(c^2*x^2) -
1)*arcsin(1/(c*x)) + 2*b^2*c*arcsin(1/(c*x))^2 + 4*a^2*c*(1/(c^2*x^2) - 1)
- 2*b^2*c*(1/(c^2*x^2) - 1) + 4*a*b*c*arcsin(1/(c*x)) - b^2*c + 4*b^2*sqrt(
-1/(c^2*x^2) + 1)*arcsin(1/(c*x))/x + 4*a*b*sqrt(-1/(c^2*x^2) + 1)/x)*c
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^2/x^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^2/x^3, x)
```

# 3.22 $\int \frac{(a+b\csc^{-1}(cx))^2}{x^4} dx$

**Optimal**. Leaf size=102

$$\frac{2b^2}{27x^3}+\frac{4b^2c^2}{9x}-\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))-\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{9x^2}-\frac{(a+b\csc^{-1}(cx))^2}{3x^3}$$

```
[Out] 2/27*b^2/x^3+4/9*b^2*c^2/x-1/3*(a+b*arccsc(c*x))^2/x^3-4/9*b*c^3*(a+b*arccs
c(c*x))*(1-1/c^2/x^2)^(1/2)-2/9*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/x
^2
```

**Rubi** [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5331, 4489, 3391, 3377, 2717}

$$-\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{9x^2}-\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))-\frac{(a+b\csc^{-1}(cx))^2}{3x^3}+\frac{4b^2c^2}{9x}+\frac{2b^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^2/x^4,x]
```

```
[Out] (2*b^2)/(27*x^3) + (4*b^2*c^2)/(9*x) - (4*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a +
b*ArcCsc[c*x]))/9 - (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(9*x^
2) - (a + b*ArcCsc[c*x])^2/(3*x^3)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^4}\,dx = -\left(c^3\text{Subst}\left(\int (a+bx)^2\cos(x)\sin^2(x)\,dx, x, \csc^{-1}(cx)\right)\right)$$
$$= -\frac{(a+b\csc^{-1}(cx))^2}{3x^3} + \frac{1}{3}\left(2bc^3\right)\text{Subst}\left(\int (a+bx)\sin^3(x)\,dx, x, \csc^{-1}(cx)\right)$$
$$= \frac{2b^2}{27x^3} - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{9x^2} - \frac{(a+b\csc^{-1}(cx))^2}{3x^3} + \frac{1}{9}\left(4bc^3\right)\text{Subst}\Big($$
$$= \frac{2b^2}{27x^3} - \frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx)) - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{9x^2} - $$
$$= \frac{2b^2}{27x^3} + \frac{4b^2c^2}{9x} - \frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx)) - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(}{9x^2}$$

**Mathematica [A]**
time = 0.15, size = 108, normalized size = 1.06

$$-\frac{9a^2+6abc\sqrt{1-\frac{1}{c^2x^2}}\,x(1+2c^2x^2)-2b^2(1+6c^2x^2)+6b\left(3a+bc\sqrt{1-\frac{1}{c^2x^2}}\,x(1+2c^2x^2)\right)\csc^{-1}(cx)+9b^2\csc^{-1}(cx)^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x^4,x]
```

```
[Out] -1/27*(9*a^2 + 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) - 2*b^2*(1 +
 6*c^2*x^2) + 6*b*(3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*ArcCs
c[c*x] + 9*b^2*ArcCsc[c*x]^2)/x^3
```

**Maple [A]**
time = 0.41, size = 154, normalized size = 1.51

| method | result |
|---|---|
| derivativedivides | $c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{3c^3x^3}-\frac{2\operatorname{arccsc}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\left(-\frac{\text{ar}}{}\right.\right.$ |
| default | $c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{3c^3x^3}-\frac{2\operatorname{arccsc}(cx)(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\left(-\frac{\text{ar}}{}\right.\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a^2/c^3/x^3+b^2*(-1/3*arccsc(c*x)^2/c^3/x^3-2/9*arccsc(c*x)*(2*c^
2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/x)+2*a*b*(-
1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^2-1)/c^2/x^2)
^(1/2)/c^4/x^4))
```

**Maxima [B]** Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(88) = 176.
time = 0.52, size = 197, normalized size = 1.93

$$\frac{2}{9}ab\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\operatorname{arccsc}(cx)}{x^3}\right)-\frac{b^2\operatorname{arccsc}(cx)^2}{3x^3}-\frac{a^2}{3x^3}-\frac{2\left(6c^5x^4\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)-3c^3x^2\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)-(6c^3x^2+c)\sqrt{cx+1}\sqrt{cx-1}-3c\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)\right)b^2}{27\sqrt{cx+1}\sqrt{cx-1}cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 2/9*a*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c -
3*arccsc(c*x)/x^3) - 1/3*b^2*arccsc(c*x)^2/x^3 - 1/3*a^2/x^3 - 2/27*(6*c^5*
x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*c^3*x^2*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)) - (6*c^3*x^2 + c)*sqrt(c*x + 1)*sqrt(c*x - 1) - 3*c*ar
ctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c*x
^3)
```

**Fricas [A]**
time = 0.40, size = 93, normalized size = 0.91

$$\frac{12\,b^2c^2x^2-9\,b^2\operatorname{arccsc}(cx)^2-18\,ab\operatorname{arccsc}(cx)-9\,a^2+2\,b^2-6\left(2\,abc^2x^2+ab+(2\,b^2c^2x^2+b^2)\operatorname{arccsc}(cx)\right)\sqrt{c^2x^2-1}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(12*b^2*c^2*x^2 - 9*b^2*arccsc(c*x)^2 - 18*a*b*arccsc(c*x) - 9*a^2 + 2
*b^2 - 6*(2*a*b*c^2*x^2 + a*b + (2*b^2*c^2*x^2 + b^2)*arccsc(c*x))*sqrt(c^2
*x^2 - 1))/x^3
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsc}\,(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x**4, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(88) = 176.
time = 0.45, size = 224, normalized size = 2.20

$$\frac{1}{27}\left(6\,b^2c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)+6\,abc^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-18\,b^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\,\arcsin\left(\frac{1}{cx}\right)-\frac{9\,b^2c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^2}{x}-18\,abc^2\sqrt{-\frac{1}{c^2x^2}+1}-\frac{18\,abc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}-\frac{9\,b^2c\arcsin\left(\frac{1}{cx}\right)^2}{x}+\frac{2\,b^2c\left(\frac{1}{c^2x^2}-1\right)}{x}-\frac{18\,abc\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{14\,b^2c}{x}-\frac{9\,a^2}{cx^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] 1/27*(6*b^2*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x)) + 6*a*b*c^2*(-1/(c
^2*x^2) + 1)^(3/2) - 18*b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x)) - 9*
b^2*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^2/x - 18*a*b*c^2*sqrt(-1/(c^2*x^2)
+ 1) - 18*a*b*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 9*b^2*c*arcsin(1/(c*x
))^2/x + 2*b^2*c*(1/(c^2*x^2) - 1)/x - 18*a*b*c*arcsin(1/(c*x))/x + 14*b^2*
c/x - 9*a^2/(c*x^3))*c
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^2/x^4,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^2/x^4, x)
```

# 3.23 $\int \frac{(a+b\csc^{-1}(cx))^2}{x^5}\,dx$

**Optimal**. Leaf size=134

$$\frac{b^2}{32x^4}+\frac{3b^2c^2}{32x^2}+\frac{3}{16}abc^4\csc^{-1}(cx)+\frac{3}{32}b^2c^4\csc^{-1}(cx)^2-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{8x^3}-\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}\,(a+$$

```
[Out] 1/32*b^2/x^4+3/32*b^2*c^2/x^2+3/16*a*b*c^4*arccsc(c*x)+3/32*b^2*c^4*arccsc(
c*x)^2-1/4*(a+b*arccsc(c*x))^2/x^4-1/8*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^
(1/2)/x^3-3/16*b*c^3*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/x
```

**Rubi** [A]
time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5331, 4489, 3391}

$$\frac{3}{16}abc^4\csc^{-1}(cx)-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{8x^3}-\frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))}{16x}-\frac{(a+b\csc^{-1}(cx))^2}{4x^4}+\frac{3}{32}b^2c^4\csc^{-1}(cx)^2+\frac{3b^2c^2}{32x^2}+\frac{b^2}{32x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^2/x^5,x]
```

```
[Out] b^2/(32*x^4) + (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcCsc[c*x])/16 + (3*b^2*c
^4*ArcCsc[c*x]^2)/32 - (b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(8*x
^3) - (3*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(16*x) - (a + b*A
rcCsc[c*x])^2/(4*x^4)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
```

```
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^5}\,dx = -\left(c^4\text{Subst}\left(\int (a+bx)^2\cos(x)\sin^3(x)\,dx, x, \csc^{-1}(cx)\right)\right)$$

$$= -\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bc^4\right)\text{Subst}\left(\int (a+bx)\sin^4(x)\,dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{b^2}{32x^4} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{8x^3} - \frac{\left(a+b\csc^{-1}(cx)\right)^2}{4x^4} + \frac{1}{8}\left(3bc^4\right)\text{Subst}\Bigg(\int$$

$$= \frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{8x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(c\right.}{16x}$$

$$= \frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{3}{16}abc^4\csc^{-1}(cx) + \frac{3}{32}b^2c^4\csc^{-1}(cx)^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}\right.}{8x^3}$$

**Mathematica** [A]
time = 0.11, size = 148, normalized size = 1.10

$$\frac{-8a^2+b^2-4abc\sqrt{1-\frac{1}{c^2x^2}}\,x+3b^2c^2x^2-6abc^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3-2b\left(8a+bc\sqrt{1-\frac{1}{c^2x^2}}\,x\left(2+3c^2x^2\right)\right)\csc^{-1}(cx)+b^2\left(-8+3c^4x^4\right)\csc^{-1}(cx)^2+6abc^4x^4\text{ArcSin}\left(\frac{1}{cx}\right)}{32x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^2/x^5,x]
```

```
[Out] (-8*a^2 + b^2 - 4*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 3*b^2*c^2*x^2 - 6*a*b*c^3
*Sqrt[1 - 1/(c^2*x^2)]*x^3 - 2*b*(8*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*
c^2*x^2))*ArcCsc[c*x] + b^2*(-8 + 3*c^4*x^4)*ArcCsc[c*x]^2 + 6*a*b*c^4*x^4*
ArcSin[1/(c*x)])/(32*x^4)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(116) = 232$.
time = 0.38, size = 274, normalized size = 2.04

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{4c^4x^4}+\frac{\operatorname{arccsc}(cx)\left(3\operatorname{arccsc}(cx)c^3x^3-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\mathrm{arc}}{}\right.\right.$ |
| default | $c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{4c^4x^4}+\frac{\operatorname{arccsc}(cx)\left(3\operatorname{arccsc}(cx)c^3x^3-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\mathrm{arc}}{}\right.\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(-1/4*a^2/c^4/x^4+b^2*(-1/4*arccsc(c*x)^2/c^4/x^4+1/16*arccsc(c*x)*(3*a
rccsc(c*x)*c^3*x^3-3*c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-2*((c^2*x^2-1)/c^2
/x^2)^(1/2))/c^3/x^3-3/32*arccsc(c*x)^2+1/128*(3*c^2*x^2+2)^2/c^4/x^4)-1/2*
a*b/c^4/x^4*arccsc(c*x)+3/16*a*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1
/2)/c/x*arctan(1/(c^2*x^2-1)^(1/2))-3/16*a*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x
^2)^(1/2)/c^3/x^3-1/8*a*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^5/x^5)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*a*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*
x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) - 1
)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c + 8*arccsc(c*x)/x^4) - 1/16*(4*(2
*(c^2*log(c*x + 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*c^2*log(c)^2
- 16*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 32*c^2
*integrate(1/4*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) - 16*c^2*integrate(1/4
*x^2*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 16*c^2*integrate(1/4*x^2*log
(x)^2/(c^2*x^7 - x^5), x) + 4*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^7 -
 x^5), x) - (2*c^4*log(c*x + 1) + 2*c^4*log(c*x - 1) - 4*c^4*log(x) + (2*c^
2*x^2 + 1)/x^4)*log(c)^2 + 16*integrate(1/4*log(c^2*x^2)/(c^2*x^7 - x^5), x
)*log(c) - 32*integrate(1/4*log(x)/(c^2*x^7 - x^5), x)*log(c) + 8*integrate
(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c
^2*x^7 - x^5), x) + 16*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x
) - 16*integrate(1/4*log(x)^2/(c^2*x^7 - x^5), x) - 4*integrate(1/4*log(c^2
*x^2)/(c^2*x^7 - x^5), x))*x^4 + 4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^
2 - log(c^2*x^2)^2)*b^2/x^4 - 1/4*a^2/x^4
```

**Fricas** [A]
time = 0.35, size = 120, normalized size = 0.90

$$\frac{3\,b^2c^2x^2 + \left(3\,b^2c^4x^4 - 8\,b^2\right)\operatorname{arccsc}\left(cx\right)^2 - 8\,a^2 + b^2 + 2\,\left(3\,abc^4x^4 - 8\,ab\right)\operatorname{arccsc}\left(cx\right) - 2\,\left(3\,abc^2x^2 + 2\,ab + \left(3\,b^2c^2x^2 + 2\,b^2\right)\operatorname{arccsc}\left(cx\right)\right)\sqrt{c^2x^2 - 1}}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*arccsc(c*x)^2 - 8*a^2 + b^2 +
 2*(3*a*b*c^4*x^4 - 8*a*b)*arccsc(c*x) - 2*(3*a*b*c^2*x^2 + 2*a*b + (3*b^2*
c^2*x^2 + 2*b^2)*arccsc(c*x))*sqrt(c^2*x^2 - 1))/x^4
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\operatorname{acsc}\left(cx\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**2/x**5,x)
```

```
[Out] Integral((a + b*acsc(c*x))**2/x**5, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(116) = 232.
time = 0.44, size = 304, normalized size = 2.27

$$-\frac{1}{256}\left(64\,b^2c^3\left(\frac{1}{c^2x^2} - 1\right)^2\arcsin\left(\frac{1}{cx}\right)^2 + 128\,abc^3\left(\frac{1}{c^2x^2} - 1\right)^2\arcsin\left(\frac{1}{cx}\right) + 128\,b^2c^3\left(\frac{1}{c^2x^2} - 1\right)\arcsin\left(\frac{1}{cx}\right)^2 - 8\,b^2c^3\left(\frac{1}{c^2x^2} - 1\right)^2 + 256\,abc^3\left(\frac{1}{c^2x^2} - 1\right)\arcsin\left(\frac{1}{cx}\right) + 40\,b^2c^3\arcsin\left(\frac{1}{cx}\right)^2 - 40\,b^2c^3\left(\frac{1}{c^2x^2} - 1\right) + 80\,abc^3\arcsin\left(\frac{1}{cx}\right) - \frac{32\,b^2c^2\left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)}{x} - 17\,b^2c^3 - \frac{32\,abc^2\left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}}}{x} + \frac{80\,b^2c^2\sqrt{-\frac{1}{c^2x^2} + 1}\arcsin\left(\frac{1}{cx}\right)}{x} + \frac{80\,abc^2\sqrt{-\frac{1}{c^2x^2} + 1}}{x} + \frac{64\,a^2}{cx^4}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] -1/256*(64*b^2*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))^2 + 128*a*b*c^3*(1/(
c^2*x^2) - 1)^2*arcsin(1/(c*x)) + 128*b^2*c^3*(1/(c^2*x^2) - 1)*arcsin(1/(c
*x))^2 - 8*b^2*c^3*(1/(c^2*x^2) - 1)^2 + 256*a*b*c^3*(1/(c^2*x^2) - 1)*arcs
in(1/(c*x)) + 40*b^2*c^3*arcsin(1/(c*x))^2 - 40*b^2*c^3*(1/(c^2*x^2) - 1) +
 80*a*b*c^3*arcsin(1/(c*x)) - 32*b^2*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/
(c*x))/x - 17*b^2*c^3 - 32*a*b*c^2*(-1/(c^2*x^2) + 1)^(3/2)/x + 80*b^2*c^2*
sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x))/x + 80*a*b*c^2*sqrt(-1/(c^2*x^2) + 1
)/x + 64*a^2/(c*x^4))*c
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^2/x^5,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^2/x^5, x)
```

# 3.24 $\int x^3(a+b\csc^{-1}(cx))^3\,dx$

**Optimal**. Leaf size=207

$$\frac{b^3\sqrt{1-\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2}+\frac{ib(a+b\csc^{-1}(cx))^2}{2c^4}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))^2}{2c^3}+\frac{b\sqrt{1-\frac{}{c}}}{}$$

```
[Out] 1/4*b^2*x^2*(a+b*arccsc(c*x))/c^2+1/2*I*b*(a+b*arccsc(c*x))^2/c^4+1/4*x^4*(
a+b*arccsc(c*x))^3-b^2*(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2
)/c^4+1/2*I*b^3*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/c^4+1/4*b^3*x*(1-1
/c^2/x^2)^(1/2)/c^3+1/2*b*x*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/c^3+1/4
*b*x^3*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.16, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5331, 4495, 4271, 3852, 8, 4269, 3798, 2221, 2317, 2438}

$$-\frac{b^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{c^4}+\frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2}+\frac{ib(a+b\csc^{-1}(cx))^2}{2c^4}+\frac{bx^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{4c}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{2c^3}+\frac{1}{4}x^4(a+b\csc^{-1}(cx))^3+\frac{ib^3\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)}{2c^4}+\frac{b^3x\sqrt{1-\frac{1}{c^2x^2}}}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (b^3*Sqrt[1 - 1/(c^2*x^2)]*x)/(4*c^3) + (b^2*x^2*(a + b*ArcCsc[c*x]))/(4*c^
2) + ((I/2)*b*(a + b*ArcCsc[c*x])^2)/c^4 + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(a +
b*ArcCsc[c*x])^2)/(2*c^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x^3*(a + b*ArcCsc[c*x]
)^2)/(4*c) + (x^4*(a + b*ArcCsc[c*x])^3)/4 - (b^2*(a + b*ArcCsc[c*x])*Log[1
 - E^((2*I)*ArcCsc[c*x])])/c^4 + ((I/2)*b^3*PolyLog[2, E^((2*I)*ArcCsc[c*x]
)])/c^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
```

```
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+b\csc^{-1}(cx)\right)^3\,dx &= -\frac{\text{Subst}\left(\int (a+bx)^3\cot(x)\csc^4(x)\,dx,x,\csc^{-1}(cx)\right)}{c^4}\\
&= \frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^3-\frac{(3b)\text{Subst}\left(\int (a+bx)^2\csc^4(x)\,dx,x,\csc^{-1}(cx)\right)}{4c^4}\\
&= \frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3(a+b\csc^{-1}(cx))^2}{4c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(\right.\\
&= \frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))^2}{2c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3(}{4}\\
&= \frac{b^3\sqrt{1-\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2}+\frac{ib(a+b\csc^{-1}(cx))^2}{2c^4}+\frac{b\sqrt{1-\frac{1}{c^2x}}}{}\\
&= \frac{b^3\sqrt{1-\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2}+\frac{ib(a+b\csc^{-1}(cx))^2}{2c^4}+\frac{b\sqrt{1-\frac{1}{c^2x}}}{}\\
&= \frac{b^3\sqrt{1-\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2}+\frac{ib(a+b\csc^{-1}(cx))^2}{2c^4}+\frac{b\sqrt{1-\frac{1}{c^2x}}}{}\\
&= \frac{b^3\sqrt{1-\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2}+\frac{ib(a+b\csc^{-1}(cx))^2}{2c^4}+\frac{b\sqrt{1-\frac{1}{c^2x}}}{}
\end{aligned}
$$

**Mathematica [A]**
time = 0.60, size = 285, normalized size = 1.38

$$\frac{2a^2bc\sqrt{1-\frac{1}{c^2x^2}}\,x+b^3c\sqrt{1-\frac{1}{c^2x^2}}\,x+ab^2c^2x^2+a^2bc^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3+a^3c^4x^4+b^2\left(3ac^4x^4+b\left(2i+2c\sqrt{1-\frac{1}{c^2x^2}}\,x+c^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3\right)\right)\csc^{-1}(cx)^2+b^3c^4x^4\csc^{-1}(cx)^3+b\csc^{-1}(cx)\left(cx\left(b^2cx+3a^2c^3x^3+2ab\sqrt{1-\frac{1}{c^2x^2}}\,(2+c^2x^2)\right)-4b^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\right)-4ab^2\log\left(\frac{1}{cx}\right)+2ib^3\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (2*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*b^2*
c^2*x^2 + a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + a^3*c^4*x^4 + b^2*(3*a*c^4*
x^4 + b*(2*I + 2*c*Sqrt[1 - 1/(c^2*x^2)]*x + c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3)
)*ArcCsc[c*x]^2 + b^3*c^4*x^4*ArcCsc[c*x]^3 + b*ArcCsc[c*x]*(c*x*(b^2*c*x +
```

```
 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2)) - 4*b^2*Log[1 -
 E^((2*I)*ArcCsc[c*x])]) - 4*a*b^2*Log[1/(c*x)] + (2*I)*b^3*PolyLog[2, E^((
2*I)*ArcCsc[c*x])])/(4*c^4)
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(213) = 426$.
time = 0.82, size = 458, normalized size = 2.21

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c^4x^4a^3}{4}+\frac{b^3\operatorname{arccsc}(cx)^3c^4x^4}{4}+\frac{b^3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{4}+\frac{b^3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+ib^3\operatorname{polylog}\left(2,-\frac{i}{cx}-\sqrt{1-\right.}{}$ |
| default | $\frac{\frac{c^4x^4a^3}{4}+\frac{b^3\operatorname{arccsc}(cx)^3c^4x^4}{4}+\frac{b^3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{4}+\frac{b^3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+ib^3\operatorname{polylog}\left(2,-\frac{i}{cx}-\sqrt{1-\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*c^4*x^4*a^3+1/4*b^3*arccsc(c*x)^3*c^4*x^4+1/4*b^3*arccsc(c*x)^2*
((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*x^3+1/2*b^3*arccsc(c*x)^2*((c^2*x^2-1)/c^2/
x^2)^(1/2)*c*x+I*b^3*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+1/4*b^3*arccsc(c
*x)*c^2*x^2+1/4*b^3*((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x+I*b^3*polylog(2,I/c/x+(
1-1/c^2/x^2)^(1/2))-b^3*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))-b^3*arc
csc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+1/2*I*b^3*arccsc(c*x)^2-1/4*I*b^3+
3/4*a*b^2*arccsc(c*x)^2*c^4*x^4+1/2*a*b^2*arccsc(c*x)*((c^2*x^2-1)/c^2/x^2)
^(1/2)*c^3*x^3+1/4*a*b^2*c^2*x^2+a*b^2*arccsc(c*x)*((c^2*x^2-1)/c^2/x^2)^(1
/2)*c*x-a*b^2*ln(1/c/x)+3*a^2*b*(1/4*c^4*x^4*arccsc(c*x)+1/12*(c^2*x^2-1)*(
c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arccsc(c*x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arccsc(c*x) + (c^2*x
^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a^2*b + 1/16
*(4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x^4*arctan2(1, sqrt(c
*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 16*integrate(3/16*(16*c^2*x^5*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 - 16*x^3*arctan2(1, sqrt(c*x +
```

```
1)*sqrt(c*x - 1))*log(c)^2 + 16*(c^2*x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1)) - x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2 - (4*x^3*arct
an2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x^3*log(c^2*x^2)^2)*sqrt(c*x + 1)*s
qrt(c*x - 1) - 4*((4*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c) + c
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^5 - (4*arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))*log(c) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^3 + 4
*(c^2*x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - x^3*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)))*log(x))*log(c^2*x^2) + 32*(c^2*x^5*arctan2(1, sqrt(c*
x + 1)*sqrt(c*x - 1))*log(c) - x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c))*log(x))/(c^2*x^2 - 1), x))*b^3 + 1/4*(2*c^4*x^4*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)) + 2*c^2*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
(c^2*x^2 + 2*log(x^2))*sqrt(c*x + 1)*sqrt(c*x - 1) - 4*arctan2(1, sqrt(c*x
+ 1)*sqrt(c*x - 1)))*a*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c^4)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arccsc(c*x)^3 + 3*a*b^2*x^3*arccsc(c*x)^2 + 3*a^2*b*x^3*ar
ccsc(c*x) + a^3*x^3, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a + b\,\mathrm{acsc}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))**3, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^3*x^3, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b \operatorname{asin}\left(\frac{1}{c x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asin(1/(c*x)))^3,x)
```

```
[Out] int(x^3*(a + b*asin(1/(c*x)))^3, x)
```

# 3.25 $\int x^2(a+b\csc^{-1}(cx))^3\,dx$

**Optimal**. Leaf size=220

$$\frac{b^2x(a+b\csc^{-1}(cx))}{c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\csc^{-1}(cx))^2}{2c}+\frac{1}{3}x^3(a+b\csc^{-1}(cx))^3+\frac{b(a+b\csc^{-1}(cx))^2\tanh^{-}}{c^3}$$

```
[Out] b^2*x*(a+b*arccsc(c*x))/c^2+1/3*x^3*(a+b*arccsc(c*x))^3+b*(a+b*arccsc(c*x))
^2*arctanh(I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+b^3*arctanh((1-1/c^2/x^2)^(1/2))/
c^3-I*b^2*(a+b*arccsc(c*x))*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c^3+I*b^2
*(a+b*arccsc(c*x))*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+b^3*polylog(3,-
I/c/x-(1-1/c^2/x^2)^(1/2))/c^3-b^3*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3
+1/2*b*x^2*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.15, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5331, 4495, 4271, 3855, 4268, 2611, 2320, 6724}

$$-\frac{ib^2\text{Li}_2\left(-e^{i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{c^3}+\frac{ib^2\text{Li}_2\left(e^{i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{c^3}+\frac{b^2x(a+b\csc^{-1}(cx))}{c^2}+\frac{b\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))^2}{c^3}+\frac{bx^2\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{2c}+\frac{1}{3}x^3(a+b\csc^{-1}(cx))^3+\frac{b^3\text{Li}_3\left(-e^{i\csc^{-1}(cx)}\right)}{c^3}-\frac{b^3\text{Li}_3\left(e^{i\csc^{-1}(cx)}\right)}{c^3}+\frac{b^3\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (b^2*x*(a + b*ArcCsc[c*x]))/c^2 + (b*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcCs
c[c*x])^2)/(2*c) + (x^3*(a + b*ArcCsc[c*x])^3)/3 + (b*(a + b*ArcCsc[c*x])^2
*ArcTanh[E^(I*ArcCsc[c*x])])/c^3 + (b^3*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c^3
 - (I*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, -E^(I*ArcCsc[c*x])])/c^3 + (I*b^2*
(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCsc[c*x])])/c^3 + (b^3*PolyLog[3, -E
^(I*ArcCsc[c*x])])/c^3 - (b^3*PolyLog[3, E^(I*ArcCsc[c*x])])/c^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2 \left(a+b\csc^{-1}(cx)\right)^3 \, dx = -\frac{\text{Subst}\left(\int (a+bx)^3 \cot(x)\csc^3(x)\,dx, x, \csc^{-1}(cx)\right)}{c^3}$$

$$= \frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^3 - \frac{b\text{Subst}\left(\int (a+bx)^2 \csc^3(x)\,dx, x, \csc^{-1}(cx)\right)}{c^3}$$

$$= \frac{b^2x(a+b\csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3\left(a+b\csc^{-1}(\right.$$

$$= \frac{b^2x(a+b\csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3\left(a+b\csc^{-1}(\right.$$

$$= \frac{b^2x(a+b\csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3\left(a+b\csc^{-1}(\right.$$

$$= \frac{b^2x(a+b\csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3\left(a+b\csc^{-1}(\right.$$

$$= \frac{b^2x(a+b\csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^2(a+b\csc^{-1}(cx))^2}{2c} + \frac{1}{3}x^3\left(a+b\csc^{-1}(\right.$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 580 vs. $2(220) = 440$.
time = 7.31, size = 580, normalized size = 2.64

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (a^3*x^3)/3 + (a^2*b*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + a^2*b*x^3*
ArcCsc[c*x] + (a^2*b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])])/(2*c^3) +
 (a*b^2*((-8*I)*PolyLog[2, -E^(I*ArcCsc[c*x])] + 2*c^3*x^3*(2 + 4*ArcCsc[c*
x]^2 - 2*Cos[2*ArcCsc[c*x]] - (3*ArcCsc[c*x]*Log[1 - E^(I*ArcCsc[c*x])])/(c
*x) + (3*ArcCsc[c*x]*Log[1 + E^(I*ArcCsc[c*x])])/(c*x) + ((4*I)*PolyLog[2,
E^(I*ArcCsc[c*x])])/(c^3*x^3) + 2*ArcCsc[c*x]*Sin[2*ArcCsc[c*x]] + ArcCsc[c
*x]*Log[1 - E^(I*ArcCsc[c*x])]*Sin[3*ArcCsc[c*x]] - ArcCsc[c*x]*Log[1 + E^(
I*ArcCsc[c*x])]*Sin[3*ArcCsc[c*x]])))/(8*c^3) + (b^3*(24*ArcCsc[c*x]*Cot[Ar
cCsc[c*x]/2] + 4*ArcCsc[c*x]^3*Cot[ArcCsc[c*x]/2] + 6*ArcCsc[c*x]^2*Csc[Arc
Csc[c*x]/2]^2 + (ArcCsc[c*x]^3*Csc[ArcCsc[c*x]/2]^4)/(c*x) - 24*ArcCsc[c*x]
```

```
^2*Log[1 - E^(I*ArcCsc[c*x])] + 24*ArcCsc[c*x]^2*Log[1 + E^(I*ArcCsc[c*x])]
 - 48*Log[Tan[ArcCsc[c*x]/2]] - (48*I)*ArcCsc[c*x]*PolyLog[2, -E^(I*ArcCsc[
c*x])] + (48*I)*ArcCsc[c*x]*PolyLog[2, E^(I*ArcCsc[c*x])] + 48*PolyLog[3, -
E^(I*ArcCsc[c*x])] - 48*PolyLog[3, E^(I*ArcCsc[c*x])] - 6*ArcCsc[c*x]^2*Sec
[ArcCsc[c*x]/2]^2 + 16*c^3*x^3*ArcCsc[c*x]^3*Sin[ArcCsc[c*x]/2]^4 + 24*ArcC
sc[c*x]*Tan[ArcCsc[c*x]/2] + 4*ArcCsc[c*x]^3*Tan[ArcCsc[c*x]/2]))/(48*c^3)
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(270) = 540$.
time = 1.00, size = 603, normalized size = 2.74

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c^3x^3a^3}{3}+\frac{b^3\operatorname{arccsc}(cx)^3c^3x^3}{3}+\frac{b^3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}{2}+b^3\operatorname{arccsc}(cx)cx-\frac{b^3\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{2}-i}{}$ |
| default | $\frac{\frac{c^3x^3a^3}{3}+\frac{b^3\operatorname{arccsc}(cx)^3c^3x^3}{3}+\frac{b^3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}{2}+b^3\operatorname{arccsc}(cx)cx-\frac{b^3\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{2}-i}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/3*c^3*x^3*a^3+1/3*b^3*arccsc(c*x)^3*c^3*x^3+1/2*b^3*arccsc(c*x)^2*
((c^2*x^2-1)/c^2/x^2)^(1/2)*c^2*x^2+b^3*arccsc(c*x)*c*x-1/2*b^3*arccsc(c*x)
^2*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*a*b^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1
/2))-b^3*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))+1/2*b^3*arccsc(c*x)^2*ln(1+I/
c/x+(1-1/c^2/x^2)^(1/2))-I*b^3*arccsc(c*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(
1/2))+b^3*polylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2))+2*b^3*arctanh(I/c/x+(1-1/c^
2/x^2)^(1/2))+a*b^2*arccsc(c*x)^2*c^3*x^3+a*b^2*arccsc(c*x)*((c^2*x^2-1)/c^
2/x^2)^(1/2)*c^2*x^2-a*b^2*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+a*b^
2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-I*a*b^2*polylog(2,-I/c/x-(1-1
/c^2/x^2)^(1/2))+I*b^3*arccsc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+a*b
^2*c*x+a^2*b*c^3*x^3*arccsc(c*x)+1/2*a^2*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2
)^(1/2)+1/2*a^2*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*ln(c*x+
(c^2*x^2-1)^(1/2)))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 1/4*b^3*x^3*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 1/2*a*b^2*c^2*(2*(c^2*x^3
 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 - 12*b^3*c^
2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1),
x)*log(c)^2 + 12*b^3*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x
 - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^
4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) +
 12*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*
a*b^2*c^2*integrate(1/4*x^4*log(x)/(c^2*x^2 - 1), x)*log(c) + 1/3*a^3*x^3 +
 12*b^3*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c
^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*b^3*c^2*integrate(1/4*x^4*arctan(1/(s
qrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*inte
grate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^2 - 1), x) +
 4*b^3*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^
2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)^2/(c^
2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)*log(x)/(c^2*x^
2 - 1), x) - 12*a*b^2*c^2*integrate(1/4*x^4*log(x)^2/(c^2*x^2 - 1), x) + 3/
2*a*b^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 + 12*b^3*i
ntegrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x)*
log(c)^2 - 12*b^3*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))
*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*b^3*integrate(1/4*x^2*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) - 12*a*b^2*i
ntegrate(1/4*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*a*b^2*integrate
(1/4*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) + 1/4*(4*x^3*arccsc(c*x) + (2*sqrt
(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) +
1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*a^2*b + 4*b^3*integra
te(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1
)))^2/(c^2*x^2 - 1), x) - b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2
*log(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*b^3*integrate(1/4*x^2*arctan(1/(sqrt
(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*b^3*in
tegrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^2 -
 1), x) - 12*a*b^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))
)^2/(c^2*x^2 - 1), x) - 4*b^3*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqr
t(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x) + 3*a*b^2*integrate(1/4*x^2*log
(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*a*b^2*integrate(1/4*x^2*log(c^2*x^2)*log
(x)/(c^2*x^2 - 1), x) + 12*a*b^2*integrate(1/4*x^2*log(x)^2/(c^2*x^2 - 1),
x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arccsc(c*x)^3 + 3*a*b^2*x^2*arccsc(c*x)^2 + 3*a^2*b*x^2*ar
ccsc(c*x) + a^3*x^2, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+b\operatorname{acsc}(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))**3,x)
[Out] Integral(x**2*(a + b*acsc(c*x))**3, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)^3*x^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asin(1/(c*x)))^3,x)
[Out] int(x^2*(a + b*asin(1/(c*x)))^3, x)
```

# 3.26 $\int x(a+b\csc^{-1}(cx))^3\, dx$

**Optimal**. Leaf size=126

$$\frac{3ib(a+b\csc^{-1}(cx))^2}{2c^2}+\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3-\frac{3b^2(a+b\csc^{-1}(cx))\log}{c^2}$$

```
[Out] 3/2*I*b*(a+b*arccsc(c*x))^2/c^2+1/2*x^2*(a+b*arccsc(c*x))^3-3*b^2*(a+b*arcc
sc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/c^2+3/2*I*b^3*polylog(2,(I/c/x
+(1-1/c^2/x^2)^(1/2))^2)/c^2+3/2*b*x*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2
)/c
```

**Rubi** [A]
time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5331, 4495, 4269, 3798, 2221, 2317, 2438}

$$-\frac{3b^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{c^2}+\frac{3bx\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2}{2c}+\frac{3ib(a+b\csc^{-1}(cx))^2}{2c^2}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3+\frac{3ib^3\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (((3*I)/2)*b*(a + b*ArcCsc[c*x])^2)/c^2 + (3*b*Sqrt[1 - 1/(c^2*x^2)]*x*(a +
 b*ArcCsc[c*x])^2)/(2*c) + (x^2*(a + b*ArcCsc[c*x])^3)/2 - (3*b^2*(a + b*Ar
cCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/c^2 + (((3*I)/2)*b^3*PolyLog[2,
E^((2*I)*ArcCsc[c*x])])/c^2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx &= -\frac{\text{Subst}\left(\int (a+bx)^3\cot(x)\csc^2(x)\,dx,x,\csc^{-1}(cx)\right)}{c^2} \\
&= \frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3-\frac{(3b)\text{Subst}\left(\int (a+bx)^2\csc^2(x)\,dx,x,\csc^{-1}(cx)\right)}{2c^2} \\
&= \frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3-\frac{(3b^2)\,\text{Subst}\left(\int (a\right.}{} \\
&= \frac{3ib(a+b\csc^{-1}(cx))^2}{2c^2}+\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(c\right. \\
&= \frac{3ib(a+b\csc^{-1}(cx))^2}{2c^2}+\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(c\right. \\
&= \frac{3ib(a+b\csc^{-1}(cx))^2}{2c^2}+\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(c\right. \\
&= \frac{3ib(a+b\csc^{-1}(cx))^2}{2c^2}+\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(c\right.
\end{aligned}
$$

**Mathematica [A]**
time = 0.33, size = 182, normalized size = 1.44

$$
\frac{3b^2\left(ac^2x^2+b\left(i+c\sqrt{1-\frac{1}{c^2x^2}}\,x\right)\right)\csc^{-1}(cx)^2+b^3c^2x^2\csc^{-1}(cx)^3+3b\csc^{-1}(cx)\left(acx\left(2b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)-2b^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)\right)+a\left(acx\left(3b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)-6b^2\log\left(\frac{1}{cx}\right)\right)+3ib^3\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (3*b^2*(a*c^2*x^2 + b*(I + c*Sqrt[1 - 1/(c^2*x^2)]*x))*ArcCsc[c*x]^2 + b^3*
c^2*x^2*ArcCsc[c*x]^3 + 3*b*ArcCsc[c*x]*(a*c*x*(2*b*Sqrt[1 - 1/(c^2*x^2)] +
 a*c*x) - 2*b^2*Log[1 - E^((2*I)*ArcCsc[c*x])]) + a*(a*c*x*(3*b*Sqrt[1 - 1/
(c^2*x^2)] + a*c*x) - 6*b^2*Log[1/(c*x)]) + (3*I)*b^3*PolyLog[2, E^((2*I)*A
rcCsc[c*x])])/(2*c^2)
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(142) = 284.
time = 0.75, size = 323, normalized size = 2.56

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{\frac{c^2x^2a^3}{2}+\frac{b^3\,\mathrm{arccsc}(cx)^3c^2x^2}{2}+\frac{3b^3\,\mathrm{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{3ib^3\,\mathrm{arccsc}(cx)^2}{2}-3b^3\,\mathrm{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)-}{}$ |
| default | $\frac{\frac{c^2x^2a^3}{2}+\frac{b^3\,\mathrm{arccsc}(cx)^3c^2x^2}{2}+\frac{3b^3\,\mathrm{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{3ib^3\,\mathrm{arccsc}(cx)^2}{2}-3b^3\,\mathrm{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)-}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*c^2*x^2*a^3+1/2*b^3*arccsc(c*x)^3*c^2*x^2+3/2*b^3*arccsc(c*x)^2*
((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x+3/2*I*b^3*arccsc(c*x)^2-3*b^3*arccsc(c*x)*l
n(1-I/c/x-(1-1/c^2/x^2)^(1/2))-3*b^3*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(
1/2))+3*I*b^3*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+3*I*b^3*polylog(2,-I/c/x
-(1-1/c^2/x^2)^(1/2))+3/2*a*b^2*arccsc(c*x)^2*c^2*x^2+3*a*b^2*arccsc(c*x)*(
(c^2*x^2-1)/c^2/x^2)^(1/2)*c*x-3*a*b^2*ln(1/c/x)+3*a^2*b*(1/2*c^2*x^2*arccs
c(c*x)+1/2/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*(c^2*x^2-1)))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arccsc(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arccsc(c*x) + x*sqrt(-
1/(c^2*x^2) + 1)/c)*a^2*b + 3*(x*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x)/c + log
(x)/c^2)*a*b^2 + 1/8*(4*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 8*integrate(3/8
*(8*c^2*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 - 8*x*arctan2(
1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 + 8*(c^2*x^3*arctan2(1, sqrt(c*x +
 1)*sqrt(c*x - 1)) - x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2 -
(4*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x*log(c^2*x^2)^2)*sqrt(c*x
 + 1)*sqrt(c*x - 1) - 4*((2*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log
(c) + c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^3 - (2*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1))*log(c) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*
x + 2*(c^2*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - x*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1)))*log(x))*log(c^2*x^2) + 16*(c^2*x^3*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1))*log(c) - x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
*log(c))*log(x))/(c^2*x^2 - 1), x))*b^3
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
[Out] integral(b^3*x*arccsc(c*x)^3 + 3*a*b^2*x*arccsc(c*x)^2 + 3*a^2*b*x*arccsc(c
*x) + a^3*x, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b\operatorname{acsc}(cx))^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))**3,x)
[Out] Integral(x*(a + b*acsc(c*x))**3, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)^3*x, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a + b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asin(1/(c*x)))^3,x)
[Out] int(x*(a + b*asin(1/(c*x)))^3, x)
```

## 3.27 $\int (a+b\csc^{-1}(cx))^3\,dx$

**Optimal**. Leaf size=144

$$x(a+b\csc^{-1}(cx))^3+\frac{6b(a+b\csc^{-1}(cx))^2\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c}-\frac{6ib^2(a+b\csc^{-1}(cx))\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a+b*arccsc(c*x))^3+6*b*(a+b*arccsc(c*x))^2*arctanh(I/c/x+(1-1/c^2/x^2)^(
1/2))/c-6*I*b^2*(a+b*arccsc(c*x))*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c+6
*I*b^2*(a+b*arccsc(c*x))*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c+6*b^3*polyl
og(3,-I/c/x-(1-1/c^2/x^2)^(1/2))/c-6*b^3*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2
))/c
```

**Rubi** [A]
time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5325, 4495, 4268, 2611, 2320, 6724}

$$-\frac{6ib^2\text{Li}_2\left(-e^{i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{c}+\frac{6ib^2\text{Li}_2\left(e^{i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{c}+x(a+b\csc^{-1}(cx))^3+\frac{6b\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))^2}{c}+\frac{6b^3\text{Li}_3\left(-e^{i\csc^{-1}(cx)}\right)}{c}-\frac{6b^3\text{Li}_3\left(e^{i\csc^{-1}(cx)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] x*(a + b*ArcCsc[c*x])^3 + (6*b*(a + b*ArcCsc[c*x])^2*ArcTanh[E^(I*ArcCsc[c*
x])])/c - ((6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, -E^(I*ArcCsc[c*x])])/c
+ ((6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCsc[c*x])])/c + (6*b^3*
PolyLog[3, -E^(I*ArcCsc[c*x])])/c - (6*b^3*PolyLog[3, E^(I*ArcCsc[c*x])])/c
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4495

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ
[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 5325

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Sub
st[Int[(a + b*x)^n*Csc[x]*Cot[x], x], x, ArcCsc[c*x]], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\csc^{-1}(cx)\right)^3\,dx = -\frac{\text{Subst}\left(\int (a+bx)^3\cot(x)\csc(x)\,dx, x, \csc^{-1}(cx)\right)}{c}$$

$$= x\left(a+b\csc^{-1}(cx)\right)^3 - \frac{(3b)\text{Subst}\left(\int (a+bx)^2\csc(x)\,dx, x, \csc^{-1}(cx)\right)}{c}$$

$$= x\left(a+b\csc^{-1}(cx)\right)^3 + \frac{6b\left(a+b\csc^{-1}(cx)\right)^2\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c} + \frac{(6b^2)\,\text{Subst}\left(\int\right.}{}$$

$$= x\left(a+b\csc^{-1}(cx)\right)^3 + \frac{6b\left(a+b\csc^{-1}(cx)\right)^2\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6ib^2(a+b\csc}{}$$

$$= x\left(a+b\csc^{-1}(cx)\right)^3 + \frac{6b\left(a+b\csc^{-1}(cx)\right)^2\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6ib^2(a+b\csc}{}$$

$$= x\left(a+b\csc^{-1}(cx)\right)^3 + \frac{6b\left(a+b\csc^{-1}(cx)\right)^2\tanh^{-1}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6ib^2(a+b\csc}{}$$

**Mathematica [A]**
time = 0.19, size = 265, normalized size = 1.84

$$\frac{a^3cx+3a^2bcx\csc^{-1}(cx)+3ab^2cx\csc^{-1}(cx)^2+b^3cx\csc^{-1}(cx)^3-6ab^2\csc^{-1}(cx)\log\left(1-e^{i\csc^{-1}(cx)}\right)-3b^3\csc^{-1}(cx)^2\log\left(1-e^{i\csc^{-1}(cx)}\right)+6ab^2\csc^{-1}(cx)\log\left(1+e^{i\csc^{-1}(cx)}\right)+3b^3\csc^{-1}(cx)^2\log\left(1+e^{i\csc^{-1}(cx)}\right)+3a^2b\log\left(c\left(1+\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)-6ib^2(a+b\csc^{-1}(cx))\,\mathrm{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)+6ib^2(a+b\csc^{-1}(cx))\,\mathrm{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)+6b^3\mathrm{PolyLog}\left(3,-e^{i\csc^{-1}(cx)}\right)-6b^3\mathrm{PolyLog}\left(3,e^{i\csc^{-1}(cx)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] (a^3*c*x + 3*a^2*b*c*x*ArcCsc[c*x] + 3*a*b^2*c*x*ArcCsc[c*x]^2 + b^3*c*x*Ar
cCsc[c*x]^3 - 6*a*b^2*ArcCsc[c*x]*Log[1 - E^(I*ArcCsc[c*x])] - 3*b^3*ArcCsc
[c*x]^2*Log[1 - E^(I*ArcCsc[c*x])] + 6*a*b^2*ArcCsc[c*x]*Log[1 + E^(I*ArcCs
c[c*x])] + 3*b^3*ArcCsc[c*x]^2*Log[1 + E^(I*ArcCsc[c*x])] + 3*a^2*b*Log[c*(
1 + Sqrt[1 - 1/(c^2*x^2)])*x] - (6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, -E
^(I*ArcCsc[c*x])] + (6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCsc[c*
x])] + 6*b^3*PolyLog[3, -E^(I*ArcCsc[c*x])] - 6*b^3*PolyLog[3, E^(I*ArcCsc[
c*x])])/c
```

**Maple [B]** Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(202) = 404.
time = 0.55, size = 412, normalized size = 2.86

| method | result |
|---|---|
| derivativedivides | $\frac{cx\,a^3+\mathrm{arccsc}(cx)^3b^3cx+3b^3\mathrm{arccsc}(cx)^2\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-3b^3\mathrm{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)-6ib^3}{}$ |
| default | $\frac{cx\,a^3+\mathrm{arccsc}(cx)^3b^3cx+3b^3\mathrm{arccsc}(cx)^2\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-3b^3\mathrm{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)-6ib^3}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*x*a^3+arccsc(c*x)^3*b^3*c*x+3*b^3*arccsc(c*x)^2*ln(1+I/c/x+(1-1/c^2/
x^2)^(1/2))-3*b^3*arccsc(c*x)^2*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))-6*I*arccsc(
c*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))*b^3+6*I*arccsc(c*x)*polylog(2,I/
c/x+(1-1/c^2/x^2)^(1/2))*b^3+6*b^3*polylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2))-6*
b^3*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))+3*arccsc(c*x)^2*a*b^2*c*x-6*I*dilo
g(1+I/c/x+(1-1/c^2/x^2)^(1/2))*a*b^2+6*a*b^2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^
2/x^2)^(1/2))-6*a*b^2*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+6*I*dilog
(1-I/c/x-(1-1/c^2/x^2)^(1/2))*a*b^2+3*arccsc(c*x)*a^2*b*c*x+3*ln(c*x+c*x*(1
-1/c^2/x^2)^(1/2))*a^2*b)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccsc(c*x))^3,x, algorithm="maxima")

[Out] -3/2*a*b^2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 12*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x)*log(c)^2 + b^3*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3/4*b^3*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*a*b^2*c^2*integrate(1/4*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^2 - 1), x) + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*a*b^2*c^2*integrate(1/4*x^2*log(x)^2/(c^2*x^2 - 1), x) - 3/2*a*b^2*(log(c*x + 1)/c - log(c*x - 1)/c)*log(c)^2 + 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x)*log(c)^2 - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) - 12*a*b^2*integrate(1/4*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 24*a*b^2*integrate(1/4*log(x)/(c^2*x^2 - 1), x)*log(c) + a^3*x + 12*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^2 - 1), x) - 3*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^2 - 1), x) - 12*a*b^2*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^2 - 1), x) - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x) + 3*a*b^2*integrate(1/4*log(c^2*x^2)^2/(c^2*x^2 - 1), x) - 12*a*b^2*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 12*a*b^2*integrate(1/4*log(x)^2/(c^2*x^2 - 1), x) + 3/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*a^2*b/c

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccsc(c*x))^3,x, algorithm="fricas")

```
[Out] integral(b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x) +
a^3, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsc}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^3, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^3, x)
```

## 3.28 $\int \frac{(a+b\csc^{-1}(cx))^3}{x}\,dx$

**Optimal**. Leaf size=124

$$\frac{i(a+b\csc^{-1}(cx))^4}{4b}-(a+b\csc^{-1}(cx))^3\log\left(1-e^{2i\csc^{-1}(cx)}\right)+\frac{3}{2}ib(a+b\csc^{-1}(cx))^2\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

```
[Out] 1/4*I*(a+b*arccsc(c*x))^4/b-(a+b*arccsc(c*x))^3*ln(1-(I/c/x+(1-1/c^2/x^2)^(
1/2))^2)+3/2*I*b*(a+b*arccsc(c*x))^2*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^
2)-3/2*b^2*(a+b*arccsc(c*x))*polylog(3,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-3/4*I
*b^3*polylog(4,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

**Rubi** [A]
time = 0.11, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5331, 3798, 2221, 2611, 6744, 2320, 6724}

$$-\frac{3}{2}b^2\text{Li}_3\left(e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))+\frac{3}{2}ib\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))^2+\frac{i(a+b\csc^{-1}(cx))^4}{4b}-\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))^3-\frac{3}{4}ib^3\text{Li}_4\left(e^{2i\csc^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3/x,x]
```

```
[Out] ((I/4)*(a + b*ArcCsc[c*x])^4)/b - (a + b*ArcCsc[c*x])^3*Log[1 - E^((2*I)*Ar
cCsc[c*x])] + ((3*I)/2)*b*(a + b*ArcCsc[c*x])^2*PolyLog[2, E^((2*I)*ArcCsc[
c*x])] - (3*b^2*(a + b*ArcCsc[c*x])*PolyLog[3, E^((2*I)*ArcCsc[c*x])])/2 -
((3*I)/4)*b^3*PolyLog[4, E^((2*I)*ArcCsc[c*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}\,dx &= -\text{Subst}\left(\int (a+bx)^3\cot(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b} + 2i\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^3}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b} - \left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right) + (3b)\text{Subst}\left(\int (a\right.\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b} - \left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right) + \frac{3}{2}ib\left(a+b\csc^{-1}(\right.\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b} - \left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right) + \frac{3}{2}ib\left(a+b\csc^{-1}(\right.\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b} - \left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right) + \frac{3}{2}ib\left(a+b\csc^{-1}(\right.\\
&= \frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b} - \left(a+b\csc^{-1}(cx)\right)^3\log\left(1-e^{2i\csc^{-1}(cx)}\right) + \frac{3}{2}ib\left(a+b\csc^{-1}(\right.
\end{aligned}
$$

**Mathematica** [A]
time = 0.13, size = 242, normalized size = 1.95

$$
a^3\log(cx) + \frac{3}{2}ia^2b\left(\csc^{-1}(cx)\left(\csc^{-1}(cx) + 2i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\right) + \text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\right) + \frac{1}{8}iab^2\left(\pi^3 - 8\csc^{-1}(cx)^3 + 24i\csc^{-1}(cx)^2\log\left(1-e^{-2i\csc^{-1}(cx)}\right) - 24\csc^{-1}(cx)\text{PolyLog}\left(2, e^{-2i\csc^{-1}(cx)}\right) + 12i\text{PolyLog}\left(3, e^{-2i\csc^{-1}(cx)}\right)\right) + \frac{1}{64}ib^3\left(\pi^4 - 16\csc^{-1}(cx)^4 + 64i\csc^{-1}(cx)^3\log\left(1-e^{-2i\csc^{-1}(cx)}\right) - 96\csc^{-1}(cx)^2\text{PolyLog}\left(2, e^{-2i\csc^{-1}(cx)}\right) + 96i\csc^{-1}(cx)\text{PolyLog}\left(3, e^{-2i\csc^{-1}(cx)}\right) + 48\text{PolyLog}\left(4, e^{-2i\csc^{-1}(cx)}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x,x]
```

```
[Out] a^3*Log[c*x] + ((3*I)/2)*a^2*b*(ArcCsc[c*x]*(ArcCsc[c*x] + (2*I)*Log[1 - E^
((2*I)*ArcCsc[c*x])]) + PolyLog[2, E^((2*I)*ArcCsc[c*x])]) + (I/8)*a*b^2*(P
i^3 - 8*ArcCsc[c*x]^3 + (24*I)*ArcCsc[c*x]^2*Log[1 - E^((-2*I)*ArcCsc[c*x])
] - 24*ArcCsc[c*x]*PolyLog[2, E^((-2*I)*ArcCsc[c*x])] + (12*I)*PolyLog[3, E
^((-2*I)*ArcCsc[c*x])]) + (I/64)*b^3*(Pi^4 - 16*ArcCsc[c*x]^4 + (64*I)*ArcC
sc[c*x]^3*Log[1 - E^((-2*I)*ArcCsc[c*x])] - 96*ArcCsc[c*x]^2*PolyLog[2, E^(
(-2*I)*ArcCsc[c*x])] + (96*I)*ArcCsc[c*x]*PolyLog[3, E^((-2*I)*ArcCsc[c*x])
] + 48*PolyLog[4, E^((-2*I)*ArcCsc[c*x])])
```

**Maple** [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(169) = 338.
time = 0.40, size = 666, normalized size = 5.37

| method | result |
|---|---|
| derivativedivides | $a^3\ln(cx) + \frac{3ia^2b\,\text{arccsc}(cx)^2}{2} - b^3\text{arccsc}(cx)^3\ln\left(1 - \frac{i}{cx} - \sqrt{1-\frac{1}{c^2x^2}}\right) + 3ia^2b\,\text{polylog}\left(2\right.$ |

| default | $a^3 \ln(cx) + \frac{3ia^2 b \operatorname{arccsc}(cx)^2}{2} - b^3 \operatorname{arccsc}(cx)^3 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 3ia^2 b \operatorname{polylog}\left(2, \right.$ |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*ln(c*x)+3/2*I*a^2*b*arccsc(c*x)^2-b^3*arccsc(c*x)^3*ln(1-I/c/x-(1-1/c^2
/x^2)^(1/2))-6*I*b^3*polylog(4,-I/c/x-(1-1/c^2/x^2)^(1/2))-6*b^3*arccsc(c*x
)*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))+6*I*a*b^2*arccsc(c*x)*polylog(2,I/c/
x+(1-1/c^2/x^2)^(1/2))-b^3*arccsc(c*x)^3*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+6*
I*a*b^2*arccsc(c*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-6*b^3*arccsc(c*x)
*polylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2))+3*I*a^2*b*polylog(2,I/c/x+(1-1/c^2/x
^2)^(1/2))+I*a*b^2*arccsc(c*x)^3-3*a*b^2*arccsc(c*x)^2*ln(1+I/c/x+(1-1/c^2/
x^2)^(1/2))-3*a*b^2*arccsc(c*x)^2*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+1/4*I*b^3
*arccsc(c*x)^4+3*I*b^3*arccsc(c*x)^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-6
*a*b^2*polylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2))-6*a*b^2*polylog(3,I/c/x+(1-1/c
^2/x^2)^(1/2))+3*I*b^3*arccsc(c*x)^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-
3*a^2*b*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-3*a^2*b*arccsc(c*x)*ln(
1-I/c/x-(1-1/c^2/x^2)^(1/2))+3*I*a^2*b*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2)
)-6*I*b^3*polylog(4,I/c/x+(1-1/c^2/x^2)^(1/2))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] -3/2*a*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 - 12*b^3*c^2*
integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^3 - x), x)
*log(c)^2 + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x -
 1)))*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*
arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^3 - x), x)*log(c) + 1
2*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*a*
b^2*c^2*integrate(1/4*x^2*log(x)/(c^2*x^3 - x), x)*log(c) + b^3*arctan2(1,
sqrt(c*x + 1)*sqrt(c*x - 1))^3*log(x) - 3/4*b^3*arctan2(1, sqrt(c*x + 1)*sq
rt(c*x - 1))*log(c^2*x^2)^2*log(x) + 24*b^3*c^2*integrate(1/4*x^2*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) - 12*b
^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(
c^2*x^3 - x), x) + 12*a*b^2*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*s
qrt(c*x - 1)))^2/(c^2*x^3 - x), x) - 3*a*b^2*c^2*integrate(1/4*x^2*log(c^2*
x^2)^2/(c^2*x^3 - x), x) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)*log(
x)/(c^2*x^3 - x), x) - 12*a*b^2*c^2*integrate(1/4*x^2*log(x)^2/(c^2*x^3 - x
), x) + 12*a^2*b*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1
```

```
)))/(c^2*x^3 - x), x) + 3/2*a*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*
log(c)^2 + 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^
2*x^3 - x), x)*log(c)^2 - 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt
(c*x - 1)))*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 24*b^3*integrate(1/4*ar
ctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^3 - x), x)*log(c) - 12*
a*b^2*integrate(1/4*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 24*a*b^2*integr
ate(1/4*log(x)/(c^2*x^3 - x), x)*log(c) + 12*b^3*integrate(1/4*sqrt(c*x + 1
)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2*log(x)/(c^2*x^3 -
 x), x) - 3*b^3*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2*lo
g(x)/(c^2*x^3 - x), x) - 24*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(
c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + 12*b^3*integrate(1/4*arc
tan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^3 - x), x) - 12*a*b^2*
integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^3 - x), x) +
 3*a*b^2*integrate(1/4*log(c^2*x^2)^2/(c^2*x^3 - x), x) - 12*a*b^2*integrat
e(1/4*log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + 12*a*b^2*integrate(1/4*log(x)
^2/(c^2*x^3 - x), x) - 12*a^2*b*integrate(1/4*arctan(1/(sqrt(c*x + 1)*sqrt(
c*x - 1)))/(c^2*x^3 - x), x) + a^3*log(x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x) +
 a^3)/x, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acsc}\left(cx\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x,x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)^3/x, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^3/x,x)
[Out] int((a + b*asin(1/(c*x)))^3/x, x)
```

## 3.29 $\int \frac{(a+b\csc^{-1}(cx))^3}{x^2}\,dx$

**Optimal**. Leaf size=80

$$6b^3c\sqrt{1-\frac{1}{c^2x^2}}+\frac{6b^2(a+b\csc^{-1}(cx))}{x}-3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-\frac{(a+b\csc^{-1}(cx))^3}{x}$$

```
[Out] 6*b^2*(a+b*arccsc(c*x))/x-(a+b*arccsc(c*x))^3/x+6*b^3*c*(1-1/c^2/x^2)^(1/2)
-3*b*c*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5331, 3377, 2718}

$$\frac{6b^2(a+b\csc^{-1}(cx))}{x}-3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-\frac{(a+b\csc^{-1}(cx))^3}{x}+6b^3c\sqrt{1-\frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3/x^2,x]
```

```
[Out] 6*b^3*c*Sqrt[1 - 1/(c^2*x^2)] + (6*b^2*(a + b*ArcCsc[c*x]))/x - 3*b*c*Sqrt[
1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2 - (a + b*ArcCsc[c*x])^3/x
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^2}\,dx = -\left(c\text{Subst}\left(\int (a+bx)^3\cos(x)\,dx, x, \csc^{-1}(cx)\right)\right)$$
$$= -\frac{(a+b\csc^{-1}(cx))^3}{x} + (3bc)\text{Subst}\left(\int (a+bx)^2\sin(x)\,dx, x, \csc^{-1}(cx)\right)$$
$$= -3bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2 - \frac{(a+b\csc^{-1}(cx))^3}{x} + (6b^2c)\,\text{Subst}\left(\int (a+\right.$$
$$= \frac{6b^2(a+b\csc^{-1}(cx))}{x} - 3bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2 - \frac{(a+b\csc^{-1}(cx))^3}{x} -$$
$$= 6b^3c\sqrt{1-\frac{1}{c^2x^2}} + \frac{6b^2(a+b\csc^{-1}(cx))}{x} - 3bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2 - \frac{(a}{}$$

**Mathematica** [A]
time = 0.13, size = 135, normalized size = 1.69

$$-\frac{a^3-6ab^2+3a^2bc\sqrt{1-\frac{1}{c^2x^2}}\,x-6b^3c\sqrt{1-\frac{1}{c^2x^2}}\,x+3b\left(a^2-2b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}\,x\right)\csc^{-1}(cx)+3b^2\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}\,x\right)\csc^{-1}(cx)^2+b^3\csc^{-1}(cx)^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^2,x]
```

```
[Out] -((a^3 - 6*a*b^2 + 3*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1 - 1/(
c^2*x^2)]*x + 3*b*(a^2 - 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[c*
x] + 3*b^2*(a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[c*x]^2 + b^3*ArcCsc[c*x
]^3)/x)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(76) = 152$.
time = 0.29, size = 199, normalized size = 2.49

| method | result |
|---|---|
| derivativedivides | $c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{cx}-3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arccsc}(cx)}{cx}\right)+3a\,b^2\left(-\right.\right.$ |
| default | $c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{cx}-3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arccsc}(cx)}{cx}\right)+3a\,b^2\left(-\right.\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a^3/c/x+b^3*(-arccsc(c*x)^3/c/x-3*arccsc(c*x)^2*((c^2*x^2-1)/c^2/x^2)^(
1/2)+6*((c^2*x^2-1)/c^2/x^2)^(1/2)+6/c/x*arccsc(c*x))+3*a*b^2*(-arccsc(c*x)
^2/c/x+2/c/x-2*arccsc(c*x)*((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a^2*b*(-1/c/x*arc
csc(c*x)-1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^2-1)))
```

**Maxima [A]**
time = 0.26, size = 147, normalized size = 1.84

$$-\frac{b^3 \operatorname{arccsc}(cx)^3}{x} - 3\left(c\sqrt{-\frac{1}{c^2x^2}+1} + \frac{\operatorname{arccsc}(cx)}{x}\right)a^2b - 6\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arccsc}(cx) - \frac{1}{x}\right)ab^2 - 3\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arccsc}(cx)^2 - 2c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{2\operatorname{arccsc}(cx)}{x}\right)b^3 - \frac{3ab^2\operatorname{arccsc}(cx)^2}{x} - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*arccsc(c*x)^3/x - 3*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*a^2*b -
 6*(c*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x) - 1/x)*a*b^2 - 3*(c*sqrt(-1/(c^2*x
^2) + 1)*arccsc(c*x)^2 - 2*c*sqrt(-1/(c^2*x^2) + 1) - 2*arccsc(c*x)/x)*b^3
- 3*a*b^2*arccsc(c*x)^2/x - a^3/x
```

**Fricas [A]**
time = 0.42, size = 98, normalized size = 1.22

$$-\frac{b^3\operatorname{arccsc}(cx)^3 + 3ab^2\operatorname{arccsc}(cx)^2 + a^3 - 6ab^2 + 3(a^2b - 2b^3)\operatorname{arccsc}(cx) + 3(b^3\operatorname{arccsc}(cx)^2 + 2ab^2\operatorname{arccsc}(cx) + a^2b - 2b^3)\sqrt{c^2x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] -(b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + a^3 - 6*a*b^2 + 3*(a^2*b - 2*
b^3)*arccsc(c*x) + 3*(b^3*arccsc(c*x)^2 + 2*a*b^2*arccsc(c*x) + a^2*b - 2*b
^3)*sqrt(c^2*x^2 - 1))/x
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsc}(cx))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x**2, x)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(76) = 152.
time = 0.48, size = 195, normalized size = 2.44

$$-\left(3b^3\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)^2 + 6ab^2\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right) + \frac{b^3\arcsin\left(\frac{1}{cx}\right)^3}{cx} + 3a^2b\sqrt{-\frac{1}{c^2x^2}+1} - 6b^3\sqrt{-\frac{1}{c^2x^2}+1} + \frac{3ab^2\arcsin\left(\frac{1}{cx}\right)^2}{cx} + \frac{3a^2b\arcsin\left(\frac{1}{cx}\right)}{cx} - \frac{6b^3\arcsin\left(\frac{1}{cx}\right)}{cx} + \frac{a^3}{cx} - \frac{6ab^2}{cx}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] -(3*b^3*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x))^2 + 6*a*b^2*sqrt(-1/(c^2*x^2
) + 1)*arcsin(1/(c*x)) + b^3*arcsin(1/(c*x))^3/(c*x) + 3*a^2*b*sqrt(-1/(c^2
*x^2) + 1) - 6*b^3*sqrt(-1/(c^2*x^2) + 1) + 3*a*b^2*arcsin(1/(c*x))^2/(c*x)
 + 3*a^2*b*arcsin(1/(c*x))/(c*x) - 6*b^3*arcsin(1/(c*x))/(c*x) + a^3/(c*x)
- 6*a*b^2/(c*x))*c
```

**Mupad [B]**
time = 0.79, size = 155, normalized size = 1.94

$$\frac{b^3\left(6\,\mathrm{asin}\left(\frac{1}{c\,x}\right)-\mathrm{asin}\left(\frac{1}{c\,x}\right)^3\right)}{x}-\frac{a^3}{x}-3\,a^2\,b\,c\left(\sqrt{1-\frac{1}{c^2\,x^2}}+\frac{\mathrm{asin}\left(\frac{1}{c\,x}\right)}{c\,x}\right)-b^3\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}\,\left(3\,\mathrm{asin}\left(\frac{1}{c\,x}\right)^2-6\right)-3\,a\,b^2\,c\left(2\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}+\frac{\mathrm{asin}\left(\frac{1}{c\,x}\right)^2-2}{c\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^3/x^2,x)
```

```
[Out] (b^3*(6*asin(1/(c*x)) - asin(1/(c*x))^3))/x - a^3/x - 3*a^2*b*c*((1 - 1/(c^
2*x^2))^(1/2) + asin(1/(c*x))/(c*x)) - b^3*c*(1 - 1/(c^2*x^2))^(1/2)*(3*asi
n(1/(c*x))^2 - 6) - 3*a*b^2*c*(2*asin(1/(c*x))*(1 - 1/(c^2*x^2))^(1/2) + (a
sin(1/(c*x))^2 - 2)/(c*x))
```

# 3.30 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^3}\,dx$

**Optimal**. Leaf size=125

$$\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}-\frac{3}{8}b^3c^2\csc^{-1}(cx)+\frac{3b^2(a+b\csc^{-1}(cx))}{4x^2}-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4x}+\frac{1}{4}c^2\left(a+b\csc\right.$$

```
[Out] -3/8*b^3*c^2*arccsc(c*x)+3/4*b^2*(a+b*arccsc(c*x))/x^2+1/4*c^2*(a+b*arccsc(
c*x))^3-1/2*(a+b*arccsc(c*x))^3/x^2+3/8*b^3*c*(1-1/c^2/x^2)^(1/2)/x-3/4*b*c
*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/x
```

**Rubi** [A]
time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5331, 4489, 3392, 32, 2715, 8}

$$\frac{3b^2(a+b\csc^{-1}(cx))}{4x^2}-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4x}+\frac{1}{4}c^2\left(a+b\csc^{-1}(cx)\right)^3-\frac{(a+b\csc^{-1}(cx))^3}{2x^2}+\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}-\frac{3}{8}b^3c^2\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3/x^3,x]
```

```
[Out] (3*b^3*c*Sqrt[1 - 1/(c^2*x^2)])/(8*x) - (3*b^3*c^2*ArcCsc[c*x])/8 + (3*b^2*
(a + b*ArcCsc[c*x]))/(4*x^2) - (3*b*c*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c
*x])^2)/(4*x) + (c^2*(a + b*ArcCsc[c*x])^3)/4 - (a + b*ArcCsc[c*x])^3/(2*x^
2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^3}\,dx &= -\left(c^2\text{Subst}\left(\int (a+bx)^3\cos(x)\sin(x)\,dx, x, \csc^{-1}(cx)\right)\right)\\ &= -\frac{\left(a+b\csc^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}\left(3bc^2\right)\text{Subst}\left(\int (a+bx)^2\sin^2(x)\,dx, x, \csc^{-1}(cx)\right)\\ &= \frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{4x^2}-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4x}-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{2x^2}+\\ &= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}+\frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{4x^2}-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4x}+\frac{1}{4}\\ &= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x}-\frac{3}{8}b^3c^2\csc^{-1}(cx)+\frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{4x^2}-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+}{4x}\end{aligned}$$

## Mathematica [A]

time = 0.13, size = 186, normalized size = 1.49

$$\frac{-4a^3+6ab^2-6a^2bc\sqrt{1-\frac{1}{c^2x^2}}\,x+3b^3c\sqrt{1-\frac{1}{c^2x^2}}\,x+6b\left(-2a^2+b^2-2abc\sqrt{1-\frac{1}{c^2x^2}}\,x\right)\csc^{-1}(cx)-6b^2\left(bc\sqrt{1-\frac{1}{c^2x^2}}\,x+a\left(2-c^2x^2\right)\right)\csc^{-1}(cx)^2+2b^3\left(-2+c^2x^2\right)\csc^{-1}(cx)^3-3b\left(-2a^2+b^2\right)c^2x^2\text{ArcSin}\left(\frac{1}{cx}\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^3,x]
```

```
[Out] (-4*a^3 + 6*a*b^2 - 6*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 3*b^3*c*Sqrt[1 - 1/
(c^2*x^2)]*x + 6*b*(-2*a^2 + b^2 - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[
c*x] - 6*b^2*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*(2 - c^2*x^2))*ArcCsc[c*x]^2
+ 2*b^3*(-2 + c^2*x^2)*ArcCsc[c*x]^3 - 3*b*(-2*a^2 + b^2)*c^2*x^2*ArcSin[1/
(c*x)])/(8*x^2)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(109) = 218$.
time = 0.35, size = 338, normalized size = 2.70

| method | result |
|---|---|
| derivativedivides | $c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(\frac{\operatorname{arccsc}(cx)^3(c^2x^2-1)}{2c^2x^2}-\frac{3\operatorname{arccsc}(cx)^2\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx}-\frac{3(c^2x^2-1)\operatorname{arccsc}(cx}{4c^2x^2}\right.\right.$ |
| default | $c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(\frac{\operatorname{arccsc}(cx)^3(c^2x^2-1)}{2c^2x^2}-\frac{3\operatorname{arccsc}(cx)^2\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx}-\frac{3(c^2x^2-1)\operatorname{arccsc}(cx}{4c^2x^2}\right.\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a^3/c^2/x^2+b^3*(1/2*arccsc(c*x)^3*(c^2*x^2-1)/c^2/x^2-3/4*arccsc
(c*x)^2*(arccsc(c*x)*c*x+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x-3/4*(c^2*x^2-1)/c
^2/x^2*arccsc(c*x)+3/8*((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x+3/8*arccsc(c*x)+1/2*
arccsc(c*x)^3)+3*a*b^2*(1/2*arccsc(c*x)^2*(c^2*x^2-1)/c^2/x^2-1/2*arccsc(c*
x)*(arccsc(c*x)*c*x+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x+1/4*arccsc(c*x)^2+1/4/
c^2/x^2)-3/2*a^2*b/c^2/x^2*arccsc(c*x)+3/4*a^2*b*(c^2*x^2-1)^(1/2)/((c^2*x^
2-1)/c^2/x^2)^(1/2)/c/x*arctan(1/(c^2*x^2-1)^(1/2))-3/4*a^2*b*(c^2*x^2-1)/(
(c^2*x^2-1)/c^2/x^2)^(1/2)/c^3/x^3)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*a^2*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) - 1/2*a^3/x^
```

```
2 - 1/8*(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan2(1,
 sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(a*b^2*c^2*(log(c*x + 1)
+ log(c*x - 1) - 2*log(x))*log(c)^2 + 16*b^3*c^2*integrate(1/8*x^2*arctan(1
/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^5 - x^3), x)*log(c)^2 - 16*b^3*c^2*i
ntegrate(1/8*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*
x^5 - x^3), x)*log(c) + 32*b^3*c^2*integrate(1/8*x^2*arctan(1/(sqrt(c*x + 1
)*sqrt(c*x - 1)))*log(x)/(c^2*x^5 - x^3), x)*log(c) - 16*a*b^2*c^2*integrat
e(1/8*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 32*a*b^2*c^2*integrate(
1/8*x^2*log(x)/(c^2*x^5 - x^3), x)*log(c) - 16*b^3*c^2*integrate(1/8*x^2*ar
ctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3),
x) + 16*b^3*c^2*integrate(1/8*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*l
og(x)^2/(c^2*x^5 - x^3), x) - 16*a*b^2*c^2*integrate(1/8*x^2*arctan(1/(sqrt
(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^5 - x^3), x) + 8*b^3*c^2*integrate(1/8*x
^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^5 - x^3), x)
 + 4*a*b^2*c^2*integrate(1/8*x^2*log(c^2*x^2)^2/(c^2*x^5 - x^3), x) - 16*a*
b^2*c^2*integrate(1/8*x^2*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) + 16*a*b^
2*c^2*integrate(1/8*x^2*log(x)^2/(c^2*x^5 - x^3), x) - (c^2*log(c*x + 1) +
c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*a*b^2*log(c)^2 - 16*b^3*integrate(
1/8*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^5 - x^3), x)*log(c)^2 +
16*b^3*integrate(1/8*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(
c^2*x^5 - x^3), x)*log(c) - 32*b^3*integrate(1/8*arctan(1/(sqrt(c*x + 1)*sq
rt(c*x - 1)))*log(x)/(c^2*x^5 - x^3), x)*log(c) + 16*a*b^2*integrate(1/8*lo
g(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 32*a*b^2*integrate(1/8*log(x)/(c^2*
x^5 - x^3), x)*log(c) + 8*b^3*integrate(1/8*sqrt(c*x + 1)*sqrt(c*x - 1)*arc
tan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^5 - x^3), x) - 2*b^3*integrat
e(1/8*sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2/(c^2*x^5 - x^3), x) + 16*b
^3*integrate(1/8*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x
)/(c^2*x^5 - x^3), x) - 16*b^3*integrate(1/8*arctan(1/(sqrt(c*x + 1)*sqrt(c
*x - 1)))*log(x)^2/(c^2*x^5 - x^3), x) + 16*a*b^2*integrate(1/8*arctan(1/(s
qrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^5 - x^3), x) - 8*b^3*integrate(1/8*ar
ctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^5 - x^3), x) - 4*
a*b^2*integrate(1/8*log(c^2*x^2)^2/(c^2*x^5 - x^3), x) + 16*a*b^2*integrate
(1/8*log(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) - 16*a*b^2*integrate(1/8*log(x
)^2/(c^2*x^5 - x^3), x))*x^2)/x^2
```

**Fricas** [A]
time = 0.35, size = 150, normalized size = 1.20

$$\frac{2\,(b^3c^2x^2-2\,b^3)\,\mathrm{arccsc}\,(cx)^3-4\,a^3+6\,ab^2+6\,(ab^2c^2x^2-2\,ab^2)\,\mathrm{arccsc}\,(cx)^2+3\,((2\,a^2b-b^3)c^2x^2-4\,a^2b+2\,b^3)\,\mathrm{arccsc}\,(cx)-3\,\left(2\,b^3\,\mathrm{arccsc}\,(cx)^2+4\,ab^2\,\mathrm{arccsc}\,(cx)+2\,a^2b-b^3\right)\sqrt{c^2x^2-1}}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(b^3*c^2*x^2 - 2*b^3)*arccsc(c*x)^3 - 4*a^3 + 6*a*b^2 + 6*(a*b^2*c^2
*x^2 - 2*a*b^2)*arccsc(c*x)^2 + 3*((2*a^2*b - b^3)*c^2*x^2 - 4*a^2*b + 2*b^
```

```
3)*arccsc(c*x) - 3*(2*b^3*arccsc(c*x)^2 + 4*a*b^2*arccsc(c*x) + 2*a^2*b - b
^3)*sqrt(c^2*x^2 - 1))/x^2
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acsc}\,(cx))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x**3, x)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(109) = 218.
time = 0.46, size = 302, normalized size = 2.42

$$-\frac{1}{8}\left(4b^3c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^3+12ab^2c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^2+2b^3c\arcsin\left(\frac{1}{cx}\right)^3+12a^2bc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)-6b^3c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)+6ab^2c\arcsin\left(\frac{1}{cx}\right)^2+4a^3c\left(\frac{1}{c^2x^2}-1\right)-6ab^2c\left(\frac{1}{c^2x^2}-1\right)+6a^2bc\arcsin\left(\frac{1}{cx}\right)-3b^3c\arcsin\left(\frac{1}{cx}\right)+\frac{6b^3\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)^2}{x}-3ab^2c+\frac{12ab^2\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{6a^2b\sqrt{-\frac{1}{c^2x^2}+1}}{x}-\frac{3b^3\sqrt{-\frac{1}{c^2x^2}+1}}{x}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^3 + 12*a*b^2*c*(1/(c^2*x^2)
 - 1)*arcsin(1/(c*x))^2 + 2*b^3*c*arcsin(1/(c*x))^3 + 12*a^2*b*c*(1/(c^2*x^
2) - 1)*arcsin(1/(c*x)) - 6*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 6*a*b
^2*c*arcsin(1/(c*x))^2 + 4*a^3*c*(1/(c^2*x^2) - 1) - 6*a*b^2*c*(1/(c^2*x^2)
 - 1) + 6*a^2*b*c*arcsin(1/(c*x)) - 3*b^3*c*arcsin(1/(c*x)) + 6*b^3*sqrt(-1
/(c^2*x^2) + 1)*arcsin(1/(c*x))^2/x - 3*a*b^2*c + 12*a*b^2*sqrt(-1/(c^2*x^2
) + 1)*arcsin(1/(c*x))/x + 6*a^2*b*sqrt(-1/(c^2*x^2) + 1)/x - 3*b^3*sqrt(-1
/(c^2*x^2) + 1)/x)*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^3/x^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^3/x^3, x)
```

## 3.31 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^4}\,dx$

**Optimal**. Leaf size=170

$$\frac{14}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}}-\frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}+\frac{2b^2\left(a+b\csc^{-1}(cx)\right)}{9x^3}+\frac{4b^2c^2\left(a+b\csc^{-1}(cx)\right)}{3x}-\frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\ ($$

```
[Out] -2/27*b^3*c^3*(1-1/c^2/x^2)^(3/2)+2/9*b^2*(a+b*arccsc(c*x))/x^3+4/3*b^2*c^2
*(a+b*arccsc(c*x))/x-1/3*(a+b*arccsc(c*x))^3/x^3+14/9*b^3*c^3*(1-1/c^2/x^2)
^(1/2)-2/3*b*c^3*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)-1/3*b*c*(a+b*arccs
c(c*x))^2*(1-1/c^2/x^2)^(1/2)/x^2
```

**Rubi** [A]
time = 0.12, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5331, 4489, 3392, 3377, 2718, 2713}

$$\frac{4b^2c^2\left(a+b\csc^{-1}(cx)\right)}{3x}+\frac{2b^2\left(a+b\csc^{-1}(cx)\right)}{9x^3}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{3x^2}-\frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{3x^3}-\frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}+\frac{14}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3/x^4,x]
```

```
[Out] (14*b^3*c^3*Sqrt[1 - 1/(c^2*x^2)])/9 - (2*b^3*c^3*(1 - 1/(c^2*x^2))^(3/2))/
27 + (2*b^2*(a + b*ArcCsc[c*x]))/(9*x^3) + (4*b^2*c^2*(a + b*ArcCsc[c*x]))/
(3*x) - (2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2)/3 - (b*c*Sqrt
[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2)/(3*x^2) - (a + b*ArcCsc[c*x])^3/(3
*x^3)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^4}\,dx = -\left(c^3\text{Subst}\left(\int (a+bx)^3\cos(x)\sin^2(x)\,dx, x, \csc^{-1}(cx)\right)\right)$$

$$= -\frac{(a+b\csc^{-1}(cx))^3}{3x^3} + (bc^3)\,\text{Subst}\left(\int (a+bx)^2\sin^3(x)\,dx, x, \csc^{-1}(cx)\right)$$

$$= \frac{2b^2(a+b\csc^{-1}(cx))}{9x^3} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2}{3x^2} - \frac{(a+b\csc^{-1}(cx))^3}{3x^3} + \ldots$$

$$= \frac{2b^2(a+b\csc^{-1}(cx))}{9x^3} - \frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+\ldots}{3x^2}$$

$$= \frac{2}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}} - \frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2} + \frac{2b^2(a+b\csc^{-1}(cx))}{9x^3} + \frac{4b^2c^2(a+b\ldots}{3\ldots}$$

$$= \frac{14}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}} - \frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2} + \frac{2b^2(a+b\csc^{-1}(cx))}{9x^3} + \frac{4b^2c^2(a+\ldots}{\ldots}$$

**Mathematica [A]**
time = 0.20, size = 204, normalized size = 1.20

$$\frac{-9a^3-9a^2bc\sqrt{1-\frac{1}{c^2x^2}}\,x(1+2c^2x^2)+6ab^2(1+6c^2x^2)+2b^3c\sqrt{1-\frac{1}{c^2x^2}}\,x(1+20c^2x^2)+3b\left(-9a^2-6abc\sqrt{1-\frac{1}{c^2x^2}}\,x(1+2c^2x^2)+2b^2(1+6c^2x^2)\right)\csc^{-1}(cx)-9b^2\left(3a+bc\sqrt{1-\frac{1}{c^2x^2}}\,x(1+2c^2x^2)\right)\csc^{-1}(cx)^2-9b^3\csc^{-1}(cx)^3}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^4,x]
```

```
[Out] (-9*a^3 - 9*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 6*a*b^2*(1 +
6*c^2*x^2) + 2*b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 20*c^2*x^2) + 3*b*(-9*a^2
 - 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 2*b^2*(1 + 6*c^2*x^2))
*ArcCsc[c*x] - 9*b^2*(3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*Ar
cCsc[c*x]^2 - 9*b^3*ArcCsc[c*x]^3)/(27*x^3)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(148) = 296$.
time = 0.41, size = 299, normalized size = 1.76

| method | result |
|---|---|
| derivativedivides | $c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}+\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arccsc}(cx)}{3cx}+\right.\right.$ 2 a |
| default | $c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}+\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arccsc}(cx)}{3cx}+\right.\right.$ 2 a |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3*arccsc(c*x)^3/c^3/x^3-1/3*arccsc(c*x)^2*(2*
c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+4/3*((c^2*x^2-1)/c^2/x^2)^(1
/2)+4/3/c/x*arccsc(c*x)+2/9/c^3/x^3*arccsc(c*x)+2/27*(2*c^2*x^2+1)/c^2/x^2*
((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a*b^2*(-1/3*arccsc(c*x)^2/c^3/x^3-2/9*arccsc
(c*x)*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/
x)+3*a^2*b*(-1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^
2-1)/c^2/x^2)^(1/2)/c^4/x^4))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c
- 3*arccsc(c*x)/x^3) - a*b^2*arccsc(c*x)^2/x^3 + 1/12*(12*x^3*integrate(-1/
4*(12*c^2*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 - 12*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 + 12*(c^2*x^2*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)) - arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2 +
sqrt(c*x + 1)*sqrt(c*x - 1)*(4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 -
log(c^2*x^2)^2) - 4*((3*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)
- c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^2 - 3*arctan2(1, sqrt(c*x
+ 1)*sqrt(c*x - 1))*log(c) + 3*(c^2*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
 1)) - arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x) + arctan2(1, sqrt(c*
x + 1)*sqrt(c*x - 1)))*log(c^2*x^2) + 24*(c^2*x^2*arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1))*log(c) - arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c))*log
(x))/(c^2*x^6 - x^4), x) - 4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 + 3*
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2)*b^3/x^3 - 1/3*a^3/x
^3 - 2/9*(6*c^5*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*c^3*x^2*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - (6*c^3*x^2 + c)*sqrt(c*x + 1)*sqrt(c
*x - 1) - 3*c*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*a*b^2/(sqrt(c*x + 1)
*sqrt(c*x - 1)*c*x^3)
```

**Fricas** [**A**]
time = 0.50, size = 173, normalized size = 1.02

$$\frac{36\,ab^2c^2x^2 - 9\,b^3\operatorname{arccsc}(cx)^3 - 27\,ab^2\operatorname{arccsc}(cx)^2 - 9\,a^3 + 6\,ab^2 + 3\,(12\,b^3c^2x^2 - 9\,a^2b + 2\,b^3)\operatorname{arccsc}(cx) - \left(2\,(9\,a^2b - 20\,b^3)c^2x^2 + 9\,a^2b - 2\,b^3 + 9\,(2\,b^3c^2x^2 + b^3)\operatorname{arccsc}(cx)^2 + 18\,(2\,ab^2c^2x^2 + ab^2)\operatorname{arccsc}(cx)\right)\sqrt{c^2x^2 - 1}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(36*a*b^2*c^2*x^2 - 9*b^3*arccsc(c*x)^3 - 27*a*b^2*arccsc(c*x)^2 - 9*a
^3 + 6*a*b^2 + 3*(12*b^3*c^2*x^2 - 9*a^2*b + 2*b^3)*arccsc(c*x) - (2*(9*a^2
*b - 20*b^3)*c^2*x^2 + 9*a^2*b - 2*b^3 + 9*(2*b^3*c^2*x^2 + b^3)*arccsc(c*x
)^2 + 18*(2*a*b^2*c^2*x^2 + a*b^2)*arccsc(c*x))*sqrt(c^2*x^2 - 1))/x^3
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsc}(cx))^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3/x**4,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x**4, x)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(148) = 296.

time = 0.48, size = 428, normalized size = 2.52

$$\frac{1}{27}\left(9\,b^3c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)^2+18\,ab^2c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)-27\,b^3c^2\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)^2-\frac{9\,b^3c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^3}{x}+9\,a^2bc^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-2\,b^3c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-54\,ab^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)-\frac{27\,ab^2c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^2}{x}-\frac{9\,b^3c\arcsin\left(\frac{1}{cx}\right)^3}{x}-27\,a^2bc^2\sqrt{-\frac{1}{c^2x^2}+1}+42\,b^3c^2\sqrt{-\frac{1}{c^2x^2}+1}-\frac{27\,a^2bc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{6\,b^3c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}-\frac{27\,ab^2c\arcsin\left(\frac{1}{cx}\right)^2}{x}+\frac{6\,ab^2c\left(\frac{1}{c^2x^2}-1\right)}{x}-\frac{27\,a^2bc\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{42\,b^3c\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{42\,ab^2c}{x}-\frac{9\,a^3}{cx^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^4,x, algorithm="giac")
```

```
[Out] 1/27*(9*b^3*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x))^2 + 18*a*b^2*c^2*(
-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x)) - 27*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)
*arcsin(1/(c*x))^2 - 9*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^3/x + 9*a^2*
b*c^2*(-1/(c^2*x^2) + 1)^(3/2) - 2*b^3*c^2*(-1/(c^2*x^2) + 1)^(3/2) - 54*a*
b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x)) - 27*a*b^2*c*(1/(c^2*x^2) -
1)*arcsin(1/(c*x))^2/x - 9*b^3*c*arcsin(1/(c*x))^3/x - 27*a^2*b*c^2*sqrt(-1
/(c^2*x^2) + 1) + 42*b^3*c^2*sqrt(-1/(c^2*x^2) + 1) - 27*a^2*b*c*(1/(c^2*x^
2) - 1)*arcsin(1/(c*x))/x + 6*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 2
7*a*b^2*c*arcsin(1/(c*x))^2/x + 6*a*b^2*c*(1/(c^2*x^2) - 1)/x - 27*a^2*b*c*
arcsin(1/(c*x))/x + 42*b^3*c*arcsin(1/(c*x))/x + 42*a*b^2*c/x - 9*a^3/(c*x^
3))*c
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^3/x^4,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^3/x^4, x)
```

## 3.32 $\int \frac{(a+b\csc^{-1}(cx))^3}{x^5}\,dx$

**Optimal**. Leaf size=208

$$\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3}+\frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x}-\frac{45}{256}b^3c^4\csc^{-1}(cx)+\frac{3b^2(a+b\csc^{-1}(cx))}{32x^4}+\frac{9b^2c^2(a+b\csc^{-1}(cx))}{32x^2}-\cdots 3$$

```
[Out] -45/256*b^3*c^4*arccsc(c*x)+3/32*b^2*(a+b*arccsc(c*x))/x^4+9/32*b^2*c^2*(a+
b*arccsc(c*x))/x^2+3/32*c^4*(a+b*arccsc(c*x))^3-1/4*(a+b*arccsc(c*x))^3/x^4
+3/128*b^3*c*(1-1/c^2/x^2)^(1/2)/x^3+45/256*b^3*c^3*(1-1/c^2/x^2)^(1/2)/x-3
/16*b*c*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/x^3-9/32*b*c^3*(a+b*arccsc(
c*x))^2*(1-1/c^2/x^2)^(1/2)/x
```

**Rubi** [A]
time = 0.12, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5331, 4489, 3392, 32, 2715, 8}

$$\frac{9b^2c^2(a+b\csc^{-1}(cx))}{32x^2}+\frac{3b^2(a+b\csc^{-1}(cx))}{32x^4}+\frac{3}{32}c^4(a+b\csc^{-1}(cx))^3-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2}{16x^3}-\frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2}{32x}-\frac{(a+b\csc^{-1}(cx))^3}{4x^4}-\frac{45}{256}b^3c^4\csc^{-1}(cx)+\frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3}+\frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])^3/x^5,x]
```

```
[Out] (3*b^3*c*Sqrt[1 - 1/(c^2*x^2)])/(128*x^3) + (45*b^3*c^3*Sqrt[1 - 1/(c^2*x^2
)])/(256*x) - (45*b^3*c^4*ArcCsc[c*x])/256 + (3*b^2*(a + b*ArcCsc[c*x]))/(3
2*x^4) + (9*b^2*c^2*(a + b*ArcCsc[c*x]))/(32*x^2) - (3*b*c*Sqrt[1 - 1/(c^2*
x^2)]*(a + b*ArcCsc[c*x])^2)/(16*x^3) - (9*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*(a +
 b*ArcCsc[c*x])^2)/(32*x) + (3*c^4*(a + b*ArcCsc[c*x])^3)/32 - (a + b*ArcCs
c[c*x])^3/(4*x^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4489

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\csc^{-1}(cx))^3}{x^5}\,dx &= -\left(c^4\text{Subst}\left(\int (a+bx)^3\cos(x)\sin^3(x)\,dx, x, \csc^{-1}(cx)\right)\right)\\
&= -\frac{(a+b\csc^{-1}(cx))^3}{4x^4} + \frac{1}{4}(3bc^4)\,\text{Subst}\left(\int (a+bx)^2\sin^4(x)\,dx, x, \csc^{-1}(cx)\right)\\
&= \frac{3b^2(a+b\csc^{-1}(cx))}{32x^4} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2}{16x^3} - \frac{(a+b\csc^{-1}(cx))^3}{4x^4} +\\
&= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3} + \frac{3b^2(a+b\csc^{-1}(cx))}{32x^4} + \frac{9b^2c^2(a+b\csc^{-1}(cx))}{32x^2} - \frac{3bc\sqrt{1-\frac{}{c}}}{}\\
&= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3} + \frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x} + \frac{3b^2(a+b\csc^{-1}(cx))}{32x^4} + \frac{9b^2c^2(a+b\csc}{32x^2}\\
&= \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3} + \frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x} - \frac{45}{256}b^3c^4\csc^{-1}(cx) + \frac{3b^2(a+b\csc^{-1}(c}{32x^4}
\end{aligned}
$$

**Mathematica** [A]
time = 0.23, size = 283, normalized size = 1.36

$$
\frac{-64a^3+24ab^2-48a^2bc\sqrt{1-\frac{1}{c^2x^2}}\,x+6b^3c\sqrt{1-\frac{1}{c^2x^2}}\,x+72ab^2c^2x^2-72a^2bc^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3+45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3+24b\left(-8a^2+b^2(1+3c^2x^2)-2abc\sqrt{1-\frac{1}{c^2x^2}}\,x(2+3c^2x^2)\right)\csc^{-1}(cx)-24b^2\left(bc\sqrt{1-\frac{1}{c^2x^2}}\,x(2+3c^2x^2)+a(8-3c^4x^4)\right)\csc^{-1}(cx)^2+8b^3(-8+3c^4x^4)\csc^{-1}(cx)^3+9b(8a^2-5b^2)\,c^4x^4\,\text{ArcSin}\left(\frac{1}{cx}\right)}{256x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])^3/x^5,x]
```

```
[Out] (-64*a^3 + 24*a*b^2 - 48*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 6*b^3*c*Sqrt[1 -
 1/(c^2*x^2)]*x + 72*a*b^2*c^2*x^2 - 72*a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3
 + 45*b^3*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + 24*b*(-8*a^2 + b^2*(1 + 3*c^2*x^2
) - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2))*ArcCsc[c*x] - 24*b^2*(
b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2) + a*(8 - 3*c^4*x^4))*ArcCsc[c*x
]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcCsc[c*x]^3 + 9*b*(8*a^2 - 5*b^2)*c^4*x^4*Ar
cSin[1/(c*x)])/(256*x^4)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(182) = 364$.
time = 0.52, size = 485, normalized size = 2.33

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arccsc}(cx)^2\left(3\operatorname{arccsc}(cx)c^3x^3-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{ar}}{3}\right.\right.$ |
| default | $c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arccsc}(cx)^2\left(3\operatorname{arccsc}(cx)c^3x^3-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{ar}}{3}\right.\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(-1/4*a^3/c^4/x^4+b^3*(-1/4*arccsc(c*x)^3/c^4/x^4+3/32*arccsc(c*x)^2*(3
*arccsc(c*x)*c^3*x^3-3*c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-2*((c^2*x^2-1)/c
^2/x^2)^(1/2))/c^3/x^3+3/32*arccsc(c*x)/c^4/x^4+3/256*(3*c^2*x^2+2)/c^3/x^3
*((c^2*x^2-1)/c^2/x^2)^(1/2)+27/256*arccsc(c*x)-9/32*(c^2*x^2-1)/c^2/x^2*ar
ccsc(c*x)+9/64*((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x-3/16*arccsc(c*x)^3)+3*a*b^2*
(-1/4*arccsc(c*x)^2/c^4/x^4+1/16*arccsc(c*x)*(3*arccsc(c*x)*c^3*x^3-3*c^2*x
^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-2*((c^2*x^2-1)/c^2/x^2)^(1/2))/c^3/x^3-3/32*
arccsc(c*x)^2+1/128*(3*c^2*x^2+2)^2/c^4/x^4)-3/4*a^2*b*arccsc(c*x)/c^4/x^4+
9/32*a^2*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*arctan(1/(c^2*
x^2-1)^(1/2))-9/32*a^2*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^3/x^3-3/
16*a^2*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^5/x^5)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] -3/32*a^2*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^
2*x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) -
 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c + 8*arccsc(c*x)/x^4) - 1/4*a^3/
x^4 - 1/16*(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(2*(c^2*log(c*x + 1) +
 c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*a*b^2*c^2*log(c)^2 + 64*b^3*c^2*i
ntegrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^7 - x^5),
x)*log(c)^2 - 64*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*
x - 1)))*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 128*b^3*c^2*integrate(1/
16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^7 - x^5), x)*l
og(c) - 64*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*lo
g(c) + 128*a*b^2*c^2*integrate(1/16*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) -
```

```
 64*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(
c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 64*b^3*c^2*integrate(1/16*x^2*arctan(
1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^7 - x^5), x) - 64*a*b^2*c^
2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^7 - x
^5), x) + 16*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x -
1)))*log(c^2*x^2)/(c^2*x^7 - x^5), x) + 16*a*b^2*c^2*integrate(1/16*x^2*log
(c^2*x^2)^2/(c^2*x^7 - x^5), x) - 64*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x
^2)*log(x)/(c^2*x^7 - x^5), x) + 64*a*b^2*c^2*integrate(1/16*x^2*log(x)^2/(
c^2*x^7 - x^5), x) - (2*c^4*log(c*x + 1) + 2*c^4*log(c*x - 1) - 4*c^4*log(x
) + (2*c^2*x^2 + 1)/x^4)*a*b^2*log(c)^2 - 64*b^3*integrate(1/16*arctan(1/(s
qrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^7 - x^5), x)*log(c)^2 + 64*b^3*integrat
e(1/16*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^7 - x^5)
, x)*log(c) - 128*b^3*integrate(1/16*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))
)*log(x)/(c^2*x^7 - x^5), x)*log(c) + 64*a*b^2*integrate(1/16*log(c^2*x^2)/
(c^2*x^7 - x^5), x)*log(c) - 128*a*b^2*integrate(1/16*log(x)/(c^2*x^7 - x^5
), x)*log(c) + 16*b^3*integrate(1/16*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(
sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^7 - x^5), x) - 4*b^3*integrate(1/16*
sqrt(c*x + 1)*sqrt(c*x - 1)*log(c^2*x^2)^2/(c^2*x^7 - x^5), x) + 64*b^3*int
egrate(1/16*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^
2*x^7 - x^5), x) - 64*b^3*integrate(1/16*arctan(1/(sqrt(c*x + 1)*sqrt(c*x -
 1)))*log(x)^2/(c^2*x^7 - x^5), x) + 64*a*b^2*integrate(1/16*arctan(1/(sqrt
(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^7 - x^5), x) - 16*b^3*integrate(1/16*arc
tan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^7 - x^5), x) - 16*
a*b^2*integrate(1/16*log(c^2*x^2)^2/(c^2*x^7 - x^5), x) + 64*a*b^2*integrat
e(1/16*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) - 64*a*b^2*integrate(1/16*lo
g(x)^2/(c^2*x^7 - x^5), x))*x^4)/x^4
```

**Fricas [A]**
time = 0.42, size = 225, normalized size = 1.08

$$\frac{72\,ab^2c^2x^2 + 8\,(3\,b^3c^4x^4 - 8\,b^3)\operatorname{arccsc}(cx)^3 - 64\,a^3 + 24\,ab^2 + 24\,(3\,ab^2c^4x^4 - 8\,ab^2)\operatorname{arccsc}(cx)^2 + 3\,(3\,(8\,a^2b - 5\,b^3)c^4x^4 + 24\,b^3c^2x^2 - 64\,a^2b + 8\,b^3)\operatorname{arccsc}(cx) - 3\,\left(3\,(8\,a^2b - 5\,b^3)c^2x^2 + 16\,a^2b - 2\,b^3 + 8\,(3\,b^3c^2x^2 + 2\,b^3)\operatorname{arccsc}(cx)^2 + 16\,(3\,ab^2c^2x^2 + 2\,ab^2)\operatorname{arccsc}(cx)\right)\sqrt{c^2x^2 - 1}}{256\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/256*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*arccsc(c*x)^3 - 64*a^3
+ 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 - 8*a*b^2)*arccsc(c*x)^2 + 3*(3*(8*a^2*b -
 5*b^3)*c^4*x^4 + 24*b^3*c^2*x^2 - 64*a^2*b + 8*b^3)*arccsc(c*x) - 3*(3*(8*
a^2*b - 5*b^3)*c^2*x^2 + 16*a^2*b - 2*b^3 + 8*(3*b^3*c^2*x^2 + 2*b^3)*arccs
c(c*x)^2 + 16*(3*a*b^2*c^2*x^2 + 2*a*b^2)*arccsc(c*x))*sqrt(c^2*x^2 - 1))/x
^4
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsc}(cx))^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*acsc(c*x))**3/x**5, x)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(182) = 364.
time = 0.45, size = 576, normalized size = 2.77

$$-\frac{1}{256}\left(64\,b^3c^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)^3+192\,ab^2c^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)^2+128\,b^3c^3\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^3+192\,a^2bc^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)-24\,b^3c^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)+384\,ab^2c^3\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^2+40\,b^3c^3\arcsin\left(\frac{1}{cx}\right)^3-24\,ab^2c^3\left(\frac{1}{c^2x^2}-1\right)^2+384\,a^2bc^3\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)-120\,b^3c^3\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)+120\,ab^2c^3\arcsin\left(\frac{1}{cx}\right)^2-\frac{48\,b^3c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)^2}{x}-120\,ab^2c^3\left(\frac{1}{c^2x^2}-1\right)+120\,a^2bc^3\arcsin\left(\frac{1}{cx}\right)-51\,b^3c^3\arcsin\left(\frac{1}{cx}\right)-\frac{96\,ab^2c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{120\,b^3c^2\sqrt{-\frac{1}{c^2x^2}+1}\,\arcsin\left(\frac{1}{cx}\right)^2}{x}-51\,ab^2c^3-\frac{48\,a^2bc^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}{x}+\frac{6\,b^3c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}}{x}+\frac{240\,ab^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\,\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{120\,a^2bc^2\sqrt{-\frac{1}{c^2x^2}+1}}{x}-\frac{51\,b^3c^2\sqrt{-\frac{1}{c^2x^2}+1}}{x}+\frac{64\,a^3}{cx^4}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] -1/256*(64*b^3*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))^3 + 192*a*b^2*c^3*(1
/(c^2*x^2) - 1)^2*arcsin(1/(c*x))^2 + 128*b^3*c^3*(1/(c^2*x^2) - 1)*arcsin(
1/(c*x))^3 + 192*a^2*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x)) - 24*b^3*c^3
*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x)) + 384*a*b^2*c^3*(1/(c^2*x^2) - 1)*arcs
in(1/(c*x))^2 + 40*b^3*c^3*arcsin(1/(c*x))^3 - 24*a*b^2*c^3*(1/(c^2*x^2) -
1)^2 + 384*a^2*b*c^3*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) - 120*b^3*c^3*(1/(c^
2*x^2) - 1)*arcsin(1/(c*x)) + 120*a*b^2*c^3*arcsin(1/(c*x))^2 - 48*b^3*c^2*
(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x))^2/x - 120*a*b^2*c^3*(1/(c^2*x^2) -
 1) + 120*a^2*b*c^3*arcsin(1/(c*x)) - 51*b^3*c^3*arcsin(1/(c*x)) - 96*a*b^2
*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x))/x + 120*b^3*c^2*sqrt(-1/(c^2*
x^2) + 1)*arcsin(1/(c*x))^2/x - 51*a*b^2*c^3 - 48*a^2*b*c^2*(-1/(c^2*x^2) +
 1)^(3/2)/x + 6*b^3*c^2*(-1/(c^2*x^2) + 1)^(3/2)/x + 240*a*b^2*c^2*sqrt(-1/
(c^2*x^2) + 1)*arcsin(1/(c*x))/x + 120*a^2*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x -
 51*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 64*a^3/(c*x^4))*c
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))^3/x^5,x)
```

```
[Out] int((a + b*asin(1/(c*x)))^3/x^5, x)
```

# 3.33 $\int \frac{x}{a+b\csc^{-1}(cx)}\,dx$

**Optimal**. Leaf size=15

$$\mathrm{Int}\left(\frac{x}{a+b\csc^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x/(a+b*arccsc(c*x)),x)
```

**Rubi** [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/(a + b*ArcCsc[c*x]),x]
```

```
[Out] Defer[Int][x/(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx = \int \frac{x}{a+b\csc^{-1}(cx)}\,dx$$

**Mathematica** [A]
time = 2.65, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/(a + b*ArcCsc[c*x]),x]
```

```
[Out] Integrate[x/(a + b*ArcCsc[c*x]), x]
```

**Maple** [A]
time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\,\mathrm{arccsc}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccsc(c*x)),x)
[Out] int(x/(a+b*arccsc(c*x)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate(x/(b*arccsc(c*x) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(x/(b*arccsc(c*x) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \operatorname{acsc}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acsc(c*x)),x)
[Out] Integral(x/(a + b*acsc(c*x)), x)
```

**Giac** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asin(1/(c*x))),x)
[Out] int(x/(a + b*asin(1/(c*x))), x)
```

# 3.34 $\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$

**Optimal**. Leaf size=13

$$\text{Int}\left(\frac{1}{a+b\csc^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(1/(a+b*arccsc(c*x)),x)
```

**Rubi** [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])^(-1),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx=\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$$

**Mathematica** [A]
time = 2.41, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])^(-1),x]
```

```
[Out] Integrate[(a + b*ArcCsc[c*x])^(-1), x]
```

**Maple** [A]
time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{arccsc}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccsc(c*x)),x)
```

```
[Out] int(1/(a+b*arccsc(c*x)),x)
```

**Maxima** [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccsc(c*x) + a), x)
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccsc(c*x) + a), x)
```

**Sympy** [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{acsc}{\left(c x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acsc(c*x)),x)
```

```
[Out] Integral(1/(a + b*acsc(c*x)), x)
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccsc(c*x) + a), x)
```

**Mupad** [**A**]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asin(1/(c*x))),x)
[Out] int(1/(a + b*asin(1/(c*x))), x)
```

## 3.35 $\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$

**Optimal**. Leaf size=17

$$\mathrm{Int}\left(\frac{1}{x\left(a+b\csc^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arccsc(c*x)),x)
```

**Rubi** [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(a + b*ArcCsc[c*x])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*ArcCsc[c*x])), x]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx = \int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$$

**Mathematica** [A]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(a + b*ArcCsc[c*x])),x]
```

```
[Out] Integrate[1/(x*(a + b*ArcCsc[c*x])), x]
```

**Maple** [A]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arccsc}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccsc(c*x)),x)
[Out] int(1/x/(a+b*arccsc(c*x)),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccsc(c*x) + a)*x), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x*arccsc(c*x) + a*x), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\operatorname{acsc}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acsc(c*x)),x)
[Out] Integral(1/(x*(a + b*acsc(c*x))), x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arccsc(c*x) + a)*x), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asin(1/(c*x)))),x)
[Out] int(1/(x*(a + b*asin(1/(c*x)))), x)
```

# 3.36 $\int \frac{1}{x^2(a+b\csc^{-1}(cx))}\,dx$

**Optimal**. Leaf size=47

$$-\frac{c\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}-\frac{c\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}$$

```
[Out] -c*Ci(a/b+arccsc(c*x))*cos(a/b)/b-c*Si(a/b+arccsc(c*x))*sin(a/b)/b
```

**Rubi** [A]
time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5331, 3384, 3380, 3383}

$$-\frac{c\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}-\frac{c\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -((c*Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]])/b) - (c*Sin[a/b]*SinIntegral[
a/b + ArcCsc[c*x]])/b
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
```

```
0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx &= -\left(c\text{Subst}\left(\int \frac{\cos(x)}{a+bx}\,dx,x,\csc^{-1}(cx)\right)\right)\\ &= -\left(\left(c\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\csc^{-1}(cx)\right)\right)-\left(c\sin\left(\frac{a}{b}\right)\right)\text{Sub}\\ &= -\frac{c\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}-\frac{c\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}\end{aligned}$$

**Mathematica** [**A**]
time = 0.06, size = 43, normalized size = 0.91

$$-\frac{c\left(\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -((c*(Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]] + Sin[a/b]*SinIntegral[a/b +
ArcCsc[c*x]]))/b)
```

**Maple** [**A**]
time = 0.24, size = 48, normalized size = 1.02

| method | result | size |
|---|---|---|
| derivativedivides | $c\left(-\frac{\text{sinIntegral}\left(\frac{a}{b}+\text{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{\text{cosineIntegral}\left(\frac{a}{b}+\text{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}\right)$ | 48 |
| default | $c\left(-\frac{\text{sinIntegral}\left(\frac{a}{b}+\text{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{\text{cosineIntegral}\left(\frac{a}{b}+\text{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}\right)$ | 48 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-Si(a/b+arccsc(c*x))*sin(a/b)/b-Ci(a/b+arccsc(c*x))*cos(a/b)/b)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccsc(c*x) + a)*x^2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^2*arccsc(c*x) + a*x^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{acsc}(cx))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acsc(c*x)),x)
```

```
[Out] Integral(1/(x**2*(a + b*acsc(c*x))), x)
```

**Giac [A]**
time = 0.56, size = 54, normalized size = 1.15

$$-c\left(\frac{\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b} + \arcsin\left(\frac{1}{cx}\right)\right)}{b} + \frac{\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b} + \arcsin\left(\frac{1}{cx}\right)\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] -c*(cos(a/b)*cos_integral(a/b + arcsin(1/(c*x)))/b + sin(a/b)*sin_integral(
a/b + arcsin(1/(c*x)))/b)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asin(1/(c*x)))),x)
```

```
[Out] int(1/(x^2*(a + b*asin(1/(c*x)))), x)
```

# 3.37 $\int \frac{1}{x^3(a+b\csc^{-1}(cx))}\,dx$

**Optimal**. Leaf size=63

$$\frac{c^2\text{CosIntegral}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}-\frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}$$

```
[Out] -1/2*c^2*cos(2*a/b)*Si(2*a/b+2*arccsc(c*x))/b+1/2*c^2*Ci(2*a/b+2*arccsc(c*x
))*sin(2*a/b)/b
```

**Rubi** [A]
time = 0.11, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5331, 4491, 12, 3384, 3380, 3383}

$$\frac{c^2\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}-\frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*ArcCsc[c*x])),x]
```

```
[Out] (c^2*CosIntegral[(2*a)/b + 2*ArcCsc[c*x]]*Sin[(2*a)/b])/(2*b) - (c^2*Cos[(2
*a)/b]*SinIntegral[(2*a)/b + 2*ArcCsc[c*x]])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+b\csc^{-1}(cx)\right)}\,dx &= -\left(c^2\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{a+bx}\,dx,x,\csc^{-1}(cx)\right)\right)\\ &= -\left(c^2\text{Subst}\left(\int \frac{\sin(2x)}{2(a+bx)}\,dx,x,\csc^{-1}(cx)\right)\right)\\ &= -\left(\frac{1}{2}c^2\text{Subst}\left(\int \frac{\sin(2x)}{a+bx}\,dx,x,\csc^{-1}(cx)\right)\right)\\ &= -\left(\frac{1}{2}\left(c^2\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\csc^{-1}(cx)\right)\right)+\frac{1}{2}\left(c^2\sin\left(\right.\right.\\ &= \frac{c^2\text{Ci}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}-\frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}\end{aligned}$$

**Mathematica** [**A**]
time = 0.06, size = 56, normalized size = 0.89

$$-\frac{c^2\left(-\text{CosIntegral}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)+\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -1/2*(c^2*(-(CosIntegral[(2*a)/b + 2*ArcCsc[c*x]]*Sin[(2*a)/b]) + Cos[(2*a)
/b]*SinIntegral[(2*a)/b + 2*ArcCsc[c*x]]))/b
```

**Maple** [**A**]
time = 0.20, size = 58, normalized size = 0.92

| method | result | size |
|---|---|---|
| derivativedivides | $c^{2}\left(-\frac{\operatorname{sinIntegral}\left(\frac{2a}{b}+2\,\mathrm{arccsc}(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b}+\frac{\operatorname{cosineIntegral}\left(\frac{2a}{b}+2\,\mathrm{arccsc}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$ | 58 |
| default | $c^{2}\left(-\frac{\operatorname{sinIntegral}\left(\frac{2a}{b}+2\,\mathrm{arccsc}(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b}+\frac{\operatorname{cosineIntegral}\left(\frac{2a}{b}+2\,\mathrm{arccsc}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$ | 58 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
[Out] c^2*(-1/2*Si(2*a/b+2*arccsc(c*x))*cos(2*a/b)/b+1/2*Ci(2*a/b+2*arccsc(c*x))*
sin(2*a/b)/b)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccsc(c*x) + a)*x^3), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^3*arccsc(c*x) + a*x^3), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(a+b\operatorname{acsc}(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*acsc(c*x)),x)
[Out] Integral(1/(x**3*(a + b*acsc(c*x))), x)
```

**Giac [A]**
time = 0.42, size = 95, normalized size = 1.51

$$\frac{1}{2}\left(\frac{2\,c\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arcsin\left(\frac{1}{cx}\right)\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{2\,c\cos\left(\frac{a}{b}\right)^{2}\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arcsin\left(\frac{1}{cx}\right)\right)}{b}+\frac{c\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arcsin\left(\frac{1}{cx}\right)\right)}{b}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*(2*c*cos(a/b)*cos_integral(2*a/b + 2*arcsin(1/(c*x)))*sin(a/b)/b - 2*c*
cos(a/b)^2*sin_integral(2*a/b + 2*arcsin(1/(c*x)))/b + c*sin_integral(2*a/b
 + 2*arcsin(1/(c*x)))/b)*c
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*asin(1/(c*x)))),x)
```

```
[Out] int(1/(x^3*(a + b*asin(1/(c*x)))), x)
```

# 3.38 $\int \frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx$

**Optimal**. Leaf size=117

$$-\frac{c^3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b}+\frac{c^3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b}-\frac{c^3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}\right.}{4b}$$

```
[Out] -1/4*c^3*Ci(a/b+arccsc(c*x))*cos(a/b)/b+1/4*c^3*Ci(3*a/b+3*arccsc(c*x))*cos
(3*a/b)/b-1/4*c^3*Si(a/b+arccsc(c*x))*sin(a/b)/b+1/4*c^3*Si(3*a/b+3*arccsc(
c*x))*sin(3*a/b)/b
```

**Rubi** [A]
time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5331, 4491, 3384, 3380, 3383}

$$-\frac{c^3\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b}+\frac{c^3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b}-\frac{c^3\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b}+\frac{c^3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -1/4*(c^3*Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]])/b + (c^3*Cos[(3*a)/b]*Co
sIntegral[(3*a)/b + 3*ArcCsc[c*x]])/(4*b) - (c^3*Sin[a/b]*SinIntegral[a/b +
 ArcCsc[c*x]])/(4*b) + (c^3*Sin[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcCsc[c*x
]])/(4*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 5331

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx &= -\left(c^3\text{Subst}\left(\int \frac{\cos(x)\sin^2(x)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right)\\ &= -\left(c^3\text{Subst}\left(\int \left(\frac{\cos(x)}{4(a+bx)}-\frac{\cos(3x)}{4(a+bx)}\right)\,dx, x, \csc^{-1}(cx)\right)\right)\\ &= -\left(\frac{1}{4}c^3\text{Subst}\left(\int \frac{\cos(x)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right)+\frac{1}{4}c^3\text{Subst}\left(\int \frac{\cos(3x)}{a+bx}\,dx, x, \csc^{-}\right.\\ &= -\left(\frac{1}{4}\left(c^3\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \csc^{-1}(cx)\right)\right)+\frac{1}{4}\left(c^3\cos\left(\frac{3a}{b}\right)\right.\\ &= -\frac{c^3\cos\left(\frac{a}{b}\right)\text{Ci}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b}+\frac{c^3\cos\left(\frac{3a}{b}\right)\text{Ci}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b}-\frac{c^3\sin\left(\frac{a}{b}\right)\text{S}}{}\end{aligned}$$

**Mathematica [A]**
time = 0.13, size = 91, normalized size = 0.78

$$-\frac{c^3\left(\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)-\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\csc^{-1}(cx)\right)\right)+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)-\sin\left(\frac{3a}{b}\right)\text{Si}\left(3\left(\frac{a}{b}+\csc^{-1}(cx)\right)\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*ArcCsc[c*x])),x]
```

```
[Out] -1/4*(c^3*(Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]] - Cos[(3*a)/b]*CosIntegr
al[3*(a/b + ArcCsc[c*x])] + Sin[a/b]*SinIntegral[a/b + ArcCsc[c*x]] - Sin[(
3*a)/b]*SinIntegral[3*(a/b + ArcCsc[c*x])]))/b
```

**Maple [A]**
time = 0.21, size = 102, normalized size = 0.87

| method | result |
|---|---|
| derivativedivides | $c^3\left(-\frac{\text{sinIntegral}\left(\frac{a}{b}+\text{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{4b}-\frac{\text{cosineIntegral}\left(\frac{a}{b}+\text{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b}+\frac{\text{sinIntegral}\left(\frac{3a}{b}+3\,\text{arccsc}(cx)\right)\text{s}}{4b}\right.$ |
| default | $c^3\left(-\frac{\text{sinIntegral}\left(\frac{a}{b}+\text{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{4b}-\frac{\text{cosineIntegral}\left(\frac{a}{b}+\text{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b}+\frac{\text{sinIntegral}\left(\frac{3a}{b}+3\,\text{arccsc}(cx)\right)\text{s}}{4b}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/4*Si(a/b+arccsc(c*x))*sin(a/b)/b-1/4*Ci(a/b+arccsc(c*x))*cos(a/b)/b
+1/4*Si(3*a/b+3*arccsc(c*x))*sin(3*a/b)/b+1/4*Ci(3*a/b+3*arccsc(c*x))*cos(3
*a/b)/b)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccsc(c*x) + a)*x^4), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^4*arccsc(c*x) + a*x^4), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+b\,\text{acsc}\,(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*acsc(c*x)),x)
```

```
[Out] Integral(1/(x**4*(a + b*acsc(c*x))), x)
```

**Giac [A]**
time = 0.42, size = 200, normalized size = 1.71

$$\frac{1}{4}\left(\frac{4c^2\cos\left(\frac{a}{b}\right)^3\operatorname{Ci}\left(\frac{3a}{b}+3\arcsin\left(\frac{1}{cx}\right)\right)}{b}+\frac{4c^2\cos\left(\frac{a}{b}\right)^2\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3a}{b}+3\arcsin\left(\frac{1}{cx}\right)\right)}{b}-\frac{3c^2\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{3a}{b}+3\arcsin\left(\frac{1}{cx}\right)\right)}{b}-\frac{c^2\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b}+\arcsin\left(\frac{1}{cx}\right)\right)}{b}-\frac{c^2\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3a}{b}+3\arcsin\left(\frac{1}{cx}\right)\right)}{b}-\frac{c^2\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arcsin\left(\frac{1}{cx}\right)\right)}{b}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*(4*c^2*cos(a/b)^3*cos_integral(3*a/b + 3*arcsin(1/(c*x)))/b + 4*c^2*cos
(a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arcsin(1/(c*x)))/b - 3*c^2*cos(a/b)
*cos_integral(3*a/b + 3*arcsin(1/(c*x)))/b - c^2*cos(a/b)*cos_integral(a/b
+ arcsin(1/(c*x)))/b - c^2*sin(a/b)*sin_integral(3*a/b + 3*arcsin(1/(c*x)))
/b - c^2*sin(a/b)*sin_integral(a/b + arcsin(1/(c*x)))/b)*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*asin(1/(c*x)))),x)
```

```
[Out] int(1/(x^4*(a + b*asin(1/(c*x)))), x)
```

# 3.39 $\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3\,dx$

**Optimal**. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\csc^{-1}(cx)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arccsc(c*x))^3,x)
```

**Rubi** [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcCsc[c*x])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3\,dx=\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3\,dx$$

**Mathematica** [A]
time = 4.88, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^3, x]
```

**Maple** [A]
time = 1.76, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\mathrm{arccsc}(cx)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccsc(c*x))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arccsc(c*x))^3,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/4*(4*b^3*d^m*x*x^m*arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))^3 - 3*b^3*d^m*x*x^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
))*log(c^2*x^2)^2 - 4*(m + 1)*integrate(-3/4*((a*b^2*d^m*m + a*b^2*d^m - (a
*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*log(c^2*x^2)^2 + 4*((b^3*arctan2(1
, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*d^m*m - ((b^3*arctan2(1, sqrt(c*x +
 1)*sqrt(c*x - 1)) + a*b^2)*c^2*d^m*m + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1)) + a*b^2)*c^2*d^m)*x^2 + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1)) + a*b^2)*d^m)*x^m*log(x)^2 + 8*((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1)) + a*b^2)*d^m*m*log(c) - ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
 + a*b^2)*c^2*d^m*m*log(c) + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
 a*b^2)*c^2*d^m*log(c))*x^2 + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
+ a*b^2)*d^m*log(c))*x^m*log(x) + (4*b^3*d^m*x^m*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1))^2 - b^3*d^m*x^m*log(c^2*x^2)^2)*sqrt(c*x + 1)*sqrt(c*x - 1) -
 4*((a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + a^2*b*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1)) - (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
a*b^2)*log(c)^2)*d^m*m + (((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a
*b^2)*c^2*log(c)^2 - (a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + a^2
*b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*c^2)*d^m*m + ((b^3*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*c^2*log(c)^2 - (a*b^2*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1))^2 + a^2*b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*
c^2)*d^m)*x^2 + (a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + a^2*b*ar
ctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1)) + a*b^2)*log(c)^2)*d^m)*x^m - 4*(((b^3*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)) + a*b^2)*d^m*m - ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)
) + a*b^2)*c^2*d^m*m + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2
)*c^2*d^m)*x^2 + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*d^m)
*x^m*log(x) + ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*d^m*m*
log(c) - ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*c^2*d^m*m*l
og(c) + (b^3*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + (b^3*arctan2(1,
sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*c^2*log(c))*d^m)*x^2 + (b^3*arctan2(1
, sqrt(c*x + 1)*sqrt(c*x - 1)) + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
)) + a*b^2)*log(c))*d^m)*x^m)*log(c^2*x^2))/((c^2*m + c^2)*x^2 - m - 1), x)
)/(m + 1)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x) +
 a^3)*(d*x)^m, x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b\operatorname{acsc}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsc(c*x))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*acsc(c*x))**3, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^3*(d*x)^m, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*asin(1/(c*x)))^3,x)
```

```
[Out] int((d*x)^m*(a + b*asin(1/(c*x)))^3, x)
```

# 3.40 $\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx$

**Optimal**. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\csc^{-1}(cx)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arccsc(c*x))^2,x)
```

**Rubi** [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcCsc[c*x])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx$$

**Mathematica** [A]
time = 3.20, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^2, x]
```

**Maple** [A]
time = 1.74, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\mathrm{arccsc}(cx)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccsc(c*x))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arccsc(c*x))^2,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/4*(4*b^2*d^m*x*x^m*arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))^2 - b^2*d^m*x*x^m*log(c^2*x^2)^2 + 4*(m + 1)*integrate((2
*sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*d^m*x^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1)) + (b^2*d^m*m + b^2*d^m - (b^2*c^2*d^m*m + b^2*c^2*d^m)*x^2)*x^m*log(x
)^2 + 2*(b^2*d^m*m*log(c) + b^2*d^m*log(c) - (b^2*c^2*d^m*m*log(c) + b^2*c^
2*d^m*log(c))*x^2)*x^m*log(x) + ((b^2*log(c)^2 - 2*a*b*arctan2(1, sqrt(c*x
+ 1)*sqrt(c*x - 1)))*d^m*m - ((b^2*c^2*log(c)^2 - 2*a*b*c^2*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1)))*d^m*m + (b^2*c^2*log(c)^2 - 2*a*b*c^2*arctan2(1,
sqrt(c*x + 1)*sqrt(c*x - 1)))*d^m)*x^2 + (b^2*log(c)^2 - 2*a*b*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1)))*d^m)*x^m - ((b^2*d^m*m + b^2*d^m - (b^2*c^2*d^
m*m + b^2*c^2*d^m)*x^2)*x^m*log(x) + (b^2*d^m*m*log(c) - (b^2*c^2*d^m*m*log
(c) + (b^2*c^2*log(c) + b^2*c^2)*d^m)*x^2 + (b^2*log(c) + b^2)*d^m)*x^m)*lo
g(c^2*x^2))/((c^2*m + c^2)*x^2 - m - 1), x))/(m + 1)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2)*(d*x)^m, x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b \operatorname{acsc}(cx)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsc(c*x))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*acsc(c*x))**2, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)^2*(d*x)^m, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int((d*x)^m*(a + b*asin(1/(c*x)))^2, x)
```

# 3.41 $\int (dx)^m \left(a+b\csc^{-1}(cx)\right) \, dx$

**Optimal**. Leaf size=66

$$\frac{(dx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{d(1+m)}+\frac{b(dx)^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{c^2x^2}\right)}{cm(1+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arccsc(c*x))/d/(1+m)+b*(d*x)^m*hypergeom([1/2, -1/2*m],[1-
1/2*m],1/c^2/x^2)/c/m/(1+m)
```

**Rubi** [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5329, 346, 371}

$$\frac{(dx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{d(m+1)}+\frac{b(dx)^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{c^2x^2}\right)}{cm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(d*(1 + m)) + (b*(d*x)^m*Hypergeometric
2F1[1/2, -1/2*m, 1 - m/2, 1/(c^2*x^2)])/(c*m*(1 + m))
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5329

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m + 1
))), Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{(dx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{d(1+m)} + \frac{(bd)\int \frac{(dx)^{-1+m}}{\sqrt{1-\frac{1}{c^2x^2}}}\,dx}{c(1+m)}$$

$$= \frac{(dx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{d(1+m)} - \frac{\left(b\left(\frac{1}{x}\right)^m (dx)^m\right)\text{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{c(1+m)}$$

$$= \frac{(dx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{d(1+m)} + \frac{b(dx)^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{c^2x^2}\right)}{cm(1+m)}$$

**Mathematica** [**A**]
time = 0.11, size = 83, normalized size = 1.26

$$\frac{(dx)^m\left((1+m)x(a+b\csc^{-1}(cx)) + \frac{b\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{c\sqrt{1-\frac{1}{c^2x^2}}}\right)}{(1+m)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((d*x)^m*((1 + m)*x*(a + b*ArcCsc[c*x]) + (b*Sqrt[1 - c^2*x^2]*Hypergeometr
ic2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(c*Sqrt[1 - 1/(c^2*x^2)])))/(1 +
 m)^2
```

**Maple** [**F**]
time = 1.83, size = 0, normalized size = 0.00

$$\int (dx)^m\,(a+b\,\mathrm{arccsc}(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccsc(c*x)),x)
```

```
[Out] int((d*x)^m*(a+b*arccsc(c*x)),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] (d^m*x*x^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + (c^2*d^m*m + c^2*d^m)*
integrate(-sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(c^2*m - (c^4*m + c^4)*x^2 + c^2
), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*(d*x)^m, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b\operatorname{acsc}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsc(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*acsc(c*x)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(d*x)^m, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\,x)^m \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*asin(1/(c*x))),x)
```

```
[Out] int((d*x)^m*(a + b*asin(1/(c*x))), x)
```

# 3.42 $\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$

**Optimal**. Leaf size=19

$$\operatorname{Int}\left(\frac{(dx)^m}{a+b\csc^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arccsc(c*x)),x)
```

**Rubi** [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcCsc[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$$

**Mathematica** [A]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcCsc[c*x]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcCsc[c*x]), x]
```

**Maple** [A]
time = 2.69, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\,\mathrm{arccsc}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arccsc(c*x)),x)
[Out] int((d*x)^m/(a+b*arccsc(c*x)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arccsc(c*x) + a), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arccsc(c*x) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\operatorname{acsc}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*acsc(c*x)),x)
[Out] Integral((d*x)**m/(a + b*acsc(c*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arccsc(c*x) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\operatorname{asin}\left(\frac{1}{c\,x}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*asin(1/(c*x))),x)
[Out] int((d*x)^m/(a + b*asin(1/(c*x))), x)
```

# 3.43 $\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$

**Optimal**. Leaf size=19

$$\text{Int}\left(\frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arccsc(c*x))^2,x)
```

**Rubi** [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcCsc[c*x])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$$

**Mathematica** [A]
time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcCsc[c*x])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcCsc[c*x])^2, x]
```

**Maple** [A]
time = 1.41, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\,\mathrm{arccsc}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arccsc(c*x))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arccsc(c*x))^2,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

```
[Out] (4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
 a)*d^m*x*x^m - (4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log(
c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b^
2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*l
og(x))*log(c^2*x^2))*integrate(4*((b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)
) + a)*d^m*m - ((b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a)*c^2*d^m*m +
 2*(b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a)*c^2*d^m)*x^2 + (b*arctan
2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a)*d^m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m
/(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 4*b^3*log(c)^2 + 8*a*b^
2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*c^2*log(c)^2 +
 (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*a*b^2*arctan2(1, sqrt(c
*x + 1)*sqrt(c*x - 1)) + a^2*b)*c^2)*x^2 - (b^3*c^2*x^2 - b^3)*log(c^2*x^2)
^2 - 4*(b^3*c^2*x^2 - b^3)*log(x)^2 + 4*(b^3*c^2*x^2*log(c) - b^3*log(c) +
(b^3*c^2*x^2 - b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log(c) - b^3*log(
c))*log(x)), x))/(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*log
(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a*b
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b^3*
log(x))*log(c^2*x^2))
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b\,\mathrm{acsc}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*acsc(c*x))**2,x)
```

```
[Out] Integral((d*x)**m/(a + b*acsc(c*x))**2, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arccsc(c*x) + a)^2, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*asin(1/(c*x)))^2,x)
```

```
[Out] int((d*x)^m/(a + b*asin(1/(c*x)))^2, x)
```

# 3.44 $\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right) \, dx$

**Optimal**. Leaf size=167

$$\frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}\;x}{6c^3}+\frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\;x^2}{2c}+\frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\;x^3}{12c}-\frac{bd^4\csc^{-1}(cx)}{4e}+\frac{(d+ex)^4\left(a+b\csc^{-1}\right.}{4e}$$

```
[Out] -1/4*b*d^4*arccsc(c*x)/e+1/4*(e*x+d)^4*(a+b*arccsc(c*x))/e+1/2*b*d*(2*c^2*d
^2+e^2)*arctanh((1-1/c^2/x^2)^(1/2))/c^3+1/6*b*e*(9*c^2*d^2+e^2)*x*(1-1/c^2
/x^2)^(1/2)/c^3+1/2*b*d*e^2*x^2*(1-1/c^2/x^2)^(1/2)/c+1/12*b*e^3*x^3*(1-1/c
^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.30, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5335, 1582, 1489, 1821, 858, 222, 272, 65, 214}

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{bde^2x^2\sqrt{1-\frac{1}{c^2x^2}}}{2c}+\frac{be^3x^3\sqrt{1-\frac{1}{c^2x^2}}}{12c}+\frac{bex\sqrt{1-\frac{1}{c^2x^2}}\,(9c^2d^2+e^2)}{6c^3}+\frac{bd(2c^2d^2+e^2)\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{2c^3}-\frac{bd^4\csc^{-1}(cx)}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*(9*c^2*d^2 + e^2)*Sqrt[1 - 1/(c^2*x^2)]*x)/(6*c^3) + (b*d*e^2*Sqrt[1 -
 1/(c^2*x^2)]*x^2)/(2*c) + (b*e^3*Sqrt[1 - 1/(c^2*x^2)]*x^3)/(12*c) - (b*d^
4*ArcCsc[c*x])/(4*e) + ((d + e*x)^4*(a + b*ArcCsc[c*x]))/(4*e) + (b*d*(2*c^
2*d^2 + e^2)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(2*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right) dx = \frac{(d+ex)^4 \left(a+b\csc^{-1}(cx)\right)}{4e} + \frac{b\int \frac{(d+ex)^4}{\sqrt{1-\frac{1}{c^2x^2}}\, x^2}dx}{4ce}$$

$$= \frac{(d+ex)^4 \left(a+b\csc^{-1}(cx)\right)}{4e} + \frac{b\int \frac{\left(e+\frac{d}{x}\right)^4 x^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{4ce}$$

$$= \frac{(d+ex)^4 \left(a+b\csc^{-1}(cx)\right)}{4e} - \frac{b\text{Subst}\left(\int \frac{(e+dx)^4}{x^4\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{4ce}$$

$$= \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\, x^3}{12c} + \frac{(d+ex)^4 \left(a+b\csc^{-1}(cx)\right)}{4e} + \frac{b\text{Subst}\left(\int \frac{-12de^3-2e^2(\cdots}{x\cdots}\right.}{\cdots}$$

$$= \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\, x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\, x^3}{12c} + \frac{(d+ex)^4 \left(a+b\csc^{-1}(cx)\right)}{4e} - \cdots$$

$$= \frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}\, x}{6c^3} + \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\, x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\, x^3}{12c} \cdots$$

$$= \frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}\, x}{6c^3} + \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\, x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\, x^3}{12c} \cdots$$

$$= \frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}\, x}{6c^3} + \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\, x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\, x^3}{12c} \cdots$$

$$= \frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}\, x}{6c^3} + \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\, x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\, x^3}{12c} \cdots$$

$$= \frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}\, x}{6c^3} + \frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\, x^2}{2c} + \frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\, x^3}{12c} \cdots$$

**Mathematica [A]**
time = 0.19, size = 165, normalized size = 0.99

$$\frac{3ac^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)+be\sqrt{1-\frac{1}{c^2x^2}}\,x(2e^2+c^2(18d^2+6dex+e^2x^2))+3bc^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)\csc^{-1}(cx)+6bd(2c^2d^2+e^2)\log\left(\left(1+\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (3*a*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*e*Sqrt[1 - 1/(c^
2*x^2)]*x*(2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)) + 3*b*c^3*x*(4*d^3 + 6
*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcCsc[c*x] + 6*b*d*(2*c^2*d^2 + e^2)*Log
[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(12*c^3)
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(147) = 294$.
time = 0.23, size = 413, normalized size = 2.47

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{(ecx+cd)^4 a}{4c^3 e}+\frac{bc\,\mathrm{arccsc}(cx)d^4}{4e}+b\,\mathrm{arccsc}(cx)d^3cx+\frac{3bce\,\mathrm{arccsc}(cx)d^2x^2}{2}+bc\,e^2\mathrm{arccsc}(cx)d\,x^3+\frac{bc\,e^3\mathrm{arccsc}(cx)x^4}{4}-\frac{b\sqrt{c^2x^2-\cdots}}{\cdots}}{\cdots}$ |
| default | $\frac{\frac{(ecx+cd)^4 a}{4c^3 e}+\frac{bc\,\mathrm{arccsc}(cx)d^4}{4e}+b\,\mathrm{arccsc}(cx)d^3cx+\frac{3bce\,\mathrm{arccsc}(cx)d^2x^2}{2}+bc\,e^2\mathrm{arccsc}(cx)d\,x^3+\frac{bc\,e^3\mathrm{arccsc}(cx)x^4}{4}-\frac{b\sqrt{c^2x^2-\cdots}}{\cdots}}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/4*(c*e*x+c*d)^4*a/c^3/e+1/4*b*c/e*arccsc(c*x)*d^4+b*arccsc(c*x)*d^3*
c*x+3/2*b*c*e*arccsc(c*x)*d^2*x^2+b*c*e^2*arccsc(c*x)*d*x^3+1/4*b*c*e^3*arc
csc(c*x)*x^4-1/4*b/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^4*ar
ctan(1/(c^2*x^2-1)^(1/2))+b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c
/x*d^3*ln(c*x+(c^2*x^2-1)^(1/2))+3/2*b/c^2*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x
^2)^(1/2)/x*d^2+1/2*b/c^2*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d+1/1
2*b/c^2*e^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x+1/2*b/c^3*e^2*(c^2*x^
2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x+(c^2*x^2-1)^(1/2))+1/6*b/
c^4*e^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

**Maxima [A]**
time = 0.28, size = 267, normalized size = 1.60

$$\frac{1}{4}ax^4e^3+adx^3e^2+\frac{3}{2}ad^2x^2e+ad^3x+\frac{3}{2}\left(x^2\,\mathrm{arccsc}(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2e+\frac{\left(2\,cx\,\mathrm{arccsc}(cx)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^3}{2\,c}+\frac{1}{4}\left(4\,x^3\,\mathrm{arccsc}(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde^2+\frac{1}{12}\left(3\,x^4\,\mathrm{arccsc}(cx)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e^3 + a*d*x^3*e^2 + 3/2*a*d^2*x^2*e + a*d^3*x + 3/2*(x^2*arccsc(c
*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)*b*d^2*e + 1/2*(2*c*x*arccsc(c*x) + log(sq
rt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^3/c + 1/4
*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^
2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/
c^2)/c)*b*d*e^2 + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/
2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*e^3
```

**Fricas** [A]
time = 0.47, size = 283, normalized size = 1.69

$$\frac{3\,ac^4x^4e^3+12\,ac^4dx^3e^2+18\,ac^4d^2x^2e+12\,ac^4d^3x+3\,(4\,bc^4d^3x-4\,bc^4d^3+(bc^4x^4-bc^4)e^3+4\,(bc^4dx^3-bc^4d)e^2+6\,(bc^4d^2x^2-bc^4d^2)e)\operatorname{arccsc}(cx)-6\,(4\,bc^4d^3+6\,bc^4d^2e+4\,bc^4de^2+bc^4e^3)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-6\,(2\,bc^3d^3+bcde^2)\log\left(-cx+\sqrt{c^2x^2-1}\right)+(6\,bc^2dxe^2+18\,bc^2d^2e+(bc^2x^2+2\,b)e^3)\sqrt{c^2x^2-1}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4*e^3 + 12*a*c^4*d*x^3*e^2 + 18*a*c^4*d^2*x^2*e + 12*a*c^4*
d^3*x + 3*(4*b*c^4*d^3*x - 4*b*c^4*d^3 + (b*c^4*x^4 - b*c^4)*e^3 + 4*(b*c^4
*d*x^3 - b*c^4*d)*e^2 + 6*(b*c^4*d^2*x^2 - b*c^4*d^2)*e)*arccsc(c*x) - 6*(4
*b*c^4*d^3 + 6*b*c^4*d^2*e + 4*b*c^4*d*e^2 + b*c^4*e^3)*arctan(-c*x + sqrt(
c^2*x^2 - 1)) - 6*(2*b*c^3*d^3 + b*c*d*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) +
 (6*b*c^2*d*x*e^2 + 18*b*c^2*d^2*e + (b*c^2*x^2 + 2*b)*e^3)*sqrt(c^2*x^2 -
1))/c^4
```

**Sympy** [A]
time = 5.99, size = 362, normalized size = 2.17

$$ad^3x+\frac{3ad^2ex^2}{2}+ade^2x^3+\frac{ae^3x^4}{4}+bd^3x\operatorname{acsc}(cx)+\frac{3bd^2ex^2\operatorname{acsc}(cx)}{2}+bde^2x^3\operatorname{acsc}(cx)+\frac{be^3x^4\operatorname{acsc}(cx)}{4}+\frac{bd^3\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}+\frac{3bd^2e\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}+\frac{bde^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{c}+\frac{be^3\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b*d**3*x*acs
c(c*x) + 3*b*d**2*e*x**2*acsc(c*x)/2 + b*d*e**2*x**3*acsc(c*x) + b*e**3*x**
4*acsc(c*x)/4 + b*d**3*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin
(c*x), True))/c + 3*b*d**2*e*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**
2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c) + b*d*e**2*Piecewise((x*sq
rt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x
**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)
/(2*c**2), True))/c + b*e**3*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*
sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2
+ 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. $2(147) = 294$.
time = 2.53, size = 1130, normalized size = 6.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b*e^3*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a*e
^3*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 24*b*d*e^2*x^3*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^3*arcsin(1/(c*x))/c + 24*a*d*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1
)^3/c + 2*b*e^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 72*b*d^2*e*x^2*(sq
rt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + 72*a*d^2*e*x^2*(sqrt(-1/(c^
2*x^2) + 1) + 1)^2/c + 24*b*d*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 +
96*b*d^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c + 12*b*e^3*x^2*(s
qrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 96*a*d^3*x*(sqrt(-1/(c^2
*x^2) + 1) + 1)/c + 12*a*e^3*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 144*b
*d^2*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 72*b*d*e^2*x*(sqrt(-1/(c^2*x^2)
 + 1) + 1)*arcsin(1/(c*x))/c^3 + 72*a*d*e^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/
c^3 + 192*b*d^3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - 192*b*d^3*log(1/(abs(
c)*abs(x)))/c^2 + 18*b*e^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 144*b*d^2*e
*arcsin(1/(c*x))/c^3 + 144*a*d^2*e/c^3 + 96*b*d*e^2*log(sqrt(-1/(c^2*x^2) +
 1) + 1)/c^4 - 96*b*d*e^2*log(1/(abs(c)*abs(x)))/c^4 + 18*b*e^3*arcsin(1/(c
*x))/c^5 + 96*b*d^3*arcsin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) +
18*a*e^3/c^5 + 96*a*d^3/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 144*b*d^2*e/
(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 72*b*d*e^2*arcsin(1/(c*x))/(c^5*x*(s
qrt(-1/(c^2*x^2) + 1) + 1)) + 72*a*d*e^2/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1
)) - 18*b*e^3/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 72*b*d^2*e*arcsin(1/(c
*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 72*a*d^2*e/(c^5*x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2) - 24*b*d*e^2/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)
^2) + 12*b*e^3*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 1
2*a*e^3/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 24*b*d*e^2*arcsin(1/(c*x
))/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 24*a*d*e^2/(c^7*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3) - 2*b*e^3/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) +
 3*b*e^3*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 3*a*e^3
/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x)^3,x)
[Out] int((a + b*asin(1/(c*x)))*(d + e*x)^3, x)
```

# 3.45 $\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right) dx$

**Optimal**. Leaf size=123

$$\frac{bde\sqrt{1-\frac{1}{c^2x^2}}\,x}{c}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c}-\frac{bd^3\csc^{-1}(cx)}{3e}+\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{b(6c^2d^2+e^2)\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

```
[Out] -1/3*b*d^3*arccsc(c*x)/e+1/3*(e*x+d)^3*(a+b*arccsc(c*x))/e+1/6*b*(6*c^2*d^2
+e^2)*arctanh((1-1/c^2/x^2)^(1/2))/c^3+b*d*e*x*(1-1/c^2/x^2)^(1/2)/c+1/6*b*
e^2*x^2*(1-1/c^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.19, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5335, 1582, 1489, 1821, 858, 222, 272, 65, 214}

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{bdex\sqrt{1-\frac{1}{c^2x^2}}}{c}+\frac{be^2x^2\sqrt{1-\frac{1}{c^2x^2}}}{6c}+\frac{b(6c^2d^2+e^2)\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}-\frac{bd^3\csc^{-1}(cx)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*d*e*Sqrt[1 - 1/(c^2*x^2)]*x)/c + (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x^2)/(6*c)
 - (b*d^3*ArcCsc[c*x])/(3*e) + ((d + e*x)^3*(a + b*ArcCsc[c*x]))/(3*e) + (b
*(6*c^2*d^2 + e^2)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(6*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right) dx = \frac{(d+ex)^3 (a+b\csc^{-1}(cx))}{3e} + \frac{b\int \frac{(d+ex)^3}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}dx}{3ce}$$

$$= \frac{(d+ex)^3 (a+b\csc^{-1}(cx))}{3e} + \frac{b\int \frac{\left(e+\frac{d}{x}\right)^3 x}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3ce}$$

$$= \frac{(d+ex)^3 (a+b\csc^{-1}(cx))}{3e} - \frac{b\text{Subst}\left(\int \frac{(e+dx)^3}{x^3\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{3ce}$$

$$= \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c} + \frac{(d+ex)^3 (a+b\csc^{-1}(cx))}{3e} + \frac{b\text{Subst}\left(\int \frac{-6de^2-e(6d}{x^2\sqrt{}}\right.}{6}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c} + \frac{(d+ex)^3 (a+b\csc^{-1}(cx))}{3e} -$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c} + \frac{(d+ex)^3 (a+b\csc^{-1}(cx))}{3e} -$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c} - \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3 (a+}{3}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c} - \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3 (a+}{3}$$

$$= \frac{bde\sqrt{1-\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c} - \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3 (a+}{3}$$

**Mathematica** [A]
time = 0.11, size = 122, normalized size = 0.99

$$\frac{c^2x\left(be\sqrt{1-\frac{1}{c^2x^2}}\,(6d+ex)+2ac(3d^2+3dex+e^2x^2)\right)+2bc^3x(3d^2+3dex+e^2x^2)\csc^{-1}(cx)+b(6c^2d^2+e^2)\log\left(\left(1+\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (c^2*x*(b*e*Sqrt[1 - 1/(c^2*x^2)]*(6*d + e*x) + 2*a*c*(3*d^2 + 3*d*e*x + e^
2*x^2)) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcCsc[c*x] + b*(6*c^2*d^2
+ e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(109) = 218$.
time = 0.23, size = 315, normalized size = 2.56

| method | result |
|---|---|
| derivativedivides | $\frac{(ecx+cd)^3a}{3c^2e}+\frac{bc\,\mathrm{arccsc}(cx)d^3}{3e}+b\,\mathrm{arccsc}(cx)d^2cx+bce\,\mathrm{arccsc}(cx)d\,x^2+\frac{bc\,e^2\mathrm{arccsc}(cx)x^3}{3}-\frac{b\sqrt{c^2x^2-1}\,d^3\arctan\left(\frac{1}{\sqrt{c^2x^2 \ldots}}\right.}{3e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$ … |
| default | $\frac{(ecx+cd)^3a}{3c^2e}+\frac{bc\,\mathrm{arccsc}(cx)d^3}{3e}+b\,\mathrm{arccsc}(cx)d^2cx+bce\,\mathrm{arccsc}(cx)d\,x^2+\frac{bc\,e^2\mathrm{arccsc}(cx)x^3}{3}-\frac{b\sqrt{c^2x^2-1}\,d^3\arctan\left(\frac{1}{\sqrt{c^2x^2 \ldots}}\right.}{3e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$ … |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/3*(c*e*x+c*d)^3*a/c^2/e+1/3*b*c/e*arccsc(c*x)*d^3+b*arccsc(c*x)*d^2*
c*x+b*c*e*arccsc(c*x)*d*x^2+1/3*b*c*e^2*arccsc(c*x)*x^3-1/3*b/e*(c^2*x^2-1)
^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*arctan(1/(c^2*x^2-1)^(1/2))+b*(c^2
*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))
+b/c^2*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d+1/6*b/c^2*e^2*(c^2*x^2
-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)+1/6*b/c^3*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1
)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2)))
```

**Maxima** [A]
time = 0.28, size = 198, normalized size = 1.61

$$\frac{1}{3}ax^3e^2+adx^2e+ad^2x+\left(x^2\,\mathrm{arccsc}(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bde+\frac{\left(2cx\,\mathrm{arccsc}(cx)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^2}{2c}+\frac{1}{12}\left(4x^3\,\mathrm{arccsc}(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e^2 + a*d*x^2*e + a*d^2*x + (x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2
) + 1)/c)*b*d*e + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1)
- log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^2/c + 1/12*(4*x^3*arccsc(c*x) + (2*
sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2
) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*e^2
```

**Fricas** [**A**]
time = 0.41, size = 208, normalized size = 1.69

$$\frac{2\,ac^3x^3e^2 + 6\,ac^3dx^2e + 6\,ac^3d^2x + 2\left(3\,bc^3d^2x - 3\,bc^3d^2 + (bc^3x^3 - bc^3)e^2 + 3\,(bc^3dx^2 - bc^3d)e\right)\operatorname{arccsc}(cx) - 4\left(3\,bc^3d^2 + 3\,bc^3de + bc^3e^2\right)\arctan\left(-cx + \sqrt{c^2x^2-1}\right) - \left(6\,bc^2d^2 + be^2\right)\log\left(-cx + \sqrt{c^2x^2-1}\right) + \sqrt{c^2x^2-1}\,(bcxe^2 + 6\,bcde)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3*e^2 + 6*a*c^3*d*x^2*e + 6*a*c^3*d^2*x + 2*(3*b*c^3*d^2*x -
 3*b*c^3*d^2 + (b*c^3*x^3 - b*c^3)*e^2 + 3*(b*c^3*d*x^2 - b*c^3*d)*e)*arccs
c(c*x) - 4*(3*b*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*arctan(-c*x + sqrt(c^2*x
^2 - 1)) - (6*b*c^2*d^2 + b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x
^2 - 1)*(b*c*x*e^2 + 6*b*c*d*e))/c^3
```

**Sympy** [**A**]
time = 4.90, size = 228, normalized size = 1.85

$$ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\operatorname{acsc}(cx) + bdex^2\operatorname{acsc}(cx) + \frac{be^2x^3\operatorname{acsc}(cx)}{3} + \frac{bd^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for}\ |c^2x^2| > 1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} + \frac{bde\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for}\ |c^2x^2| > 1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{c} + \frac{be^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} & \text{for}\ |c^2x^2| > 1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*acsc(c*x) + b*d*e*x**2*acs
c(c*x) + b*e**2*x**3*acsc(c*x)/3 + b*d**2*Piecewise((acosh(c*x), Abs(c**2*x
**2) > 1), (-I*asin(c*x), True))/c + b*d*e*Piecewise((sqrt(c**2*x**2 - 1)/c
, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/c + b*e**2*Piecewi
se((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1),
 (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*a
sin(c*x)/(2*c**2), True))/(3*c)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(109) = 218.
time = 1.97, size = 602, normalized size = 4.89

$$\frac{1}{24}\left(\frac{be^2x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{ae^2x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c} - \frac{24\,bdex^2\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{be^2x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c^2} - \frac{24\,adex^2\left(\frac{1}{c^2x^2}-1\right)}{c} + \frac{12\,bd^2x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{12\,ad^2x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c} + \frac{3\,be^2x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{cx}\right)}{c^3} + \frac{24\,bdex\sqrt{-\frac{1}{c^2x^2}+1}}{c^2} + \frac{3\,ae^2x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^3} + \frac{24\,bd^2\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{24\,bd^2\log\left(\frac{1}{|c||x|}\right)}{c^2} + \frac{24\,bde\arcsin\left(\frac{1}{cx}\right)}{c^3} + \frac{24\,ade}{c^3} + \frac{4\,be^2\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4} - \frac{4\,be^2\log\left(\frac{1}{|c||x|}\right)}{c^4} + \frac{12\,bd^2\arcsin\left(\frac{1}{cx}\right)}{c^3x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} + \frac{12\,ad^2}{c^3x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} + \frac{3\,be^2\arcsin\left(\frac{1}{cx}\right)}{c^5x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} + \frac{3\,ae^2}{c^5x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} - \frac{be^2}{c^6x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2} + \frac{be^2\arcsin\left(\frac{1}{cx}\right)}{c^7x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3} + \frac{ae^2}{c^7x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/24*(b*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + a*e^2*x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c - 24*b*d*e*x^2*(1/(c^2*x^2) - 1)*arcsin(
1/(c*x))/c + b*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 - 24*a*d*e*x^2*(1
/(c^2*x^2) - 1)/c + 12*b*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))
/c + 12*a*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c + 3*b*e^2*x*(sqrt(-1/(c^2*x^
2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 24*b*d*e*x*sqrt(-1/(c^2*x^2) + 1)/c^2 +
3*a*e^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 24*b*d^2*log(sqrt(-1/(c^2*x^2)
 + 1) + 1)/c^2 - 24*b*d^2*log(1/(abs(c)*abs(x)))/c^2 + 24*b*d*e*arcsin(1/(c
*x))/c^3 + 24*a*d*e/c^3 + 4*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 4*b
*e^2*log(1/(abs(c)*abs(x)))/c^4 + 12*b*d^2*arcsin(1/(c*x))/(c^3*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) + 12*a*d^2/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3*b*
e^2*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3*a*e^2/(c^5*x*(
sqrt(-1/(c^2*x^2) + 1) + 1)) - b*e^2/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^
2) + b*e^2*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + a*e^2
/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))*c
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x)^2,x)
```

```
[Out] int((a + b*asin(1/(c*x)))*(d + e*x)^2, x)
```

# 3.46 $\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=83

$$\frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}-\frac{bd^2\csc^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+\frac{bd\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] -1/2*b*d^2*arccsc(c*x)/e+1/2*(e*x+d)^2*(a+b*arccsc(c*x))/e+b*d*arctanh((1-1
/c^2/x^2)^(1/2))/c+1/2*b*e*x*(1-1/c^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.12, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5335, 1582, 1410, 1821, 858, 222, 272, 65, 214}

$$\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+\frac{bd\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}+\frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{2c}-\frac{bd^2\csc^{-1}(cx)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c) - (b*d^2*ArcCsc[c*x])/(2*e) + ((d + e*x
)^2*(a + b*ArcCsc[c*x]))/(2*e) + (b*d*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1410

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol
] :> -Subst[Int[(d + e/x^n)^q*((a + c/x^(2*n))^p/x^2), x], x, 1/x] /; FreeQ
[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)dx = \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e} + \frac{b\int \frac{(d+ex)^2}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}dx}{2ce}$$

$$= \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e} + \frac{b\int \frac{\left(e+\frac{d}{x}\right)^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{2ce}$$

$$= \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e} - \frac{b\text{Subst}\left(\int \frac{(e+dx)^2}{x^2\sqrt{1-\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{2ce}$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e} + \frac{b\text{Subst}\left(\int \frac{-2de-d^2x}{x\sqrt{1-\frac{x^2}{c^2}}}\right.}{2ce}$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e} - \frac{(bd)\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c}}}\right.}{c}$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{bd^2\csc^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e} - \frac{(bd)\text{Su}}{}$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{bd^2\csc^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e} + (bcd)\text{Su}$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{bd^2\csc^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e} + \frac{bd\tanh}{}$$

**Mathematica** [A]
time = 0.15, size = 113, normalized size = 1.36

$$adx+\frac{1}{2}aex^2+\frac{bex\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c}+bdx\csc^{-1}(cx)+\frac{1}{2}bex^2\csc^{-1}(cx)+\frac{bd\sqrt{1-\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + b*d*x*
ArcCsc[c*x] + (b*e*x^2*ArcCsc[c*x])/2 + (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTan
h[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

**Maple** [A]
time = 0.16, size = 139, normalized size = 1.67

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+b\,\mathrm{arccsc}(cx)dcx+\frac{bc\,\mathrm{arccsc}(cx)e\,x^2}{2}+\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{c^2x^2-1}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{b\left(c^2x^2-1\right)e}{2c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}}{c}$ |
| default | $\frac{\frac{a\left(d\,c^2x+\frac{1}{2}c^2e\,x^2\right)}{c}+b\,\mathrm{arccsc}(cx)dcx+\frac{bc\,\mathrm{arccsc}(cx)e\,x^2}{2}+\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{c^2x^2-1}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{b\left(c^2x^2-1\right)e}{2c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}}{c}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c*(d*c^2*x+1/2*c^2*e*x^2)+b*arccsc(c*x)*d*c*x+1/2*b*c*arccsc(c*x)*e*
x^2+b/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*(c^2*x^2-1)^(1/2)*d*ln(c*x+(c^2*x^2-1
)^(1/2))+1/2*b/c^2/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c^2*x^2-1)*e)
```

**Maxima** [A]
time = 0.27, size = 94, normalized size = 1.13

$$\frac{1}{2}\,ax^2e+adx+\frac{1}{2}\left(x^2\,\mathrm{arccsc}\left(cx\right)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)be+\frac{\left(2\,cx\,\mathrm{arccsc}\left(cx\right)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2*e + a*d*x + 1/2*(x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)*b*
e + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1
/(c^2*x^2) + 1) + 1))*b*d/c
```

**Fricas** [A]
time = 0.48, size = 134, normalized size = 1.61

$$\frac{ac^2x^2e+2\,ac^2dx-2\,bcd\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}\,be+\left(2\,bc^2dx-2\,bc^2d+\left(bc^2x^2-bc^2\right)e\right)\mathrm{arccsc}\left(cx\right)-2\left(2\,bc^2d+bc^2e\right)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c^2*x^2*e + 2*a*c^2*d*x - 2*b*c*d*log(-c*x + sqrt(c^2*x^2 - 1)) + sq
rt(c^2*x^2 - 1)*b*e + (2*b*c^2*d*x - 2*b*c^2*d + (b*c^2*x^2 - b*c^2)*e)*arc
csc(c*x) - 2*(2*b*c^2*d + b*c^2*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/c^2
```

**Sympy [A]**
time = 3.25, size = 104, normalized size = 1.25

$$adx+\frac{aex^2}{2}+bdx\operatorname{acsc}(cx)+\frac{bex^2\operatorname{acsc}(cx)}{2}+\frac{bd\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}+\frac{be\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x + a*e*x**2/2 + b*d*x*acsc(c*x) + b*e*x**2*acsc(c*x)/2 + b*d*Piecewise
((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c + b*e*Piecewise(
(sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, Tru
e))/(2*c)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(73) = 146.
time = 0.59, size = 341, normalized size = 4.11

$$\frac{1}{8}\left(\frac{bex^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{aex^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c}+\frac{4bdx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{4adx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c}+\frac{2bex\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{8bd\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{8bd\log\left(\frac{1}{|c||x|}\right)}{c^2}+\frac{2be\arcsin\left(\frac{1}{cx}\right)}{c^3}+\frac{2ae}{c^3}+\frac{4bd\arcsin\left(\frac{1}{cx}\right)}{c^3x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}+\frac{4ad}{c^3x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}-\frac{2be}{c^4x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}+\frac{be\arcsin\left(\frac{1}{cx}\right)}{c^5x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}+\frac{ae}{c^5x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + a*e*x^2*(sq
rt(-1/(c^2*x^2) + 1) + 1)^2/c + 4*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin
(1/(c*x))/c + 4*a*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c + 2*b*e*x*(sqrt(-1/(c^
2*x^2) + 1) + 1)/c^2 + 8*b*d*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - 8*b*d*lo
g(1/(abs(c)*abs(x)))/c^2 + 2*b*e*arcsin(1/(c*x))/c^3 + 2*a*e/c^3 + 4*b*d*ar
csin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 4*a*d/(c^3*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) - 2*b*e/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + b*e*arc
sin(1/(c*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + a*e/(c^5*x^2*(sqrt(
-1/(c^2*x^2) + 1) + 1)^2))*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x),x)
[Out] int((a + b*asin(1/(c*x)))*(d + e*x), x)
```

# 3.47 $\int (a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=31

$$ax+bx\csc^{-1}(cx)+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] a*x+b*x*arccsc(c*x)+b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

**Rubi** [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5323, 272, 65, 214}

$$ax+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}+bx\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCsc[c*x],x]
```

```
[Out] a*x + b*x*ArcCsc[c*x] + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5323

```
Int[ArcCsc[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsc[c*x], x] + Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\csc^{-1}(cx)\right)\,dx &= ax+b\int \csc^{-1}(cx)\,dx\\
&= ax+bx\csc^{-1}(cx)+\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x}\,dx}{c}\\
&= ax+bx\csc^{-1}(cx)-\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{2c}\\
&= ax+bx\csc^{-1}(cx)+(bc)\text{Subst}\left(\int \frac{1}{c^2-c^2x^2}\,dx,x,\sqrt{1-\frac{1}{c^2x^2}}\right)\\
&= ax+bx\csc^{-1}(cx)+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}
\end{aligned}
$$

**Mathematica** [A]
time = 0.04, size = 58, normalized size = 1.87

$$
ax+bx\csc^{-1}(cx)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCsc[c*x],x]
```

```
[Out] a*x + b*x*ArcCsc[c*x] + (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

**Maple** [A]
time = 0.08, size = 37, normalized size = 1.19

| method | result | size |
|---|---|---|

| | | |
|---|---|---|
| default | $ax + bx\,\mathrm{arccsc}(cx) + \frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$ | 37 |
| derivativedivides | $\frac{acx+cxb\,\mathrm{arccsc}(cx)+\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)b}{c}$ | 40 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccsc(c*x),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arccsc(c*x)+b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

**Maxima** [A]
time = 0.26, size = 53, normalized size = 1.71

$$ax + \frac{\left(2\,cx\,\mathrm{arccsc}\,(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(
-1/(c^2*x^2) + 1) + 1))*b/c
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(29) = 58.
time = 0.43, size = 64, normalized size = 2.06

$$\frac{acx - 2\,bc\arctan\left(-cx+\sqrt{c^2x^2-1}\right) + (bcx - bc)\,\mathrm{arccsc}\,(cx) - b\log\left(-cx+\sqrt{c^2x^2-1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="fricas")
```

```
[Out] (a*c*x - 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arccsc(c*x)
 - b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

**Sympy** [A]
time = 1.78, size = 32, normalized size = 1.03

$$ax + b\left(x\,\mathrm{acsc}\,(cx) + \frac{\begin{cases} \mathrm{acosh}\,(cx) & \text{for } |c^2x^2| > 1 \\ -i\,\mathrm{asin}\,(cx) & \text{otherwise} \end{cases}}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acsc(c*x),x)
```

```
[Out] a*x + b*(x*acsc(c*x) + Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin
(c*x), True))/c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.
time = 0.42, size = 62, normalized size = 2.00

$$\frac{1}{2}\,bc\left(\frac{2\,x\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsc(c*x),x, algorithm="giac")
```

```
[Out] 1/2*b*c*(2*x*arcsin(1/(c*x))/c + (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sq
rt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

**Mupad** [B]
time = 0.00, size = 33, normalized size = 1.06

$$a\,x+b\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right)+\frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asin(1/(c*x)),x)
```

```
[Out] a*x + b*x*asin(1/(c*x)) + (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

# 3.48 $\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx$

**Optimal**. Leaf size=257

$$\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}$$

```
[Out] -(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e+(a+b*arccsc(c*x))*
ln(1-I*(I/c/x+(1-1/c^2/x^2)^(1/2))*(e-(-c^2*d^2+e^2)^(1/2))/c/d)/e+(a+b*arc
csc(c*x))*ln(1-I*(I/c/x+(1-1/c^2/x^2)^(1/2))*(e+(-c^2*d^2+e^2)^(1/2))/c/d)/
e+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e-I*b*polylog(2,I*(I/c/x
+(1-1/c^2/x^2)^(1/2))*(e-(-c^2*d^2+e^2)^(1/2))/c/d)/e-I*b*polylog(2,I*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(e+(-c^2*d^2+e^2)^(1/2))/c/d)/e
```

**Rubi** [A]
time = 0.29, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5333, 2598}

$$\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(\sqrt{e^2-c^2d^2}+e\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}-\frac{\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{e}-\frac{ib\text{Li}_2\left(\frac{i\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}-\frac{ib\text{Li}_2\left(\frac{i\left(e+\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}+\frac{ib\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x),x]
```

```
[Out] ((a + b*ArcCsc[c*x])*Log[1 - (I*(e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcCsc[c*
x]))/(c*d)])/e + ((a + b*ArcCsc[c*x])*Log[1 - (I*(e + Sqrt[-(c^2*d^2) + e^2
])*E^(I*ArcCsc[c*x]))/(c*d)])/e - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*Arc
Csc[c*x])])/e - (I*b*PolyLog[2, (I*(e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcCsc
[c*x]))/(c*d)])/e - (I*b*PolyLog[2, (I*(e + Sqrt[-(c^2*d^2) + e^2])*E^(I*Ar
cCsc[c*x]))/(c*d)])/e + ((I/2)*b*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/e
```

Rule 2598

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rule 5333

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :> S
imp[(a + b*ArcCsc[c*x])*(Log[1 - I*(e - Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcCs
c[c*x])/(c*d))]/e), x] + (Dist[b/(c*e), Int[Log[1 - I*(e - Sqrt[(-c^2)*d^2
+ e^2])*(E^(I*ArcCsc[c*x])/(c*d))]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] + Di
st[b/(c*e), Int[Log[1 - I*(e + Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcCsc[c*x])/(
c*d))]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] - Dist[b/(c*e), Int[Log[1 - E^(2
```

```
*I*ArcCsc[c*x])]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] + Simp[(a + b*ArcCsc[c
*x])*(Log[1 - I*(e + Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcCsc[c*x])/(c*d))]/e),
 x] - Simp[(a + b*ArcCsc[c*x])*(Log[1 - E^(2*I*ArcCsc[c*x])]/e), x]) /; Fre
eQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} + \frac{(a+b\csc^{-1}(cx))\log\left(1\right.}{}$$

$$= \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} + \frac{(a+b\csc^{-1}(cx))\log\left(1\right.}{}$$

**Mathematica** [**A**]
time = 0.46, size = 411, normalized size = 1.60

$$\frac{a\log(d+ex)}{e}+\frac{b\left(i(\pi-2\csc^{-1}(cx))^2+32i\,\text{ArcSin}\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\text{ArcTan}\left(\frac{(cd-e)\cot\left(\frac{1}{4}(\pi+2\csc^{-1}(cx))\right)}{\sqrt{-c^2d^2+e^2}}\right)-4\left(\pi-2\csc^{-1}(cx)+4\,\text{ArcSin}\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\right)\log\left(1+\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{-i\csc^{-1}(cx)}}{cd}\right)-4\left(\pi-2\csc^{-1}(cx)-4\,\text{ArcSin}\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\right)\log\left(1+\frac{i\left(e+\sqrt{-c^2d^2+e^2}\right)e^{-i\csc^{-1}(cx)}}{cd}\right)-8\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+4(\pi-2\csc^{-1}(cx))\log\left(e+\frac{d}{x}\right)+8\csc^{-1}(cx)\log\left(e+\frac{d}{x}\right)+8i\left(\text{PolyLog}\left(2,\frac{i\left(-e+\sqrt{-c^2d^2+e^2}\right)e^{-i\csc^{-1}(cx)}}{cd}\right)+\text{PolyLog}\left(2,-\frac{i\left(e+\sqrt{-c^2d^2+e^2}\right)e^{-i\csc^{-1}(cx)}}{cd}\right)\right)+4i\left(\csc^{-1}(cx)^2+\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\right)\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(I*(Pi - 2*ArcCsc[c*x])^2 + (32*I)*ArcSin[Sqrt[1 +
e/(c*d)]/Sqrt[2]]*ArcTan[((c*d - e)*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[-(c^2
*d^2) + e^2]] - 4*(Pi - 2*ArcCsc[c*x] + 4*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]]
)*Log[1 + (I*(e - Sqrt[-(c^2*d^2) + e^2]))/(c*d*E^(I*ArcCsc[c*x]))] - 4*(Pi
 - 2*ArcCsc[c*x] - 4*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]])*Log[1 + (I*(e + Sqr
t[-(c^2*d^2) + e^2]))/(c*d*E^(I*ArcCsc[c*x]))] - 8*ArcCsc[c*x]*Log[1 - E^((
2*I)*ArcCsc[c*x])] + 4*(Pi - 2*ArcCsc[c*x])*Log[e + d/x] + 8*ArcCsc[c*x]*Lo
g[e + d/x] + (8*I)*(PolyLog[2, (I*(-e + Sqrt[-(c^2*d^2) + e^2]))/(c*d*E^(I*
ArcCsc[c*x]))] + PolyLog[2, ((-I)*(e + Sqrt[-(c^2*d^2) + e^2]))/(c*d*E^(I*A
rcCsc[c*x]))]) + (4*I)*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]))
)/(8*e)
```

**Maple** [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(316) = 632$.
time = 1.07, size = 893, normalized size = 3.47

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{ac\ln(ecx+cd)}{e}-\frac{ibc\,\mathrm{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{ibce\,\mathrm{dilog}\left(\frac{cd\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)+ie-\sqrt{c^2d^2-e^2}}{ie-\sqrt{c^2d^2-e^2}}\right)}{c^2d^2-e^2}+\frac{ibce\,\mathrm{dilog}\left(\right.}{}}{}$ |
| default | $\frac{\frac{ac\ln(ecx+cd)}{e}-\frac{ibc\,\mathrm{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{ibce\,\mathrm{dilog}\left(\frac{cd\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)+ie-\sqrt{c^2d^2-e^2}}{ie-\sqrt{c^2d^2-e^2}}\right)}{c^2d^2-e^2}+\frac{ibce\,\mathrm{dilog}\left(\right.}{}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a*c*ln(c*e*x+c*d)/e-I*b*c*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))/e+I*b*c*e/(
c^2*d^2-e^2)*dilog((c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e-(c^2*d^2-e^2)^(1/2)
)/(I*e-(c^2*d^2-e^2)^(1/2)))+I*b*c*e/(c^2*d^2-e^2)*dilog((c*d*(I/c/x+(1-1/c
^2/x^2)^(1/2))+I*e+(c^2*d^2-e^2)^(1/2))/(I*e+(c^2*d^2-e^2)^(1/2)))-b*c*e*ar
ccsc(c*x)/(c^2*d^2-e^2)*ln((c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e-(c^2*d^2-e^
2)^(1/2))/(I*e-(c^2*d^2-e^2)^(1/2)))-b*c*e*arccsc(c*x)/(c^2*d^2-e^2)*ln((c*
d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e+(c^2*d^2-e^2)^(1/2))/(I*e+(c^2*d^2-e^2)^(
1/2)))-I*b*c^3*d^2/e/(c^2*d^2-e^2)*dilog((c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I
*e-(c^2*d^2-e^2)^(1/2))/(I*e-(c^2*d^2-e^2)^(1/2)))+b*c^3*d^2/e*arccsc(c*x)/
(c^2*d^2-e^2)*ln((c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e-(c^2*d^2-e^2)^(1/2))/
(I*e-(c^2*d^2-e^2)^(1/2)))+b*c^3*d^2/e*arccsc(c*x)/(c^2*d^2-e^2)*ln((c*d*(I
/c/x+(1-1/c^2/x^2)^(1/2))+I*e+(c^2*d^2-e^2)^(1/2))/(I*e+(c^2*d^2-e^2)^(1/2)
))-I*b*c^3*d^2/e/(c^2*d^2-e^2)*dilog((c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e+(
c^2*d^2-e^2)^(1/2))/(I*e+(c^2*d^2-e^2)^(1/2)))+I*b*c/e*dilog(1+I/c/x+(1-1/c
^2/x^2)^(1/2))-b*c/e*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2)))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*e^(-1)*log(x*e + d) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
/(x*e + d), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(x*e + d), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x), x)
```

**Giac [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)}{d + e\,x} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x), x)
```

# 3.49 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2} dx$

**Optimal**. Leaf size=102

$$\frac{b\csc^{-1}(cx)}{de} - \frac{a+b\csc^{-1}(cx)}{e(d+ex)} + \frac{b\tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2-e^2}}$$

```
[Out] b*arccsc(c*x)/d/e+(-a-b*arccsc(c*x))/e/(e*x+d)+b*arctanh((c^2*d+e/x)/c/(c^2
*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d/(c^2*d^2-e^2)^(1/2)
```

**Rubi** [A]
time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5335, 1582, 1489, 858, 222, 739, 212}

$$-\frac{a+b\csc^{-1}(cx)}{e(d+ex)} + \frac{b\tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}} + \frac{b\csc^{-1}(cx)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^2,x]
```

```
[Out] (b*ArcCsc[c*x])/(d*e) - (a + b*ArcCsc[c*x])/(e*(d + e*x)) + (b*ArcTanh[(c^2
*d + e/x)/(c*Sqrt[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])])/(d*Sqrt[c^2*d^2 -
 e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{a+b\csc^{-1}(cx)}{e(d+ex)} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)}\,dx}{ce}$$

$$= -\frac{a+b\csc^{-1}(cx)}{e(d+ex)} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)x^3}\,dx}{ce}$$

$$= -\frac{a+b\csc^{-1}(cx)}{e(d+ex)} + \frac{b\text{Subst}\left(\int \frac{x}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce}$$

$$= -\frac{a+b\csc^{-1}(cx)}{e(d+ex)} - \frac{b\text{Subst}\left(\int \frac{1}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\right.}{cde}$$

$$= \frac{b\csc^{-1}(cx)}{de} - \frac{a+b\csc^{-1}(cx)}{e(d+ex)} + \frac{b\text{Subst}\left(\int \frac{1}{d^2-\frac{e^2}{c^2}-x^2}\,dx, x, \frac{d+\frac{e}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}\right)}{cd}$$

$$= \frac{b\csc^{-1}(cx)}{de} - \frac{a+b\csc^{-1}(cx)}{e(d+ex)} + \frac{b\tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2-e^2}}$$

**Mathematica [A]**
time = 0.14, size = 141, normalized size = 1.38

$$-\frac{a}{e(d+ex)} - \frac{b\csc^{-1}(cx)}{e(d+ex)} + \frac{b\text{ArcSin}\left(\frac{1}{cx}\right)}{de} + \frac{b\log(d+ex)}{d\sqrt{c^2d^2-e^2}} - \frac{b\log\left(e+c\left(cd-\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{d\sqrt{c^2d^2-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^2,x]
```

```
[Out] -(a/(e*(d + e*x))) - (b*ArcCsc[c*x])/(e*(d + e*x)) + (b*ArcSin[1/(c*x)])/(d
*e) + (b*Log[d + e*x])/(d*Sqrt[c^2*d^2 - e^2]) - (b*Log[e + c*(c*d - Sqrt[c
^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])*x])/(d*Sqrt[c^2*d^2 - e^2])
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(98) = 196$.
time = 2.32, size = 216, normalized size = 2.12

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{a c^2}{(ecx+cd)e}-\frac{b c^2 \operatorname{arccsc}(cx)}{(ecx+cd)e}+\frac{b\sqrt{c^2x^2-1}\, \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, xd}-\frac{b\sqrt{c^2x^2-1}\, \ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\sqrt{c^2x^2-}}{ecx+cd}\right.}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}}{c}$ |
| default | $\frac{-\frac{a c^2}{(ecx+cd)e}-\frac{b c^2 \operatorname{arccsc}(cx)}{(ecx+cd)e}+\frac{b\sqrt{c^2x^2-1}\, \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, xd}-\frac{b\sqrt{c^2x^2-1}\, \ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\sqrt{c^2x^2-}}{ecx+cd}\right.}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}}{c}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-a*c^2/(c*e*x+c*d)/e-b*c^2/(c*e*x+c*d)/e*arccsc(c*x)+b/e*(c^2*x^2-1)^(
1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d*arctan(1/(c^2*x^2-1)^(1/2))-b/e*(c^2*x
^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/((c^2*d^2-e^2)/e^2)^(1/2)*ln(2*
(((c^2*d^2-e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*c^2*x-e)/(c*e*x+c*d)))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x*e^2 + c^2*d*e)*integrate(x*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))
/(c^2*x^3*e^2 + c^2*d*x^2*e - x*e^2 - d*e + (c^2*x^3*e^2 + c^2*d*x^2*e - x*
e^2 - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) + arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1))*b/(x*e^2 + d*e) - a/(x*e^2 + d*e)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(94) = 188$.
time = 0.42, size = 442, normalized size = 4.33

$$\left[-\frac{ac^2d^3 - ade^2 - \sqrt{c^2d^2-e^2}\,(bxe^2+bde)\log\left(\frac{c^3d^2x+cde+\sqrt{c^2d^2-e^2}\,(c^2dx+e)+\left(c^2d^2+\sqrt{c^2d^2-e^2}\,cd-e^2\right)\sqrt{c^2x^2-1}}{xe+d}\right) + (bc^2d^3-bde^2)\operatorname{arccsc}(cx) + 2\,(bc^2d^2xe+bc^2d^3-bxe^3-bde^2)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)}{c^2d^3xe^2+c^2d^4e-dxe^4-d^2e^3}, -\frac{ac^2d^3 - ade^2 + 2\sqrt{-c^2d^2+e^2}\,(bxe^2+bde)\arctan\left(\frac{\sqrt{-c^2d^2+e^2}\left(cxe+cd-\sqrt{c^2x^2-1}\,e\right)}{c^2d^2-e^2}\right) + (bc^2d^3-bde^2)\operatorname{arccsc}(cx) + 2\,(bc^2d^2xe+bc^2d^3-bxe^3-bde^2)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)}{c^2d^3xe^2+c^2d^4e-dxe^4-d^2e^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-(a*c^2*d^3 - a*d*e^2 - sqrt(c^2*d^2 - e^2)*(b*x*e^2 + b*d*e)*log((c^3*d^2
*x + c*d*e + sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 + sqrt(c^2*d^2 -
e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(x*e + d)) + (b*c^2*d^3 - b*d*e^2)*arccs
c(c*x) + 2*(b*c^2*d^2*x*e + b*c^2*d^3 - b*x*e^3 - b*d*e^2)*arctan(-c*x + sq
rt(c^2*x^2 - 1)))/(c^2*d^3*x*e^2 + c^2*d^4*e - d*x*e^4 - d^2*e^3), -(a*c^2*
d^3 - a*d*e^2 + 2*sqrt(-c^2*d^2 + e^2)*(b*x*e^2 + b*d*e)*arctan(sqrt(-c^2*d
^2 + e^2)*(c*x*e + c*d - sqrt(c^2*x^2 - 1)*e)/(c^2*d^2 - e^2)) + (b*c^2*d^3
 - b*d*e^2)*arccsc(c*x) + 2*(b*c^2*d^2*x*e + b*c^2*d^3 - b*x*e^3 - b*d*e^2)
*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*x*e^2 + c^2*d^4*e - d*x*e^4 - d
^2*e^3)]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{acsc}\,(cx)}{(d+ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x)**2, x)
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^2,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^2, x)
```

# 3.50 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx$

**Optimal**. Leaf size=172

$$-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)}+\frac{b\csc^{-1}(cx)}{2d^2e}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}+\frac{b(2c^2d^2-e^2)\tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\left(c^2d^2-e^2\right)^{3/2}}$$

```
[Out] 1/2*b*arccsc(c*x)/d^2/e+1/2*(-a-b*arccsc(c*x))/e/(e*x+d)^2+1/2*b*(2*c^2*d^2
-e^2)*arctanh((c^2*d+e/x)/c/(c^2*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^2/(c
^2*d^2-e^2)^(3/2)-1/2*b*c*e*(1-1/c^2/x^2)^(1/2)/d/(c^2*d^2-e^2)/(e+d/x)
```

**Rubi** [A]

time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5335, 1582, 1489, 1665, 858, 222, 739, 212}

$$-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(\frac{d}{x}+e\right)}+\frac{b(2c^2d^2-e^2)\tanh^{-1}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{2d^2\left(c^2d^2-e^2\right)^{3/2}}+\frac{b\csc^{-1}(cx)}{2d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^3,x]
```

```
[Out] -1/2*(b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(d*(c^2*d^2 - e^2)*(e + d/x)) + (b*ArcCs
c[c*x])/(2*d^2*e) - (a + b*ArcCsc[c*x])/(2*e*(d + e*x)^2) + (b*(2*c^2*d^2 -
 e^2)*ArcTanh[(c^2*d + e/x)/(c*Sqrt[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])])
/(2*d^2*(c^2*d^2 - e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)^2}\,dx}{2ce}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} - \frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)^2 x^4}\,dx}{2ce}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} + \frac{b\text{Subst}\left(\int \frac{x^2}{(e+dx)^2\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} - \frac{(bc)\text{Subst}\left(\int \frac{e-\left(d-\frac{e^2}{c^2d}\right)x}{(e+dx)\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right.}{2e\left(c^2d^2-e^2\right)}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2cd^2e} - \frac{\left(bc\right.}{}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} + \frac{b\csc^{-1}(cx)}{2d^2e} - \frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} + \frac{\left(bc\left(2-\frac{e^2}{c^2d^2}\right)\right)\text{Subst}\left(\right.}{2(}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)} + \frac{b\csc^{-1}(cx)}{2d^2e} - \frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2} + \frac{b(2c^2d^2-e^2)\tanh^{-1}\left(\right.}{2d^2\left(c^2d\right.}$$

**Mathematica [A]**
time = 0.34, size = 250, normalized size = 1.45

$$\frac{1}{2}\left(-\frac{a}{e(d+ex)^2} - \frac{bce\sqrt{1-\frac{1}{c^2x^2}}\,x}{d\left(c^2d^2-e^2\right)(d+ex)} - \frac{b\csc^{-1}(cx)}{e(d+ex)^2} + \frac{b\text{ArcSin}\left(\frac{1}{cx}\right)}{d^2e} + \frac{b(2c^2d^2-e^2)\log(d+ex)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}} - \frac{b(2c^2d^2-e^2)\log\left(e+c\left(cd-\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d^2 - e^2)*
(d + e*x)) - (b*ArcCsc[c*x])/(e*(d + e*x)^2) + (b*ArcSin[1/(c*x)])/(d^2*e)
+ (b*(2*c^2*d^2 - e^2)*Log[d + e*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2
- e^2]) - (b*(2*c^2*d^2 - e^2)*Log[e + c*(c*d - Sqrt[c^2*d^2 - e^2]*Sqrt[1
- 1/(c^2*x^2)])*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2 - e^2]))/2
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(159) = 318$.

time = 4.42, size = 971, normalized size = 5.65

| method | result |
|---|---|
| derivativedivides | $-\frac{a c^3}{2(ecx+cd)^2 e} - \frac{b c^3 \operatorname{arccsc}(cx)}{2(ecx+cd)^2 e} + \frac{b c^3 \sqrt{c^2x^2-1}\, d \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{2e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x\left(c^2d^2-e^2\right)(ecx+cd)} + \frac{b c^3 \sqrt{c^2x^2-1}\, \arctan\left(\frac{1}{\sqrt{c^2x^2-}}\right.}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\left(c^2d^2-e^2\right)(ecx+cd)}$ |
| default | $-\frac{a c^3}{2(ecx+cd)^2 e} - \frac{b c^3 \operatorname{arccsc}(cx)}{2(ecx+cd)^2 e} + \frac{b c^3 \sqrt{c^2x^2-1}\, d \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{2e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x\left(c^2d^2-e^2\right)(ecx+cd)} + \frac{b c^3 \sqrt{c^2x^2-1}\, \arctan\left(\frac{1}{\sqrt{c^2x^2-}}\right.}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\left(c^2d^2-e^2\right)(ecx+cd)}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*a*c^3/(c*e*x+c*d)^2/e-1/2*b*c^3/(c*e*x+c*d)^2/e*arccsc(c*x)+1/2*b
*c^3/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d/(c^2*d^2-e^2)/(c*e
*x+c*d)*arctan(1/(c^2*x^2-1)^(1/2))+1/2*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1
)/c^2/x^2)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d)*arctan(1/(c^2*x^2-1)^(1/2))-b*c^
3/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d/(c^2*d^2-e^2)/(c*e*x+
c*d)/((c^2*d^2-e^2)/e^2)^(1/2)*ln(2*(((c^2*d^2-e^2)/e^2)^(1/2)*(c^2*x^2-1)^
(1/2)*e-d*c^2*x-e)/(c*e*x+c*d))-b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^
2)^(1/2)/(c^2*d^2-e^2)/(c*e*x+c*d)/((c^2*d^2-e^2)/e^2)^(1/2)*ln(2*(((c^2*d^
2-e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*c^2*x-e)/(c*e*x+c*d))-1/2*b*c*e*(c^
2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(c^2*d^2-e^2)/(c*e*x+c*d)*ar
ctan(1/(c^2*x^2-1)^(1/2))-1/2*b*c*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^
2)^(1/2)/d^2/(c^2*d^2-e^2)/(c*e*x+c*d)*arctan(1/(c^2*x^2-1)^(1/2))-1/2*b*c*
e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(c^2*d^2-e^2)/(c*e*x+c*d)+1/2
*b*c*e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(c^2*d^2-e^2)/(c*e
*x+c*d)/((c^2*d^2-e^2)/e^2)^(1/2)*ln(2*(((c^2*d^2-e^2)/e^2)^(1/2)*(c^2*x^2-
1)^(1/2)*e-d*c^2*x-e)/(c*e*x+c*d))+1/2*b*c*e^2*(c^2*x^2-1)^(1/2)/(((c^2*x^2-
```

```
1)/c^2/x^2)^(1/2)/d^2/(c^2*d^2-e^2)/(c*e*x+c*d)/((c^2*d^2-e^2)/e^2)^(1/2)*l
n(2*(((c^2*d^2-e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*c^2*x-e)/(c*e*x+c*d)))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(c^2*x^2*e^3 + 2*c^2*d*x*e^2 + c^2*d^2*e)*integrate(1/2*x*e^(1/2*lo
g(c*x + 1) + 1/2*log(c*x - 1))/(c^2*x^4*e^3 + 2*c^2*d*x^3*e^2 + (c^2*d^2*e
- e^3)*x^2 - 2*d*x*e^2 - d^2*e + (c^2*x^4*e^3 + 2*c^2*d*x^3*e^2 + (c^2*d^2*
e - e^3)*x^2 - 2*d*x*e^2 - d^2*e)*e^(log(c*x + 1) + log(c*x - 1))), x) + ar
ctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(x^2*e^3 + 2*d*x*e^2 + d^2*e) - 1/
2*a/(x^2*e^3 + 2*d*x*e^2 + d^2*e)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(154) = 308$.
time = 0.66, size = 1062, normalized size = 6.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*c^4*d^6 + b*c^3*d^5*e - b*c*d*x^2*e^5 - (4*b*c^2*d^3*x*e^2 + 2*b*c
^2*d^4*e - b*x^2*e^5 - 2*b*d*x*e^4 + (2*b*c^2*d^2*x^2 - b*d^2)*e^3)*sqrt(c^
2*d^2 - e^2)*log((c^3*d^2*x + c*d*e + sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (
c^2*d^2 + sqrt(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(x*e + d)) + (b
*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arccsc(c*x) + 2*(2*b*c^4*d^5*x*e +
b*c^4*d^6 - 4*b*c^2*d^3*x*e^3 + b*x^2*e^6 + 2*b*d*x*e^5 - (2*b*c^2*d^2*x^2
- b*d^2)*e^4 + (b*c^4*d^4*x^2 - 2*b*c^2*d^4)*e^2)*arctan(-c*x + sqrt(c^2*x^
2 - 1)) - (2*b*c*d^2*x - a*d^2)*e^4 + (b*c^3*d^3*x^2 - b*c*d^3)*e^3 + 2*(b*
c^3*d^4*x - a*c^2*d^4)*e^2 + (b*c^2*d^3*x*e^3 + b*c^2*d^4*e^2 - b*d*x*e^5 -
 b*d^2*e^4)*sqrt(c^2*x^2 - 1))/(2*c^4*d^7*x*e^2 + c^4*d^8*e - 4*c^2*d^5*x*e
^4 + d^2*x^2*e^7 + 2*d^3*x*e^6 - (2*c^2*d^4*x^2 - d^4)*e^5 + (c^4*d^6*x^2 -
 2*c^2*d^6)*e^3), -1/2*(a*c^4*d^6 + b*c^3*d^5*e - b*c*d*x^2*e^5 + 2*(4*b*c^
2*d^3*x*e^2 + 2*b*c^2*d^4*e - b*x^2*e^5 - 2*b*d*x*e^4 + (2*b*c^2*d^2*x^2 -
b*d^2)*e^3)*sqrt(-c^2*d^2 + e^2)*arctan(sqrt(-c^2*d^2 + e^2)*(c*x*e + c*d -
 sqrt(c^2*x^2 - 1)*e)/(c^2*d^2 - e^2)) + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d
^2*e^4)*arccsc(c*x) + 2*(2*b*c^4*d^5*x*e + b*c^4*d^6 - 4*b*c^2*d^3*x*e^3 +
b*x^2*e^6 + 2*b*d*x*e^5 - (2*b*c^2*d^2*x^2 - b*d^2)*e^4 + (b*c^4*d^4*x^2 -
2*b*c^2*d^4)*e^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (2*b*c*d^2*x - a*d^2)*
e^4 + (b*c^3*d^3*x^2 - b*c*d^3)*e^3 + 2*(b*c^3*d^4*x - a*c^2*d^4)*e^2 + (b*
```

```
c^2*d^3*x*e^3 + b*c^2*d^4*e^2 - b*d*x*e^5 - b*d^2*e^4)*sqrt(c^2*x^2 - 1))/(
2*c^4*d^7*x*e^2 + c^4*d^8*e - 4*c^2*d^5*x*e^4 + d^2*x^2*e^7 + 2*d^3*x*e^6 -
 (2*c^2*d^4*x^2 - d^4)*e^5 + (c^4*d^6*x^2 - 2*c^2*d^6)*e^3)]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acsc}(cx)}{(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x)**3, x)
```

**Giac** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^3,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^3, x)
```

# 3.51 $\int x^2\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=496

$$\frac{4bd\sqrt{d+ex}\,(1-c^2x^2)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{4b(d+ex)^{3/2}\,(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{2d^2(d+ex)^{3/2}\,\left(a+b\csc^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\,(a+b\,\mathrm{csc}}{5e^3}$$

```
[Out] 2/3*d^2*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^3-4/5*d*(e*x+d)^(5/2)*(a+b*arccsc
(c*x))/e^3+2/7*(e*x+d)^(7/2)*(a+b*arccsc(c*x))/e^3-4/35*b*(e*x+d)^(3/2)*(-c
^2*x^2+1)/c^3/e/x/(1-1/c^2/x^2)^(1/2)+4/105*b*d*(-c^2*x^2+1)*(e*x+d)^(1/2)/
c^3/e/x/(1-1/c^2/x^2)^(1/2)+4/105*b*(5*c^2*d^2-9*e^2)*EllipticE(1/2*(-c*x+1
)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)
/c^4/e^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/105*b*d*(9*c^2*d
^2-e^2)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*
(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*
x+d)^(1/2)-32/105*b*d^4*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/
(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1/c
^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]
time = 2.03, antiderivative size = 693, normalized size of antiderivative = 1.40, number of steps used = 31, number of rules used = 16, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {45, 5355, 12, 6853, 6874, 757, 858, 733, 435, 430, 972, 946, 174, 552, 551, 847}

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{32bd^4\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{32bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}+\frac{32bd^2\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{4b\sqrt{1-c^2x^2}\,(c^2d^2+3e^2)\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^4e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{4bd\sqrt{1-c^2x^2}\,(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^4e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}+\frac{4bd(1-c^2x^2)\sqrt{d+ex}}{105c^3ex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{4b(1-c^2x^2)(d+ex)^{3/2}}{35c^3ex\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (4*b*d*Sqrt[d + e*x]*(1 - c^2*x^2))/(105*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) - (
4*b*(d + e*x)^(3/2)*(1 - c^2*x^2))/(35*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*
d^2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (4*d*(d + e*x)^(5/2)*(a
+ b*ArcCsc[c*x]))/(5*e^3) + (2*(d + e*x)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^3)
 + (32*b*d^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(105*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(
d + e*x))/(c*d + e)]) - (4*b*(c^2*d^2 + 3*e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x
^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(35*c^4*e^2*
Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (32*b*d^3*Sqrt[(c*
(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)])/(105*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
 - (4*b*d*(c*d - e)*(c*d + e)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^
```

```
2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(105*c^4*e^2*
Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d^4*Sqrt[(c*(d + e*x))/(c*d
+ e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/
(c*d + e)])/(105*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2}{}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2}{}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2}{}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2}{}$$

$$= \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3} + \frac{2}{}$$

$$= \frac{16bd\sqrt{d+ex}\,(1-c^2x^2)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{4b(d+ex)^{3/2}\,(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2(d+ex)^{3/2}}{3}$$

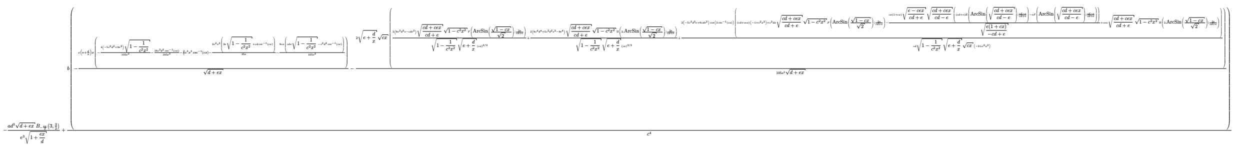
$$= \frac{4bd\sqrt{d+ex}\,(1-c^2x^2)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{4b(d+ex)^{3/2}\,(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2(d+ex)^{3/2}\,(a}{3e}$$

$$= \frac{4bd\sqrt{d+ex}\,(1-c^2x^2)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{4b(d+ex)^{3/2}\,(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2(d+ex)^{3/2}\,(a}{3e}$$

$$= \frac{4bd\sqrt{d+ex}\,(1-c^2x^2)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{4b(d+ex)^{3/2}\,(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2(d+ex)^{3/2}\,(a}{3e}$$

$$= \frac{4bd\sqrt{d+ex}\,(1-c^2x^2)}{} - \frac{4b(d+ex)^{3/2}\,(1-c^2x^2)}{} + \frac{2d^2(d+ex)^{3/2}\,(a}{}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 40.71, size = 870, normalized size = 1.75



Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] -((a*d^3*Sqrt[d + e*x]*Beta[-((e*x)/d), 3, 3/2])/(e^3*Sqrt[1 + (e*x)/d])) +
 (b*(-((c*(e + d/x)*x*((-4*(-5*c^2*d^2 + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(105
*e^2) - (16*c^3*d^3*ArcCsc[c*x])/(105*e^3) - (2*c^3*x^3*ArcCsc[c*x])/7 - (2
*c^2*x^2*(2*e*Sqrt[1 - 1/(c^2*x^2)] + c*d*ArcCsc[c*x]))/(35*e) - (8*c*x*(c*
d*e*Sqrt[1 - 1/(c^2*x^2)] - c^2*d^2*ArcCsc[c*x]))/(105*e^2)))/Sqrt[d + e*x]
) - (2*Sqrt[e + d/x]*Sqrt[c*x]*((2*(9*c^3*d^3*e - c*d*e^3)*Sqrt[(c*d + c*e*
x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(8*c
^4*d^4 + 5*c^2*d^2*e^2 - 9*e^4)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*
x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1
 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-5*c^3*d^3*e + 9*c*d*e^3)*
Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c*d + c*e*
x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e
*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]],
(c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (
c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqrt[(c*d +
c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]
*(-2 + c^2*x^2))))/(105*e^3*Sqrt[d + e*x])))/c^4
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. $2(445) = 890$.
time = 0.73, size = 1204, normalized size = 2.43 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*(1/7*(e*x+d)^(7/2)-2/5*d*(e*x+d)^(5/2)+1/3*d^2*(e*x+d)^(3/2))+b*(1
/7*arccsc(c*x)*(e*x+d)^(7/2)-2/5*arccsc(c*x)*d*(e*x+d)^(5/2)+1/3*arccsc(c*x
)*d^2*(e*x+d)^(3/2)-2/105/c^4*(-3*(c/(c*d-e))^(1/2)*c^3*(e*x+d)^(7/2)+7*(c/
(c*d-e))^(1/2)*c^3*d*(e*x+d)^(5/2)-5*(c/(c*d-e))^(1/2)*c^3*d^2*(e*x+d)^(3/2
)-4*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*E
llipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3-5
```

```
*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elli
pticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3+8*d^
3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ell
ipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/
(c*d-e))^(1/2))*c^3+(c/(c*d-e))^(1/2)*c^3*d^3*(e*x+d)^(1/2)+5*((-c*(e*x+d)+
c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^
(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e-5*((-c*(e*x+d)+c
*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(
1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e+3*(c/(c*d-e))^(1/
2)*c*e^2*(e*x+d)^(3/2)-8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*
d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d
+e))^(1/2))*c*d*e^2+9*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e
)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e)
)^(1/2))*c*d*e^2-(c/(c*d-e))^(1/2)*c*d*e^2*(e*x+d)^(1/2)-9*((-c*(e*x+d)+c*d
-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/
2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^3+9*((-c*(e*x+d)+c*d-e)/(c*
d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(
c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^3)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+
d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\operatorname{acsc}(cx))\sqrt{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))*(e*x+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))*sqrt(d + e*x), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)*x^2, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2),x)
```

```
[Out] int(x^2*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2), x)
```

# 3.52 $\int x\sqrt{d+ex}\,(a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=404

$$-\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}+\frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^2}-\frac{8bd\sqrt{d+ex}\,\sqrt{1-}}{15c^2e}$$

```
[Out] -2/3*d*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^2+2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*
x))/e^2-4/15*b*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/x/(1-1/c^2/x^2)^(1/2)-8/15*b*
d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(
1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/
2)+4/15*b*(3*c^2*d^2-e^2)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(
c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e/x/(1-1/c^
2/x^2)^(1/2)/(e*x+d)^(1/2)+8/15*b*d^3*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2)
,2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/
c/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]

time = 1.68, antiderivative size = 502, normalized size of antiderivative = 1.24, number of steps used = 24, number of rules used = 15, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.790$, Rules used = {45, 5355, 12, 6853, 6874, 757, 858, 733, 435, 430, 972, 946, 174, 552, 551}

$$-\frac{2d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}+\frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^2}+\frac{8bd^3\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}+\frac{8bd^2\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2ex\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{8bd\sqrt{1-c^2x^2}\,\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2ex\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{4b\sqrt{1-c^2x^2}\,(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^4ex\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{4b(1-c^2x^2)\sqrt{d+ex}}{15c^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*Sqrt[1 - 1/(c^2*x^2)]*x) - (2*d*
(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) + (2*(d + e*x)^(5/2)*(a + b*Ar
cCsc[c*x]))/(5*e^2) - (8*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcS
in[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^2*e*Sqrt[1 - 1/(c^2*x^2)
]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]
*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)
])/(15*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (4*b*(c*d - e)*(c*d +
 e)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1
 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) + (8*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipt
icPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c*e^2*Sqrt[1 -
 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
```

```
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
```

```
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$b$ $\int$

$$\int x\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)\,dx=-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+$$

(2

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-$$

$$=-\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)}{}$$

$$=-\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)}{}$$

$$=-\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)}{}$$

$$=-\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)}{}$$

$$=-\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{}+\frac{2(d+ex)}{}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 10.95, size = 368, normalized size = 0.91

$$\frac{1}{15}\left(\frac{4b\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}{c}+\frac{2a\sqrt{d+ex}\,(-2d^2+dex+3e^2x^2)}{e^2}+\frac{2b\sqrt{d+ex}\,(-2d^2+dex+3e^2x^2)\csc^{-1}(cx)}{e^2}-\frac{4ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-2cd(cd-e)E\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(-c^2d^2-2cde+e^2)\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+2c^2d^2\Pi\left(1+\frac{e}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)\right)}{c^3e^2\sqrt{-\frac{c}{cd+e}}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((4*b*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (2*a*Sqrt[d + e*x]*(-2*d^2
 + d*e*x + 3*e^2*x^2))/e^2 + (2*b*Sqrt[d + e*x]*(-2*d^2 + d*e*x + 3*e^2*x^2
)*ArcCsc[c*x])/e^2 - ((4*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*
e*x)/(c*d + e)]*(-2*c*d*(c*d - e)*EllipticE[I*ArcSinh[Sqrt[-(c/(c*d + e))]*
Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (-(c^2*d^2) - 2*c*d*e + e^2)*Ellipti
cF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + 2*
c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x
]], (c*d + e)/(c*d - e)]))/(c^3*e^2*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^2*x^
2)]*x))/15
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 826 vs. 2(363) = 726.
time = 0.66, size = 827, normalized size = 2.05

| method | result |
|---|---|
| derivativedivides | $-2a\left(-\frac{(ex+d)^{\frac{5}{2}}}{5}+\frac{(ex+d)^{\frac{3}{2}}d}{3}\right)-2b\left(-\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\frac{-\frac{2\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}}{15}+\frac{2d^2\sqrt{\frac{-c(ex}{c}}}{}}{}\right.$ |
| default | $-2a\left(-\frac{(ex+d)^{\frac{5}{2}}}{5}+\frac{(ex+d)^{\frac{3}{2}}d}{3}\right)-2b\left(-\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\frac{-\frac{2\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}}{15}+\frac{2d^2\sqrt{\frac{-c(ex}{c}}}{}}{}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(-1/5*(e*x+d)^(5/2)+1/3*(e*x+d)^(3/2)*d)-b*(-1/5*arccsc(c*x)*(e*x
+d)^(5/2)+1/3*arccsc(c*x)*(e*x+d)^(3/2)*d+2/15/c^3*(-(c/(c*d-e))^(1/2)*c^2*
(e*x+d)^(5/2)+d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c
*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1
/2))*c^2+2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^
(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^
```

```
2*d^2-2*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))
^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))
^(1/2)/(c/(c*d-e))^(1/2))*c^2+2*(c/(c*d-e))^(1/2)*c^2*d*(e*x+d)^(3/2)-2*((-
c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elliptic
F((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+2*((-c*(e*
x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*
x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e-(c/(c*d-e))^(1/
2)*c^2*d^2*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*
d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d
+e))^(1/2))*e^2+(c/(c*d-e))^(1/2)*e^2*(e*x+d)^(1/2))/(c/(c*d-e))^(1/2)/x/((
c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b\operatorname{acsc}(cx))\sqrt{d + ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))*(e*x+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))*sqrt(d + e*x), x)
```

**<u>Giac</u>** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)*x, x)
```

**<u>Mupad</u>** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2),x)
[Out] int(x*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2), x)
```

## 3.53 $\int \sqrt{d+ex}\,(a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=315

$$\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e}-\frac{4b\sqrt{d+ex}\,\sqrt{1-c^2x^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\,\sqrt{1}}{3c^2}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2^
(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/x/(1-
1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*d*EllipticF(1/2*(-c*x+1)^(
1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2
+1)^(1/2)/c^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/3*b*d^2*EllipticPi(1/2*
(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/
2)*(-c^2*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]
time = 0.34, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5335, 1588, 972, 733, 430, 947, 174, 552, 551, 858, 435}

$$\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e}-\frac{4bd^2\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{4bd\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{4b\sqrt{1-c^2x^2}\,\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{\frac{c(d+ex)}{cd+e}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e) - (4*b*Sqrt[d + e*x]*Sqrt[1 -
 c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2
*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*d*Sqrt[(c*(d
 + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2
]], (2*e)/(c*d + e)])/(3*c^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*
d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
```

```
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{d+ex}\,(a+b\csc^{-1}(cx))\,dx = \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e} + \frac{(2b)\int \frac{(d+ex)^{3/2}}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{3ce}$$

$$= \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{(d+ex)^{3/2}}{x\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \left(\frac{2de}{\sqrt{d+ex}\,\sqrt{-\ldots}}\ldots\right.}{\ldots}$$

$$= \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e} + \frac{\left(4bd\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\,\sqrt{-\frac{1}{c\ldots}}}\ldots}{3c\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e} - \frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\,\sqrt{1-c^2x^2}\,F\left(\sin^{-1}\left(\ldots\right.\right.}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+\ldots}}$$

$$= \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e} - \frac{4b\sqrt{d+ex}\,\sqrt{1-c^2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{1\ldots}}{\ldots}\right.\right.}{3c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+e\ldots}{cd+\ldots}}}$$

$$= \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e} - \frac{4b\sqrt{d+ex}\,\sqrt{1-c^2x^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{1\ldots}}{\ldots}\right.\right.}{\ldots}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 657 vs. 2(315) = 630.

time = 28.38, size = 657, normalized size = 2.09

$$\frac{2a(d+ex)^{3/2}}{3e}-\frac{b(cd+cex)\left(-\frac{2\left(2e\sqrt{1-\frac{1}{c^2x^2}}+cd\csc^{-1}(cx)+cex\csc^{-1}(cx)\right)}{e}+\frac{4d\sqrt{-c^2\left(1-\frac{1}{c^2x^2}\right)x^2}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(cd+e)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{cd+cex}{cd+e}}}-\frac{4(-cd+e)\sqrt{-c^2\left(1-\frac{1}{c^2x^2}\right)x^2}\,\Pi\left(2;\mathrm{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{cd+cex}{cd+e}}}+\frac{\left(c^2\left(1-\frac{1}{c^2x^2}\right)x^2(cd+cex)+c^2dx\sqrt{-c^2\left(1-\frac{1}{c^2x^2}\right)x^2}\sqrt{\frac{cd+cex}{cd+e}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)-\frac{cx(1+cx)\sqrt{\frac{e-cex}{cd+e}}\sqrt{\frac{cd+cex}{cd-e}}\left((cd+e)E\left(\mathrm{ArcSin}\left(\sqrt{\frac{cd+cex}{cd-e}}\right)|\frac{cd-e}{cd+e}\right)-eF\left(\mathrm{ArcSin}\left(\sqrt{\frac{cd+cex}{cd-e}}\right)|\frac{cd-e}{cd+e}\right)\right)}{\sqrt{\frac{e(1+cx)}{-cd+e}}}+cex\sqrt{-c^2\left(1-\frac{1}{c^2x^2}\right)x^2}\sqrt{\frac{cd+cex}{cd+e}}\,\Pi\left(2;\mathrm{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)\right)\sec(2\csc^{-1}(cx))\sin(4\csc^{-1}(cx))}{c^2\left(1-\frac{1}{c^2x^2}\right)\left(e+\frac{d}{x}\right)x^2}\right)}{3c^2\sqrt{d+ex}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (2*a*(d + e*x)^(3/2))/(3*e) - (b*(c*d + c*e*x)*((-2*(2*e*Sqrt[1 - 1/(c^2*x^
2)] + c*d*ArcCsc[c*x] + c*e*x*ArcCsc[c*x]))/e + (4*d*Sqrt[-(c^2*(1 - 1/(c^2
*x^2))*x^2)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/((c
*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*d + c*e*x)/(c*d + e)]) - (4*(-(c*d)
 + e)*Sqrt[-(c^2*(1 - 1/(c^2*x^2))*x^2)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*d + c*e*x
)/(c*d + e)]) + ((c^2*(1 - 1/(c^2*x^2))*x^2*(c*d + c*e*x) + c^2*d*x*Sqrt[-(
c^2*(1 - 1/(c^2*x^2))*x^2)]*Sqrt[(c*d + c*e*x)/(c*d + e)]*EllipticF[ArcSin[
Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/
(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(
c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c
*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d)
+ e)] + c*e*x*Sqrt[-(c^2*(1 - 1/(c^2*x^2))*x^2)]*Sqrt[(c*d + c*e*x)/(c*d +
e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])*Sec[2*Ar
cCsc[c*x]]*Sin[4*ArcCsc[c*x]])/(c^2*(1 - 1/(c^2*x^2))*(e + d/x)*x^2)))/(3*c
^2*Sqrt[d + e*x])
```

**Maple** [A]
time = 0.57, size = 386, normalized size = 1.23

| method | result |
|---|---|
| derivativedivides | $\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\frac{2\left(2d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\mathrm{EllipticE}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-}}\right.\right.}{}\right.$ |
| default | $\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\frac{2\left(2d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\mathrm{EllipticE}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-}}\right.\right.}{}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(1/3*(e*x+d)^(3/2)*a+b*(1/3*(e*x+d)^(3/2)*arccsc(c*x)+2/3/c^2*(2*d*Elli
pticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c-EllipticE(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d-d*EllipticPi((
e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^
(1/2))*c+EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))
*e-EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e)*((
-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)/(c/(c*d
-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2
)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*sqrt(x*e + d), x)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acsc}\,(cx))\,\sqrt{d + ex}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x)^(1/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))*(d + e*x)^(1/2), x)
```

# 3.54 $\int \frac{\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{x}\,dx$

**Optimal**. Leaf size=24

$$\text{Int}\left(\frac{\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{x},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{x}\,dx$$

**Mathematica** [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] $Aborted
```

**Maple** [A]
time = 3.83, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccsc}(cx))\,\sqrt{ex+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*sqrt(d)*log(x*e/(x*e + 2*sqrt(x*e + d)*sqrt(d) + 2*d)) + b*integrate(sqrt
(x*e + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) + 2*sqrt(x*e + d)*a
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*sqrt(x*e + d)/x, x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x+d)**(1/2)/x,x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x,x)
[Out] int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x, x)
```

# 3.55 $\int \frac{\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{x^2}\,dx$

**Optimal**. Leaf size=24

$$\mathrm{Int}\left(\frac{\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Defer[Int][(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{x^2}\,dx$$

**Mathematica** [A]
time = 6.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2, x]
```

**Maple** [A]
time = 2.53, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccsc}(cx))\sqrt{ex+d}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*x*e*log(x*e/(x*e + 2*sqrt(x*e + d)*sqrt(d) + 2*d)) + 2*b*sqrt(d)*x*i
ntegrate(sqrt(x*e + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x) - 2*
sqrt(x*e + d)*a*sqrt(d))/(sqrt(d)*x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*sqrt(x*e + d)/x^2, x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x+d)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x^2, x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x^2,x)
[Out] int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x^2, x)
```

# 3.56 $\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$

**Optimal**. Leaf size=372

$$-\frac{4be\sqrt{d+ex}\,(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{28bd\sqrt{d+ex}\,\sqrt{1-c^2x^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right.\right.}{15c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}}$$

```
[Out] 2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*x))/e-4/15*b*e*(-c^2*x^2+1)*(e*x+d)^(1/2)/c
^3/x/(1-1/c^2/x^2)^(1/2)-28/15*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(
1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/x/(1-1/c^2/x^2
)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/2*(-c*
x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c
^2*x^2+1)^(1/2)/c^4/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/5*b*d^3*EllipticP
i(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e
))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]

time = 0.52, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {5335, 1588, 972, 733, 430, 947, 174, 552, 551, 858, 435, 945, 1598}

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{4bd^3\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{28bd\sqrt{1-c^2x^2}\,\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{4b\sqrt{1-c^2x^2}\,(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^4x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{4be(1-c^2x^2)\sqrt{d+ex}}{15c^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (-4*b*e*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*
(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e) - (28*b*d*Sqrt[d + e*x]*Sqrt[1 -
 c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^
2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*(2*c^2*d^2
+ e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqr
t[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[d + e*x]) - (4*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellip
ticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*c*e*Sqrt[1 - 1
/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
```

```
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+\frac{(2b)\int\frac{(d+ex)^{5/2}}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{5ce}$$

$$=\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{(d+ex)^{5/2}}{x\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x}\cdots$$

$$=\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+\frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{-\cdots}}\cdots\right.}{\cdots}$$

$$=\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+\frac{\left(6bd^2\sqrt{-\frac{1}{c^2}+x^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{-\cdots}}\cdots}{5c\sqrt{1-\frac{1}{c^2x^2}}\,x\cdots}$$

$$=-\frac{4be\sqrt{d+ex}\,(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+\frac{\left(6bd\sqrt{-\cdots}\right.\cdots}{\cdots}$$

$$=-\frac{4be\sqrt{d+ex}\,(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{12bd^2\sqrt{\frac{c\cdots}{\cdots}}\cdots}{\cdots}$$

$$=-\frac{4be\sqrt{d+ex}\,(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{12bd\sqrt{d+\cdots}\cdots}{\cdots}$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 10.90, size = 333, normalized size = 0.90

$$\frac{1}{15}\left(\frac{4be\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}{c}+\frac{6a(d+ex)^{5/2}}{e}+\frac{6b(d+ex)^{5/2}\csc^{-1}(cx)}{e}-\frac{4ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-7cd(cd-e)E\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(9c^2d^2-7cde+e^2)F\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)-3c^2d^2\Pi\left(1+\frac{e}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)\right)}{c^3e\sqrt{-\frac{c}{cd+e}}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] ((4*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (6*a*(d + e*x)^(5/2))/e
+ (6*b*(d + e*x)^(5/2)*ArcCsc[c*x])/e - ((4*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d)
 + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(-7*c*d*(c*d - e)*EllipticE[I*ArcSinh[Sq
rt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (9*c^2*d^2 - 7*c*
d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d +
e)/(c*d - e)] - 3*c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d +
 e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c^3*e*Sqrt[-(c/(c*d + e))]*Sq
rt[1 - 1/(c^2*x^2)]*x))/15
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(335) = 670$.
time = 0.58, size = 798, normalized size = 2.15

| method | result |
|---|---|
| derivativedivides | $\frac{2(ex+d)^{\frac{5}{2}}a}{5}+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}}{15}+\frac{6d^2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\text{EllipticF}}{5}\right.$ |
| default | $\frac{2(ex+d)^{\frac{5}{2}}a}{5}+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}}{15}+\frac{6d^2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\text{EllipticF}}{5}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(1/5*(e*x+d)^(5/2)*a+b*(1/5*arccsc(c*x)*(e*x+d)^(5/2)+2/15/c^3*((c/(c*d
-e))^(1/2)*c^2*(e*x+d)^(5/2)+9*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*
(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c
*d-e)/(c*d+e))^(1/2))*c^2-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)
+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(
c*d+e))^(1/2))*c^2*d^2-3*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d
)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d
```

```
-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2-2*(c/(c*d-e))^(1/2)*c^2*d*(e
*x+d)^(3/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e
))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))
*c*d*e-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1
/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*
e+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2
)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1
/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*e^2*(e*x+d)^(1/2))/(c/(c
*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1
/2)))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*x*e + a*d + (b*x*e + b*d)*arccsc(c*x))*sqrt(x*e + d), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{acsc}(cx))(d + ex)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(a+b*acsc(c*x)),x)
[Out] Integral((a + b*acsc(c*x))*(d + e*x)**(3/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^(3/2)*(b*arccsc(c*x) + a), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))*(d + e*x)^(3/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))*(d + e*x)^(3/2), x)
```

# 3.57 $\int \frac{x^3 (a+b \csc^{-1}(cx))}{\sqrt{d+ex}} \, dx$

**Optimal**. Leaf size=714

$$-\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}}+\frac{4bd\sqrt{d+ex}\,(1-c^2x^2)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2d^3\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4}+\frac{2d^2(d+ex)^{3/2}\,(a+b\csc}{e^4}$$ (cut off at page edge)

```
[Out] 2*d^2*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^4-6/5*d*(e*x+d)^(5/2)*(a+b*arccsc(c
*x))/e^4+2/7*(e*x+d)^(7/2)*(a+b*arccsc(c*x))/e^4-2*d^3*(a+b*arccsc(c*x))*(e
*x+d)^(1/2)/e^4-4/35*b*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e/(1-1/c^2/x^2)^(1/2)
+4/21*b*d*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1-1/c^2/x^2)^(1/2)-24/35*b*
d^2*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)
^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))
^(1/2)+4/105*b*(2*c^2*d^2-9*e^2)*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/
2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^3/x/(1-1/c^2/x
^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+64/35*b*d^3*EllipticF(1/2*(-c*x+1)^(1/2
)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)
^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-32/105*b*d*(c*d-e)*(c*d+
e)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+
d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^
(1/2)+64/35*b*d^4*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e
))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^4/x/(1-1/c^2/x^2
)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]
time = 1.91, antiderivative size = 714, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {45, 5355, 12, 6853, 6874, 733, 435, 958, 430, 946, 174, 552, 551, 847, 858, 956, 1668}

$$-\frac{2d^3\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4}+\frac{2d^2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4}-\frac{6d(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^4}+\frac{2(d+ex)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^4}+\frac{64bd^4\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35ce^4x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}+\frac{64bd^3\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{24bd^2\sqrt{1-c^2x^2}\,\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{4b\sqrt{1-c^2x^2}\,(2c^2d^2-9e^2)\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^4e^3x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{32bd\sqrt{1-c^2x^2}\,(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^4e^3x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}+\frac{4bd(1-c^2x^2)\sqrt{d+ex}}{21c^3e^2x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{4b(1-c^2x^2)\sqrt{d+ex}}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(35*c^3*e*Sqrt[1 - 1/(c^2*x^2)]) + (4*b*
d*Sqrt[d + e*x]*(1 - c^2*x^2))/(21*c^3*e^2*Sqrt[1 - 1/(c^2*x^2)]*x) - (2*d^
3*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^4 + (2*d^2*(d + e*x)^(3/2)*(a + b*Ar
cCsc[c*x]))/e^4 - (6*d*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^4) + (2*(d
 + e*x)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^4) - (24*b*d^2*Sqrt[d + e*x]*Sqrt[1
 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(35*
```

```
c^2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*(2*c^
2*d^2 - 9*e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(105*c^4*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c
*(d + e*x))/(c*d + e)]) + (64*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 -
c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(35*c^2
*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d*(c*d - e)*(c*d + e)*S
qrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(105*c^4*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
+ e*x]) + (64*b*d^4*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipti
cPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(35*c*e^4*Sqrt[1 -
1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 956

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/(c*(2*m + 1))), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)/(Sqr
t[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m
+ 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d
*g*(4*m - 1))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = -\frac{2d^3\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} + \frac{2d^2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} - \frac{6d(d+e$$

$$= -\frac{2d^3\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} + \frac{2d^2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} - \frac{6d(d+e$$

$$= -\frac{2d^3\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} + \frac{2d^2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} - \frac{6d(d+e$$

$$= -\frac{2d^3\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} + \frac{2d^2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} - \frac{6d(d+e$$

$$= -\frac{2d^3\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} + \frac{2d^2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4} - \frac{6d(d+e$$

$$= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}} + \frac{8bd\sqrt{d+ex}\,(1-c^2x^2)}{35c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}\,(a+b\csc^{-1}$$

$$= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}} + \frac{4bd\sqrt{d+ex}\,(1-c^2x^2)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}\,(a+b\csc^{-1}$$

$$= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}} + \frac{4bd\sqrt{d+ex}\,(1-c^2x^2)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}\,(a+b\csc^{-1}$$

$$= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}} + \frac{4bd\sqrt{d+ex}\,(1-c^2x^2)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}\,(a+b\csc^{-1}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 30.53, size = 873, normalized size = 1.22

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (a*d^4*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 4, 1/2])/(e^4*Sqrt[d + e*x]) + (b
*(-((c*(e + d/x)*x*((-4*(16*c^2*d^2 + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(105*e^
3) + (32*c^3*d^3*ArcCsc[c*x])/(35*e^4) - (2*c^3*x^3*ArcCsc[c*x])/(7*e) - (4
*c^2*x^2*(e*Sqrt[1 - 1/(c^2*x^2)] - 3*c*d*ArcCsc[c*x]))/(35*e^2) + (4*c*x*(
5*c*d*e*Sqrt[1 - 1/(c^2*x^2)] - 12*c^2*d^2*ArcCsc[c*x]))/(105*e^3)))/Sqrt[d
 + e*x]) + (2*Sqrt[e + d/x]*Sqrt[c*x]*((2*(40*c^3*d^3*e + 8*c*d*e^3)*Sqrt[(
c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sq
rt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2))
 + (2*(48*c^4*d^4 + 16*c^2*d^2*e^2 + 9*e^4)*Sqrt[(c*d + c*e*x)/(c*d + e)]*S
qrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-16*c^3*d^3*e
- 9*c*d*e^3)*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqr
t[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sq
rt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/
(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(
c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x
*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[
1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d
/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(105*e^4*Sqrt[d + e*x])))/c^4
```

**Maple** [A]
time = 0.62, size = 1233, normalized size = 1.73 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^4*(-a*(-1/7*(e*x+d)^(7/2)+3/5*d*(e*x+d)^(5/2)-d^2*(e*x+d)^(3/2)+d^3*(e*
x+d)^(1/2))-b*(-1/7*arccsc(c*x)*(e*x+d)^(7/2)+3/5*arccsc(c*x)*d*(e*x+d)^(5/
2)-arccsc(c*x)*d^2*(e*x+d)^(3/2)+arccsc(c*x)*d^3*(e*x+d)^(1/2)+2/105/c^4*(-
3*(c/(c*d-e))^(1/2)*c^3*(e*x+d)^(7/2)+14*(c/(c*d-e))^(1/2)*c^3*d*(e*x+d)^(5
/2)-19*(c/(c*d-e))^(1/2)*c^3*d^2*(e*x+d)^(3/2)+16*EllipticE((e*x+d)^(1/2)*(
c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2
)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*c^3*d^3-48*d^3*((-c*(e*x+d)+c*d-e)/(c*
```

```
d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/
(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^3+24*((
-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ellipti
cF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3+8*(c/(c
*d-e))^(1/2)*c^3*d^3*(e*x+d)^(1/2)+16*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(
1/2),((c*d-e)/(c*d+e))^(1/2))*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+
d)+c*d+e)/(c*d+e))^(1/2)*c^2*d^2*e-16*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((
-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),
((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e+3*(c/(c*d-e))^(1/2)*c*e^2*(e*x+d)^(3/2)+
9*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*((-c*(
e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*c*d*e^2-((-
c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elliptic
F((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2-8*(c/(c*
d-e))^(1/2)*c*d*e^2*(e*x+d)^(1/2)+9*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/
2),((c*d-e)/(c*d+e))^(1/2))*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)
+c*d+e)/(c*d+e))^(1/2)*e^3-9*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d
)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/
(c*d+e))^(1/2))*e^3)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^
2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/sqrt(e*x + d), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2),x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2), x)
```

## 3.58 $\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx$

**Optimal**. Leaf size=530

$$-\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{2d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3}-\frac{4d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3}+\frac{2(d+ex)^{5/2}\,(a\cdots}{5e^{\cdots}}$$

```
[Out] -4/3*d*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^3+2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*
x))/e^3+2*d^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e^3-4/15*b*(-c^2*x^2+1)*(e*x+
d)^(1/2)/c^3/e/x/(1-1/c^2/x^2)^(1/2)+4/5*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2
^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2/
x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-32/15*b*d^2*EllipticF(1/2*(
-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*
(-c^2*x^2+1)^(1/2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/15*b*(c*d-
e)*(c*d+e)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*
(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^2/x/(1-1/c^2/x^2)^(1/2)/
(e*x+d)^(1/2)-32/15*b*d^3*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(
e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1
/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]
time = 1.46, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {45, 5355, 12, 6853, 6874, 733, 435, 958, 430, 946, 174, 552, 551, 847, 858}

$$\frac{2d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3}-\frac{4d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3}+\frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^3}-\frac{32bd^3\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{32bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}+\frac{4bd\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{4b\sqrt{1-c^2x^2}\,(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^4e^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{4b(1-c^2x^2)\sqrt{d+ex}}{15c^3ex\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*e*Sqrt[1 - 1/(c^2*x^2)]*x) + (2*
d^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^3 - (4*d*(d + e*x)^(3/2)*(a + b*Ar
cCsc[c*x]))/(3*e^3) + (2*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^3) + (4*
b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]]
, (2*e)/(c*d + e)])/(5*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(
c*d + e)]) - (32*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elli
pticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^2*e^2*Sqrt[1 -
 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (4*b*(c*d - e)*(c*d + e)*Sqrt[(c*(d + e*x)
)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*
e)/(c*d + e)])/(15*c^4*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d
^3*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqr
```

```
t[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sq
rt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
```

```
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \frac{2d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/}}{}$$

$$= \frac{2d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/}}{}$$

$$= \frac{2d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/}}{}$$

$$= \frac{2d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/}}{}$$

$$= \frac{2d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/}}{}$$

$$= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}\,(a+}{3e^3}$$

$$= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}\,(a+}{3e^3}$$

$$= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}\,(a+}{3e^3}$$

$$= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}\,(a+}{3e^3}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 30.50, size = 784, normalized size = 1.48

$$-\frac{ad^3\sqrt{1+\frac{ex}{d}}\,B_{-\frac{ex}{d}}\left(3,\frac{1}{2}\right)}{e^3\sqrt{d+ex}}+\frac{b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(\frac{4cd\sqrt{1-\frac{1}{c^2x^2}}}{5e^2}-\frac{16c^2d^2\csc^{-1}(cx)}{15e^3}-\frac{2c^2x^2\csc^{-1}(cx)}{5e}-\frac{4cx\left(e\sqrt{1-\frac{1}{c^2x^2}}-2cd\csc^{-1}(cx)\right)}{15e^2}\right)}{\sqrt{d+ex}}-\frac{2\sqrt{e+\frac{d}{x}}\sqrt{cx}\left(\frac{2(7c^2d^2e+e^3)\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}(cx)^{3/2}}+\frac{2(8c^3d^3+3cde^2)\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}(cx)^{3/2}}-\frac{6cde\cos\left(2\csc^{-1}(cx)\right)\left((cd+cex)(-1+c^2x^2)+c^2dx\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)-\frac{cx(1+cx)\sqrt{\frac{e-cex}{cd+e}}\sqrt{\frac{cd+cex}{cd-e}}\left((cd+e)E\left(\text{ArcSin}\left(\sqrt{\frac{cd+cex}{cd-e}}\right)|\frac{cd-e}{cd+e}\right)-eF\left(\text{ArcSin}\left(\sqrt{\frac{cd+cex}{cd-e}}\right)|\frac{cd-e}{cd+e}\right)\right)}{\sqrt{\frac{e(1+cx)}{-cd+e}}}+cex\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}\sqrt{cx}(-2+c^2x^2)}\right)}{15e^3\sqrt{d+ex}}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] -((a*d^3*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 3, 1/2])/(e^3*Sqrt[d + e*x])) +
 (b*(-((c*(e + d/x)*x*((4*c*d*Sqrt[1 - 1/(c^2*x^2)])/(5*e^2) - (16*c^2*d^2*
ArcCsc[c*x])/(15*e^3) - (2*c^2*x^2*ArcCsc[c*x])/(5*e) - (4*c*x*(e*Sqrt[1 -
1/(c^2*x^2)] - 2*c*d*ArcCsc[c*x]))/(15*e^2)))/Sqrt[d + e*x]) - (2*Sqrt[e +
d/x]*Sqrt[c*x]*((2*(7*c^2*d^2*e + e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1
 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqr
t[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(8*c^3*d^3 + 3*c*d*e^2)*
Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c
*x)^(3/2)) - (6*c*d*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^
2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d
 + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d
+ c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d +
 c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)
] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, Arc
Sin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[
e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(15*e^3*Sqrt[d + e*x])))/c^3
```

**Maple** [A]
time = 0.58, size = 850, normalized size = 1.60

| method | result |
|---|---|
| derivativedivides | $\frac{2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-\frac{2(ex+d)^{\frac{3}{2}}d}{3}+d^2\sqrt{ex+d}\right)+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\frac{2\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\operatorname{arccsc}(cx)d^2\sqrt{ex+d}+\right.}{}$ |

| | |
|---|---|
| default | $2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-\frac{2(ex+d)^{\frac{3}{2}}d}{3}+d^2\sqrt{ex+d}\right)+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\frac{2\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\operatorname{arccsc}(cx)d^2\sqrt{ex+d}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*(1/5*(e*x+d)^(5/2)-2/3*(e*x+d)^(3/2)*d+d^2*(e*x+d)^(1/2))+b*(1/5*a
rccsc(c*x)*(e*x+d)^(5/2)-2/3*arccsc(c*x)*(e*x+d)^(3/2)*d+arccsc(c*x)*d^2*(e
*x+d)^(1/2)+2/15/c^3*((c/(c*d-e))^(1/2)*c^2*(e*x+d)^(5/2)-2*(c/(c*d-e))^(1/
2)*c^2*d*(e*x+d)^(3/2)+4*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d
)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/
(c*d+e))^(1/2))*c^2+3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e
)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e)
)^(1/2))*c^2*d^2-8*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+
e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,
(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)^
(1/2)-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/
2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e
+3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*El
lipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+((-c
*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF
((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(
1/2)*e^2*(e*x+d)^(1/2))/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)
+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/sqrt(e*x + d), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2),x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2), x)
```

## 3.59 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

**Optimal**. Leaf size=344

$$-\frac{2d\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{4b\sqrt{d+ex}\,\sqrt{1-c^2x^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-}}{\sqrt{2}}\right.\right.}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^2-2*d*(a+b*arccsc(c*x))*(e*x+d)^(1/2)
/e^2-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*
(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*
d+e))^(1/2)+8/3*b*d*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e)
)^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)
^(1/2)/(e*x+d)^(1/2)+8/3*b*d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1
/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^2/x
/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]
time = 1.20, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {45, 5355, 12, 6853, 6874, 733, 435, 958, 430, 946, 174, 552, 551}

$$-\frac{2d\sqrt{d+ex}\,\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{8bd^2\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}+\frac{8bd\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2ex\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{4b\sqrt{1-c^2x^2}\,\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2ex\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{\frac{c(d+ex)}{cd+e}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-2*d*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^2 + (2*(d + e*x)^(3/2)*(a + b*Ar
cCsc[c*x]))/(3*e^2) - (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin
[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x
*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt
[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3
*c^2*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (8*b*d^2*Sqrt[(c*(d + e*x))
/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(3*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
```

```
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = -\frac{2d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}+\frac{b\int \frac{2(-2d+ex)\sqrt{\cdots}}{3e^2\sqrt{1-\cdots}}}{c}$$

$$=-\frac{2d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}+\frac{(2b)\int \frac{(-2d+ex)\cdots}{\sqrt{1-\cdots}}}{3ce\cdots}$$

$$=-\frac{2d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}+\frac{\left(2b\sqrt{1-c^2x^2}\cdots\right.}{3ce\cdots}$$

$$=-\frac{2d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}+\frac{\left(2b\sqrt{1-c^2x^2}\cdots\right.}{\cdots}$$

$$=-\frac{2d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}-\frac{\left(4bd\sqrt{1-c^2x\cdots}\right.}{3ce^2\cdots}$$

$$=-\frac{2d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}-\frac{\left(4bd^2\sqrt{1-c^2\cdots}\right.}{\cdots}$$

$$=-\frac{2d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}-\frac{4b\sqrt{d+ex}\,\sqrt{\cdots}}{3c^2\cdots}$$

$$=-\frac{2d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}-\frac{4b\sqrt{d+ex}\,\sqrt{\cdots}}{3c^2\cdots}$$

$$=-\frac{2d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}-\frac{4b\sqrt{d+ex}\,\sqrt{\cdots}}{3c^2\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 10.91, size = 289, normalized size = 0.84

$$\frac{2\left(a(-2d+ex)\sqrt{d+ex}+b(-2d+ex)\sqrt{d+ex}\,\csc^{-1}(cx)+\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left((cd-e)E\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(cd+e)F\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)-2cd\Pi\left(1+\frac{e}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)\right)}{c^2\sqrt{-\frac{c}{cd+e}}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (2*(a*(-2*d + e*x)*Sqrt[d + e*x] + b*(-2*d + e*x)*Sqrt[d + e*x]*ArcCsc[c*x]
 + ((2*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*((
c*d - e)*EllipticE[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)
/(c*d - e)] + (c*d + e)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e
*x]], (c*d + e)/(c*d - e)] - 2*c*d*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-
(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c^2*Sqrt[-(c/(c*d +
e))]*Sqrt[1 - 1/(c^2*x^2)]*x)))/(3*e^2)
```

**Maple** [A]
time = 0.57, size = 410, normalized size = 1.19

| method | result |
|---|---|
| derivativedivides | $\frac{-2a\left(-\frac{(ex+d)^{\frac{3}{2}}}{3}+d\sqrt{ex+d}\right)-2b\left(-\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\operatorname{arccsc}(cx)d\sqrt{ex+d}+\frac{2\left(d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\ \sqrt{\ }\right.\right.}{}\right.}{}$ |
| default | $\frac{-2a\left(-\frac{(ex+d)^{\frac{3}{2}}}{3}+d\sqrt{ex+d}\right)-2b\left(-\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\operatorname{arccsc}(cx)d\sqrt{ex+d}+\frac{2\left(d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\ \sqrt{\ }\right.\right.}{}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(-1/3*(e*x+d)^(3/2)+d*(e*x+d)^(1/2))-b*(-1/3*(e*x+d)^(3/2)*arccsc
(c*x)+arccsc(c*x)*d*(e*x+d)^(1/2)+2/3/c^2*(d*EllipticF((e*x+d)^(1/2)*(c/(c*
d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c+EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^
(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d-2*d*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))
^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c-EllipticF((e*x+
d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e+EllipticE((e*x+d)^(1/
2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e)*((-c*(e*x+d)+c*d+e)/(c*d+e
))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+
d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsc(c*x) + a*x)/sqrt(x*e + d), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/sqrt(e*x + d), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2),x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2), x)
```

# 3.60 $\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx$

**Optimal**. Leaf size=212

$$\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e}-\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}-\frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\ \sqrt{}\cdots}{\cdots}$$

```
[Out] 2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e-4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2
),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c
^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*
2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)
^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]
time = 0.23, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5335, 1588, 958, 733, 430, 947, 174, 552, 551}

$$\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e}-\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c^2x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{4bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)
]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e
)])/(c^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*d*Sqrt[(c*(d + e*x))
/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
```

```
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
```

```
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e} + \frac{(2b)\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{ce}$$

$$= \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{x\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\,\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{c\sqrt{1-\frac{1}{c^2x^2}}\,x} + -$$

$$= \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e} + \frac{\left(2bd\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\,\sqrt{1+cx}\,\sqrt{d+ex}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\,\sqrt{1-c^2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\right.}{c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\,\sqrt{1-c^2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\right.}{c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\,\sqrt{1-c^2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\right.}{c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

**Mathematica [A]**
time = 3.83, size = 243, normalized size = 1.15

$$\frac{2\left(\frac{a(d+ex)}{e}+\frac{b\left((d+ex)\csc^{-1}(cx)+\frac{2cd\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-c^2x^2}}\right)}{e}+\frac{2bcx^2\sqrt{1+cx}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)\left(\cos\left(\frac{1}{2}\csc^{-1}(cx)\right)-\sin\left(\frac{1}{2}\csc^{-1}(cx)\right)\right)^3\left(\cos\left(\frac{1}{2}\csc^{-1}(cx)\right)+\sin\left(\frac{1}{2}\csc^{-1}(cx)\right)\right)}{\sqrt{1-cx}\,(-1+c^2x^2)}\right)}{\sqrt{d+ex}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*((a*(d + e*x))/e + (b*((d + e*x)*ArcCsc[c*x] + (2*c*d*Sqrt[1 - 1/(c^2*x^
2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)])/Sqrt[1 - c^2*x^2]))/e + (2*b*c*x^2*Sqrt[1 + c*x]*Sq
rt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/
(c*d + e)]*(Cos[ArcCsc[c*x]/2] - Sin[ArcCsc[c*x]/2])^3*(Cos[ArcCsc[c*x]/2]
+ Sin[ArcCsc[c*x]/2]))/(Sqrt[1 - c*x]*(-1 + c^2*x^2))))/Sqrt[d + e*x]
```

**Maple [A]**
time = 0.55, size = 252, normalized size = 1.19

| method | result |
|---|---|
| derivativedivides | $\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\text{arccsc}(cx)+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\left(\text{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(e}{c^2e^2x}}}\right.}{e}$ |
| default | $\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\text{arccsc}(cx)+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\left(\text{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(e}{c^2e^2x}}}\right.}{e}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*((e*x+d)^(1/2)*a+b*((e*x+d)^(1/2)*arccsc(c*x)+2/c*((-c*(e*x+d)+c*d-e)/(
c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*(EllipticF((e*x+d)^(1/2)*(
c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))-EllipticPi((e*x+d)^(1/2)*(c/(c*d-
e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)))/((c^2*(e*x+d)
^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/(c/(c*d-e))^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/sqrt(x*e + d), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acsc}(cx)}{\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/sqrt(d + e*x), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/sqrt(e*x + d), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\operatorname{asin}\left(\frac{1}{c\,x}\right)}{\sqrt{d + e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^(1/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^(1/2), x)
```

# 3.61 $\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$

**Optimal**. Leaf size=24

$$\operatorname{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

**Mathematica** [A]
time = 4.59, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]), x]
```

**Maple** [A]
time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x)
```

**Maxima** [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*sqrt(d)*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(x*e + d)
*x), x) + a*log(x*e/(x*e + 2*sqrt(x*e + d)*sqrt(d) + 2*d)))/sqrt(d)
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*sqrt(x*e + d)/(x^2*e + d*x), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x + d)*x), x)
```

**Mupad** [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(1/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(1/2)), x)
```

# 3.62 $\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$

**Optimal**. Leaf size=24

$$\operatorname{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x)
```

**Rubi** [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

**Mathematica** [A]
time = 6.97, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]), x]
```

**Maple** [A]
time = 3.26, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x^2\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*d^(3/2)*x*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(
x*e + d)*x^2), x) - a*x*e*log(x*e/(x*e + 2*sqrt(x*e + d)*sqrt(d) + 2*d)) -
2*sqrt(x*e + d)*a*sqrt(d))/(d^(3/2)*x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*sqrt(x*e + d)/(x^3*e + d*x^2), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{x^2 \sqrt{d + ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**2*sqrt(d + e*x)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x + d)*x^2), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(1/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(1/2)), x)
```

# 3.63 $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

**Optimal**. Leaf size=551

$$-\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4}-\frac{2d(d+ex)^{3/2}\,(a+b\csc^{-1}(c$$

```
[Out] -2*d*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^4+2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*x)
)/e^4+2*d^3*(a+b*arccsc(c*x))/e^4/(e*x+d)^(1/2)+6*d^2*(a+b*arccsc(c*x))*(e*
x+d)^(1/2)/e^4-4/15*b*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1-1/c^2/x^2)^(1
/2)+32/15*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2
))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d
)/(c*d+e))^(1/2)-8*b*d^2*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c
*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c
^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/2*(-c*x+1)^(
1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2
+1)^(1/2)/c^4/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-64/5*b*d^3*EllipticPi
(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e)
)^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^4/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]
time = 1.86, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {45, 5355, 12, 6853, 6874, 733, 430, 946, 174, 552, 551, 858, 435, 945, 1598}

$$\frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4}-\frac{2d(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{e^4}+\frac{2(d+ex)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^4}-\frac{64bd^3\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{8bd^2\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}+\frac{32bd\sqrt{1-c^2x^2}\,\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{4b\sqrt{1-c^2x^2}\,(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^4e^3x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{4b(1-c^2x^2)\sqrt{d+ex}}{15c^3e^2x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (-4*b*Sqrt[d + e*x]*(1 - c^2*x^2))/(15*c^3*e^2*Sqrt[1 - 1/(c^2*x^2)]*x) + (
2*d^3*(a + b*ArcCsc[c*x]))/(e^4*Sqrt[d + e*x]) + (6*d^2*Sqrt[d + e*x]*(a +
b*ArcCsc[c*x]))/e^4 - (2*d*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/e^4 + (2*(d
 + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^4) + (32*b*d*Sqrt[d + e*x]*Sqrt[1 -
 c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^
2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (8*b*d^2*Sqr
t[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)])/(c^2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]
) - (4*b*(2*c^2*d^2 + e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*
EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*c^4*e^3*Sqrt
[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (64*b*d^3*Sqrt[(c*(d + e*x))/(c*d + e)
```

```
]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d
 + e)])/(5*c*e^4*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
```

```
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
```

```
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b\csc}{e^4}$$

$$= \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b\csc}{e^4}$$

$$= \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b\csc}{e^4}$$

$$= \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b\csc}{e^4}$$

$$= \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b\csc}{e^4}$$

$$= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx)}{e^4}$$

$$= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx)}{e^4}$$

$$= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx)}{e^4}$$

$$= -\frac{4b\sqrt{d+ex}\,(1-c^2x^2)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,(a+b\csc^{-1}(cx)}{e^4}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 30.76, size = 814, normalized size = 1.48

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
[Out] (a*d^4*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 4, -1/2])/(e^4*(d + e*x)^(3/2))
 + (b*(-((c^2*(e + d/x)^2*x^2*((32*c*d*Sqrt[1 - 1/(c^2*x^2)])/(15*e^3) - (3
2*c^2*d^2*ArcCsc[c*x])/(5*e^4) + (2*c^2*d^2*ArcCsc[c*x])/(e^3*(e + d/x)) -
(2*c^2*x^2*ArcCsc[c*x])/(5*e^2) - (2*c*x*(2*e*Sqrt[1 - 1/(c^2*x^2)] - 9*c*d
*ArcCsc[c*x]))/(15*e^3)))/(d + e*x)^(3/2)) - (2*(e + d/x)^(3/2)*(c*x)^(3/2)
*((2*(32*c^2*d^2*e + e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*E
llipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*
x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(48*c^3*d^3 + 8*c*d*e^2)*Sqrt[(c*d +
c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) -
 (16*c*d*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[
(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/S
qrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt
[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c
*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*
d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*S
qrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
- c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sq
rt[c*x]*(-2 + c^2*x^2))))/(15*e^4*(d + e*x)^(3/2))))/c^4
```

**Maple** [A]
time = 0.58, size = 880, normalized size = 1.60

| method | result |
|---|---|
| derivativedivides | $\frac{-2a\left(-\frac{(ex+d)^{\frac{5}{2}}}{5}+(ex+d)^{\frac{3}{2}}d-3d^2\sqrt{ex+d}-\frac{d^3}{\sqrt{ex+d}}\right)-2b\left(-\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d-3\,\mathrm{arc}\right.}{}$ |

| | |
|---|---|
| default | $\frac{-2a\left(-\frac{(ex+d)^{\frac{5}{2}}}{5}+(ex+d)^{\frac{3}{2}}d-3d^2\sqrt{ex+d}-\frac{d^3}{\sqrt{ex+d}}\right)-2b\left(-\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d-3\operatorname{arccs}\right.}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^4*(-a*(-1/5*(e*x+d)^(5/2)+(e*x+d)^(3/2)*d-3*d^2*(e*x+d)^(1/2)-d^3/(e*x+
d)^(1/2))-b*(-1/5*arccsc(c*x)*(e*x+d)^(5/2)+arccsc(c*x)*(e*x+d)^(3/2)*d-3*a
rccsc(c*x)*d^2*(e*x+d)^(1/2)-arccsc(c*x)*d^3/(e*x+d)^(1/2)-2/15/c^3*((c/(c*
d-e))^(1/2)*c^2*(e*x+d)^(5/2)-2*(c/(c*d-e))^(1/2)*c^2*d*(e*x+d)^(3/2)+24*d^
2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ell
ipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2+8*((-c*
(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2-48*d^2*((-
c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elliptic
Pi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-
e))^(1/2))*c^2+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)-8*((-c*(e*x+d)+c*d-e
)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)
*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+8*((-c*(e*x+d)+c*d-e)/(c*
d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(
c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+((-c*(e*x+d)+c*d-e)/(c*d-e))^(
1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))
^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*e^2*(e*x+d)^(1/2))/(c
/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)
^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\operatorname{acsc}(cx))}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)
[Out] Integral(x**3*(a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x + d)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(d + e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2),x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2), x)
```

# 3.64 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

**Optimal**. Leaf size=369

$$-\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3}-\frac{4b\sqrt{d+ex}\,\sqrt{1-c^2x}}{3c^2e^2\sqrt{1-}}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^3-2*d^2*(a+b*arccsc(c*x))/e^3/(e*x+d)
^(1/2)-4*d*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e^3-4/3*b*EllipticE(1/2*(-c*x+1)
^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/
c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+20/3*b*d*EllipticF(
1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(
1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+32/3*b*
d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(
e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d
)^(1/2)
```

**Rubi** [A]
time = 1.41, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {45, 5355, 12, 6853, 6874, 733, 430, 946, 174, 552, 551, 858, 435}

$$-\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3}+\frac{32bd^2\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}+\frac{20bd\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{4b\sqrt{1-c^2x^2}\,\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{\frac{c(d+ex)}{cd+e}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (-2*d^2*(a + b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a +
b*ArcCsc[c*x]))/e^3 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (4*
b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(3*c^2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*
d + e)]) + (20*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elliptic
F[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e^2*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[d + e*x]) + (32*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1
- c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(
3*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
```

```
e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\,(a+b\,\mathrm{cs}}{3e^3}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\,(a+b\,\mathrm{cs}}{3e^3}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\,(a+b\,\mathrm{cs}}{3e^3}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\,(a+b\,\mathrm{cs}}{3e^3}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\,(a+b\,\mathrm{cs}}{3e^3}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\,(a+b\,\mathrm{cs}}{3e^3}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\,(a+b\,\mathrm{cs}}{3e^3}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\,(a+b\,\mathrm{cs}}{3e^3}$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}\,(a+b\,\mathrm{cs}}{3e^3}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 30.56, size = 750, normalized size = 2.03

$$-\frac{ad^3\left(1+\frac{ex}{d}\right)^{3/2}B_{-\frac{ex}{d}}\left(3,-\frac{1}{2}\right)}{e^3(d+ex)^{3/2}}+\frac{b\left(-\frac{c^2\left(e+\frac{d}{x}\right)^2x^2\left(-\frac{4\sqrt{1-\frac{1}{c^2x^2}}}{3e^2}+\frac{16cd\csc^{-1}(cx)}{3e^3}-\frac{2cd\csc^{-1}(cx)}{e^2\left(e+\frac{d}{x}\right)}-\frac{2cx\csc^{-1}(cx)}{3e^2}\right)}{(d+ex)^{3/2}}+\frac{2\left(e+\frac{d}{x}\right)^{3/2}(cx)^{3/2}\left(\frac{10cde\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}(cx)^{3/2}}+\frac{2(8c^2d^2+e^2)\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}(cx)^{3/2}}-\frac{2e\cos\left(2\csc^{-1}(cx)\right)\left((cd+cex)(-1+c^2x^2)+c^2dx\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)-\frac{cx(1+cx)\sqrt{\frac{e-cex}{cd+e}}\sqrt{\frac{cd+cex}{cd-e}}\left((cd+e)E\left(\text{ArcSin}\left(\sqrt{\frac{cd+cex}{cd-e}}\right)|\frac{cd-e}{cd+e}\right)-eF\left(\text{ArcSin}\left(\sqrt{\frac{cd+cex}{cd-e}}\right)|\frac{cd-e}{cd+e}\right)\right)}{\sqrt{\frac{e(1+cx)}{-cd+e}}}+cex\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}\sqrt{cx}\,(-2+c^2x^2)}\right)}{3e^3(d+ex)^{3/2}}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] -((a*d^3*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 3, -1/2])/(e^3*(d + e*x)^(3/2
))) + (b*(-((c^2*(e + d/x)^2*x^2*((-4*Sqrt[1 - 1/(c^2*x^2)])/(3*e^2) + (16*
c*d*ArcCsc[c*x])/(3*e^3) - (2*c*d*ArcCsc[c*x])/(e^2*(e + d/x)) - (2*c*x*Arc
Csc[c*x])/(3*e^2)))/(d + e*x)^(3/2)) + (2*(e + d/x)^(3/2)*(c*x)^(3/2)*((10*
c*d*e*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*
(c*x)^(3/2)) + (2*(8*c^2*d^2 + e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 -
c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sq
rt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*e*Cos[2*ArcCsc[c*x]]*((
c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1
- c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x
*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d
+ e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]
- e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))
/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqr
t[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)
]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(3*e^3
*(d + e*x)^(3/2))))/c^3
```

**Maple** [A]
time = 0.54, size = 439, normalized size = 1.19

| method | result |
|---|---|
| derivativedivides | $2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}-2\operatorname{arccsc}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsc}(cx)d^2}{\sqrt{ex+d}}-\frac{2\left(4\right.}{}\right.$ |

| | |
|---|---|
| default | $2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}-2\operatorname{arccsc}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsc}(cx)d^2}{\sqrt{ex+d}}-\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*(1/3*(e*x+d)^(3/2)-2*d*(e*x+d)^(1/2)-d^2/(e*x+d)^(1/2))+b*(1/3*(e*
x+d)^(3/2)*arccsc(c*x)-2*arccsc(c*x)*d*(e*x+d)^(1/2)-arccsc(c*x)*d^2/(e*x+d
)^(1/2)-2/3/c^2*(4*d*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*
d+e))^(1/2))*c+EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^
(1/2))*c*d-8*d*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/
(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c-EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/
2),((c*d-e)/(c*d+e))^(1/2))*e+EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c
*d-e)/(c*d+e))^(1/2))*e)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*
d-e)/(c*d-e))^(1/2)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2
*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2(a+b\operatorname{acsc}(cx))}{(d+ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x + d)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2),x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2), x)
```

## 3.65 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

**Optimal**. Leaf size=238

$$\frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2}-\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\,\sqrt{1-c^2x^2}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c^2e\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] 2*d*(a+b*arccsc(c*x))/e^2/(e*x+d)^(1/2)+2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e
^2-4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(
e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d
)^(1/2)-8*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(
1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^2/x/(1-1/c^2/x^2)^(1
/2)/(e*x+d)^(1/2)
```

**Rubi** [A]
time = 1.20, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = $\{45, 5355, 12, 6853, 6874, 733, 430, 946, 174, 552, 551\}$

$$\frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2}+\frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}}-\frac{8bd\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{4b\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c^2ex\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*d*(a + b*ArcCsc[c*x]))/(e^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*Arc
Csc[c*x]))/e^2 - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellip
ticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c^2*e*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[d + e*x]) - (8*b*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^
2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*e^
2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{b\int \frac{2(2d+ex)}{e^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2\sqrt{d+ex}\cdots}\cdots}{c}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(2b)\int \frac{2d+ex}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2\sqrt{d+\cdots}}\cdots}{ce^2}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{\cdots}{x\sqrt{d+e\cdots}}}{ce^2\sqrt{1-\frac{1}{c^2x^2}}\cdots}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int\left(\frac{\cdots}{\sqrt{d+\cdots}}\cdots\right.}{\cdots}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int \frac{\cdots}{x\sqrt{d+\cdots}}}{ce^2\sqrt{1-\frac{1}{c^2x^2}}\cdots}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int \frac{\cdots}{x\sqrt{1-\cdots}}}{ce^2\sqrt{1-\cdots}}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\,\sqrt{1-c^2x^2}\cdots}{c^2e\sqrt{1-\frac{1}{c^2\cdots}}\cdots}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\,\sqrt{1-c^2x^2}\cdots}{c^2e\sqrt{1-\frac{1}{c^2\cdots}}\cdots}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\,\sqrt{1-c^2x^2}\cdots}{c^2e\sqrt{1-\frac{1}{c^2\cdots}}\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 11.22, size = 226, normalized size = 0.95

$$\frac{2\left(\frac{a(2d+ex)}{\sqrt{d+ex}}+\frac{b(2d+ex)\csc^{-1}(cx)}{\sqrt{d+ex}}-\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(F\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)-2\Pi\left(1+\frac{e}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)\right)}{c\sqrt{-\frac{c}{cd+e}}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*((a*(2*d + e*x))/Sqrt[d + e*x] + (b*(2*d + e*x)*ArcCsc[c*x])/Sqrt[d + e*
x] - ((2*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*
(EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d -
e)] - 2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x
]], (c*d + e)/(c*d - e)]))/(c*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^2*x^2)]*x)
))/e^2
```

**Maple** [A]
time = 0.51, size = 282, normalized size = 1.18

| method | result |
|---|---|
| derivativedivides | $-2a\left(-\sqrt{ex+d}-\frac{d}{\sqrt{ex+d}}\right)-2b\left(-\sqrt{ex+d}\,\mathrm{arccsc}(cx)-\frac{\mathrm{arccsc}(cx)d}{\sqrt{ex+d}}-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{-c(}}{}\right.$ |
| default | $-2a\left(-\sqrt{ex+d}-\frac{d}{\sqrt{ex+d}}\right)-2b\left(-\sqrt{ex+d}\,\mathrm{arccsc}(cx)-\frac{\mathrm{arccsc}(cx)d}{\sqrt{ex+d}}-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{-c(}}{}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(-(e*x+d)^(1/2)-d/(e*x+d)^(1/2))-b*(-(e*x+d)^(1/2)*arccsc(c*x)-ar
ccsc(c*x)*d/(e*x+d)^(1/2)-2/c*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+
d)+c*d+e)/(c*d+e))^(1/2)*(EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e
)/(c*d+e))^(1/2))-2*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/
d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)))/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2
*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/(c/(c*d-e))^(1/2)))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsc(c*x) + a*x)*sqrt(x*e + d)/(x^2*e^2 + 2*d*x*e + d^2), x
)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{acsc}(cx))}{(d + ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x + d)^(3/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{(d + e\,x)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2),x)
```

```
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2), x)
```

# 3.66 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

**Optimal**. Leaf size=119

$$-\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}}+\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\Pi\left(2;\mathrm{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] -2*(a+b*arccsc(c*x))/e/(e*x+d)^(1/2)+4*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1
/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/
2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]
time = 0.17, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5335, 1588, 947, 174, 552, 551}

$$\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\mathrm{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCsc[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[(c*(d + e*x))/(c*d +
 e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(
c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
```

```
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] && !GtQ[a, 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !In
tegerQ[p] && !IntegerQ[q] && PosQ[n]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} - \frac{(2b)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2\sqrt{d+ex}}\,dx}{ce}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} - \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}}\,dx}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} + \frac{\left(4b\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}}}\,d\right.}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} + \frac{\left(4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1}}\right.}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

**Mathematica** [A]
time = 0.16, size = 124, normalized size = 1.04

$$\frac{-2(-1+c^2x^2)\left(a+b\csc^{-1}(cx)\right)+4bc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}\,(-1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(-1 + c^2*x^2)*(a + b*ArcCsc[c*x]) + 4*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt
[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c
*x]/Sqrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e*x]*(-1 + c^2*x^2))
```

**Maple [A]**
time = 0.49, size = 215, normalized size = 1.81

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$ |
| default | $\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arccsc(c*x)+2/c/((c^2*(e*x+d)^2-2
*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/d/(c/(c*d-e))^(1/2)*((-c*(
e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))
^(1/2))))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*sqrt(x*e + d)/(x^2*e^2 + 2*d*x*e + d^2), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x + d)^(3/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^(3/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^(3/2), x)
```

# 3.67 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$

**Optimal**. Leaf size=24

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x)
```

**Rubi** [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

**Mathematica** [A]
time = 14.46, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)), x]
```

**Maple** [A]
time = 6.18, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x\,(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] ((b*d^(3/2)*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((x^2*e + d*x
)*sqrt(x*e + d)), x) + a*log(x*e/(x*e + 2*sqrt(x*e + d)*sqrt(d) + 2*d))*sq
rt(x*e + d) + 2*a*sqrt(d))/(sqrt(x*e + d)*d^(3/2))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*sqrt(x*e + d)/(x^3*e^2 + 2*d*x^2*e + d^2*x), x
)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x+d)**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x + d)^(3/2)*x), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,(d + e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(3/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(3/2)), x)
```

## 3.68 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$

**Optimal**. Leaf size=24

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x)
```

**Rubi** [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

**Mathematica** [A]
time = 15.66, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

**Maple** [A]
time = 5.21, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x^2\,(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(b*d^2*x^2*e + b*d^3*x)*sqrt(d)*integrate(arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1))/((x^3*e + d*x^2)*sqrt(x*e + d)), x) - 2*(3*a*x*e + a*d)*sqrt(
x*e + d)*sqrt(d) - 3*(a*x^2*e^2 + a*d*x*e)*log(x*e/(x*e + 2*sqrt(x*e + d)*s
qrt(d) + 2*d)))/((d^2*x^2*e + d^3*x)*sqrt(d))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*sqrt(x*e + d)/(x^4*e^2 + 2*d*x^3*e + d^2*x^2),
 x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}{\left(cx \right)}}{x^{2} \left(d + ex\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**2*(d + e*x)**(3/2)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x + d)^(3/2)*x^2), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(3/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(3/2)), x)
```

# 3.69 $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

**Optimal**. Leaf size=602

$$-\frac{4bd^2(1-c^2x^2)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}+\frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}}-\frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\,(a+b\csc}{e^4}$$

[Out] 2/3*d^3*(a+b*arccsc(c*x))/e^4/(e*x+d)^(3/2)+2/3*(e*x+d)^(3/2)*(a+b*arccsc(c
*x))/e^4-6*d^2*(a+b*arccsc(c*x))/e^4/(e*x+d)^(1/2)-4/3*b*d^2*(-c^2*x^2+1)/c
/e^2/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-6*d*(a+b*arccsc(c*x)
)*(e*x+d)^(1/2)/e^4+8/3*b*d^2*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*
(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/e^3/(c^2*d^2-e^2)/x/(1-
1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*(2*c^2*d^2-e^2)*EllipticE(
1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x
^2+1)^(1/2)/c^2/e^3/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))
^(1/2)+32/3*b*d*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1
/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(
1/2)/(e*x+d)^(1/2)+64/3*b*d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/
2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^4/x/
(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)

**Rubi** [A]
time = 2.15, antiderivative size = 602, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 18, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {45, 5355, 12, 6853, 6874, 759, 21, 733, 435, 972, 946, 174, 552, 551, 849, 858, 430, 1665}

$$\frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}}-\frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^4}+\frac{64bd^2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}+\frac{8bd^2\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e^3x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{4b\sqrt{1-c^2x^2}\,(2c^2d^2-e^2)\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{32bd\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^3x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}-\frac{4bd^2(1-c^2x^2)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]

[Out] (-4*b*d^2*(1 - c^2*x^2))/(3*c*e^2*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*S
qrt[d + e*x]) + (2*d^3*(a + b*ArcCsc[c*x]))/(3*e^4*(d + e*x)^(3/2)) - (6*d^
2*(a + b*ArcCsc[c*x]))/(e^4*Sqrt[d + e*x]) - (6*d*Sqrt[d + e*x]*(a + b*ArcC
sc[c*x]))/e^4 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^4) + (8*b*d^2*
Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)])/(3*e^3*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d +
 e*x))/(c*d + e)]) - (4*b*(2*c^2*d^2 - e^2)*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]
*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e^3*(c^2
*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (32*b*
d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -

```
 c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c^2*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
 + e*x]) + (64*b*d^2*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipt
icPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*e^4*Sqrt[1 -
1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] && !I
ntegerQ[p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} +$$

$$= \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} +$$

$$= \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} +$$

$$= \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} +$$

$$= \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^4} +$$

$$= -\frac{68bd^2(1-c^2x^2)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} -$$

$$= -\frac{68bd^2(1-c^2x^2)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} -$$

$$= -\frac{4bd^2(1-c^2x^2)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} -$$

$$= -\frac{4bd^2(1-c^2x^2)}{3ce^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} -$$

$$= -\frac{4bd^2(1-c^2x^2)}{} + \frac{2d^3(a+b\csc^{-1}(cx))}{} -$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 30.67, size = 887, normalized size = 1.47

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (a*d^4*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 4, -3/2])/(e^4*(d + e*x)^(5/2))
 + (b*(-((c^3*(e + d/x)^3*x^3*((-4*Sqrt[1 - 1/(c^2*x^2)])/(3*e*(-(c^2*d^2)
+ e^2)) + (32*c*d*ArcCsc[c*x])/(3*e^4) - (2*c*d*ArcCsc[c*x])/(3*e^2*(e + d/
x)^2) - (2*c*x*ArcCsc[c*x])/(3*e^3) - (2*(-2*c^2*d^2*e*Sqrt[1 - 1/(c^2*x^2)
] - 7*c^3*d^3*ArcCsc[c*x] + 7*c*d*e^2*ArcCsc[c*x]))/(3*e^3*(-(c^2*d^2) + e^
2)*(e + d/x))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*((2*(8*c^
3*d^3*e - 8*c*d*e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipt
icF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]
*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(16*c^4*d^4 - 16*c^2*d^2*e^2 - e^4)*Sqrt[(
c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x
]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3
/2)) + (2*e^3*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sq
rt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x
]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*S
qrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)
/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/
(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*
x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]
*Sqrt[c*x]*(-2 + c^2*x^2))))/(3*(c*d - e)*e^4*(c*d + e)*(d + e*x)^(5/2))))/
c^4
```

**Maple** [A]
time = 0.96, size = 1067, normalized size = 1.77 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^4*(-a*(-1/3*(e*x+d)^(3/2)+3*d*(e*x+d)^(1/2)-1/3*d^3/(e*x+d)^(3/2)+3*d^2
/(e*x+d)^(1/2))-b*(-1/3*(e*x+d)^(3/2)*arccsc(c*x)+3*arccsc(c*x)*d*(e*x+d)^(
1/2)-1/3*arccsc(c*x)*d^3/(e*x+d)^(3/2)+3*arccsc(c*x)*d^2/(e*x+d)^(1/2)+2/3/
c^2*(8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2
)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^
3*(e*x+d)^(1/2)-16*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(
```

```
c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(
c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^
3*d^2*(e*x+d)^2+2*(c/(c*d-e))^(1/2)*c^3*d^3*(e*x+d)-7*((-c*(e*x+d)+c*d-e)/(
c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c
/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2*(e*x+d)^(1/2)-((-c*(e*x+d)
+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)
^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2*(e*x+d)^(1/2)+16*
((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ellip
ticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c
*d-e))^(1/2))*c*d*e^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^3*d^4+((-c*(e*x+d)+
c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^
(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^3*(e*x+d)^(1/2)-((-c*(e*
x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*
x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^3*(e*x+d)^(1/2)+(c/
(c*d-e))^(1/2)*c*d^2*e^2)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/x
/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

**Maxima** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsc(c*x) + a*x^3)*sqrt(x*e + d)/(x^3*e^3 + 3*d*x^2*e^2 +
 3*d^2*x*e + d^3), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x + d)^(5/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2), x)
```

# 3.70 $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

**Optimal**. Leaf size=440

$$\frac{4bd(1-c^2x^2)}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}-\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(}{e^3}$$

```
[Out] -2/3*d^2*(a+b*arccsc(c*x))/e^3/(e*x+d)^(3/2)+4*d*(a+b*arccsc(c*x))/e^3/(e*x
+d)^(1/2)+4/3*b*d*(-c^2*x^2+1)/c/e/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x
+d)^(1/2)+2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e^3-4/3*b*d*EllipticE(1/2*(-c*x
+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/
2)/e^2/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4*b*El
lipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c
*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
-32/3*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2)
)*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1/c^2/x^2)^(1/2)/
(e*x+d)^(1/2)
```

**Rubi** [A]
time = 1.73, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {45, 5355, 12, 6853, 6874, 759, 21, 733, 435, 972, 946, 174, 552, 551, 849, 858, 430}

$$-\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3}-\frac{4bd\sqrt{1-c^2x^2}\,\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e^2x\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{32bd\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{4b\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c^2e^2x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}+\frac{4bd(1-c^2x^2)}{3cex\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*d*(1 - c^2*x^2))/(3*c*e*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
 + e*x]) - (2*d^2*(a + b*ArcCsc[c*x]))/(3*e^3*(d + e*x)^(3/2)) + (4*d*(a +
b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))
/e^3 - (4*b*d*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x
]/Sqrt[2]], (2*e)/(c*d + e)])/(3*e^2*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*
x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[
1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c^
2*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d*Sqrt[(c*(d + e*x))/(
c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)])/(3*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
```

```
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$b$

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = -\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \cdots$$

$($

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \cdots$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \cdots$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \cdots$$

$$= -\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,(a+b\csc^{-1}(cx))}{e^3} + \cdots$$

$$= \frac{12bd(1-c^2x^2)}{ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(c\cdots}{e^3\sqrt{d+ex}}$$

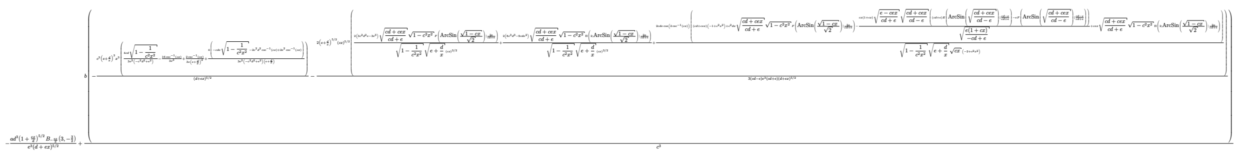
$$= \frac{12bd(1-c^2x^2)}{ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(c\cdots}{e^3\sqrt{d+ex}}$$

$$= \frac{4bd(1-c^2x^2)}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(\cdots}{e^3\sqrt{d+ex}}$$

$$= \frac{4bd(1-c^2x^2)}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(\cdots}{e^3\sqrt{d+ex}}$$

$$= \frac{4bd(1-c^2x^2)}{\cdots} - \frac{2d^2(a+b\csc^{-1}(cx))}{\cdots} + \frac{4d(a+b\csc^{-1}(\cdots}{\cdots}$$

**Mathematica** [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 30.59, size = 856, normalized size = 1.95



Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] -((a*d^3*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 3, -3/2])/(e^3*(d + e*x)^(5/2
))) + (b*(-((c^3*(e + d/x)^3*x^3*((4*c*d*Sqrt[1 - 1/(c^2*x^2)])/(3*e^2*(-(c
^2*d^2) + e^2)) - (16*ArcCsc[c*x])/(3*e^3) + (2*ArcCsc[c*x])/(3*e*(e + d/x)
^2) + (4*(-(c*d*e*Sqrt[1 - 1/(c^2*x^2)]) - 2*c^2*d^2*ArcCsc[c*x] + 2*e^2*Ar
cCsc[c*x]))/(3*e^2*(-(c^2*d^2) + e^2)*(e + d/x))))/(d + e*x)^(5/2)) - (2*(e
 + d/x)^(5/2)*(c*x)^(5/2)*((2*(3*c^2*d^2*e - 3*e^3)*Sqrt[(c*d + c*e*x)/(c*d
 + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*
d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(8*c^3*d^3
- 9*c*d*e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2,
ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqr
t[e + d/x]*(c*x)^(3/2)) + (2*c*d*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 +
c^2*x^2) + c^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elliptic
F[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e
- c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcS
in[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSi
n[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))
/(-(c*d) + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elli
pticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^
2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(3*(c*d - e)*e^3*(c*d + e
)*(d + e*x)^(5/2))))/c^3
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. $2(401) = 802$.
time = 0.93, size = 1026, normalized size = 2.33 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*((e*x+d)^(1/2)-1/3*d^2/(e*x+d)^(3/2)+2*d/(e*x+d)^(1/2))+b*((e*x+d)
^(1/2)*arccsc(c*x)-1/3*arccsc(c*x)*d^2/(e*x+d)^(3/2)+2*arccsc(c*x)*d/(e*x+d
)^(1/2)+2/3/c*(4*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*
d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/
2))*c^2*d^2*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c
*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*
```

```
d+e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-
c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),
1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-(c
/(c*d-e))^(1/2)*c^2*d*(e*x+d)^2+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*
x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-
e)/(c*d+e))^(1/2))*c*d*e*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*(
(-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2)
,((c*d-e)/(c*d+e))^(1/2))*c*d*e*(e*x+d)^(1/2)+2*(c/(c*d-e))^(1/2)*c^2*d^2*(
e*x+d)-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1
/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2*
(e*x+d)^(1/2)+8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d
+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d
+e))^(1/2)/(c/(c*d-e))^(1/2))*e^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2*d^3+(
c/(c*d-e))^(1/2)*d*e^2)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/x/(
(c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x + d)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2),x)
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2), x)
```

# 3.71 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

**Optimal**. Leaf size=314

$$-\frac{4b(1-c^2x^2)}{3c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}+\frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}}-\frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{4b\sqrt{d+ex}\,\sqrt{1-c^2x^2}\,E}{3e\left(c^2d^2-e^2\right)\sqrt{1}}$$

```
[Out] 2/3*d*(a+b*arccsc(c*x))/e^2/(e*x+d)^(3/2)-2*(a+b*arccsc(c*x))/e^2/(e*x+d)^(
1/2)-4/3*b*(-c^2*x^2+1)/c/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
+4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x
+d)^(1/2)*(-c^2*x^2+1)^(1/2)/e/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+
d)/(c*d+e))^(1/2)+8/3*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/
(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^2/x/(1-1/c
^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]
time = 1.54, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {45, 5355, 12, 6853, 6874, 759, 21, 733, 435, 972, 946, 174, 552, 551}

$$-\frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}}+\frac{4b\sqrt{1-c^2x^2}\,\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3ex\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{8b\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}-\frac{4b(1-c^2x^2)}{3cx\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (-4*b*(1 - c^2*x^2))/(3*c*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d +
e*x]) + (2*d*(a + b*ArcCsc[c*x]))/(3*e^2*(d + e*x)^(3/2)) - (2*(a + b*ArcCs
c[c*x]))/(e^2*Sqrt[d + e*x]) + (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*Ellipti
cE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*e*(c^2*d^2 - e^2)*Sq
rt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (8*b*Sqrt[(c*(d + e*
x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]
], (2*e)/(c*d + e)])/(3*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{b\int \frac{2(-2d-3ex)}{3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)^{3/2}}\,dx}{c}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{(2b)\int \frac{-2d-3ex}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)^{3/2}}\,dx}{3ce^2}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \frac{-2d-3ex}{x(d+ex)^{3/2}\sqrt{1-c^2 \cdots}}}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{\left(2b\sqrt{1-c^2x^2}\right)\int \left(-\frac{3e}{(d+ex)^{3/2}\sqrt{1-\cdots}}\cdots\right.}{3ce^2\sqrt{1-\cdots}}$$

$$= \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} - \frac{\left(4bd\sqrt{1-c^2x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-\cdots}}}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{4b(1-c^2x^2)}{c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx \cdots}{e^2\sqrt{d+ex}}$$

$$= -\frac{4b(1-c^2x^2)}{c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx \cdots}{e^2\sqrt{d+ex}}$$

$$= -\frac{4b(1-c^2x^2)}{3c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(\cdots}{e^2\sqrt{d+ex}}$$

$$= -\frac{4b(1-c^2x^2)}{3c\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(\cdots}{e^2\sqrt{d+ex}}$$

$$= -\frac{4b(1-c^2x^2)}{\cdots} + \frac{2d(a+b\csc^{-1}(cx))}{\cdots} - \frac{2(a+b\csc^{-1}(\cdots}{\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 11.08, size = 345, normalized size = 1.10

$$\frac{4bc\sqrt{1-\frac{1}{c^2x^2}}\,x}{3\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2a(2d+3ex)}{3e^2(d+ex)^{3/2}}-\frac{2b(2d+3ex)\csc^{-1}(cx)}{3e^2(d+ex)^{3/2}}+\frac{4ib\sqrt{-\frac{c}{cd+e}}\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(cdE\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)-cdF\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+2(cd+e)\Pi\left(1+\frac{e}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)\right)}{3c^2de^2\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(3*(c^2*d^2 - e^2)*Sqrt[d + e*x]) - (2*a*(2
*d + 3*e*x))/(3*e^2*(d + e*x)^(3/2)) - (2*b*(2*d + 3*e*x)*ArcCsc[c*x])/(3*e
^2*(d + e*x)^(3/2)) + (((4*I)/3)*b*Sqrt[-(c/(c*d + e))]*Sqrt[(e*(1 + c*x))/
(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(c*d*EllipticE[I*ArcSinh[Sqrt[-(c
/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] - c*d*EllipticF[I*ArcSinh
[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + 2*(c*d + e)*El
lipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d +
 e)/(c*d - e)]))/(c^2*d*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(284) = 568$.
time = 0.88, size = 900, normalized size = 2.87

| method | result |
|---|---|
| derivativedivides | $-2a\left(\frac{1}{\sqrt{ex+d}}-\frac{d}{3(ex+d)^{\frac{3}{2}}}\right)-2b\left(\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}-\frac{\operatorname{arccsc}(cx)d}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\ \mathrm{Ellip}\right.}{}\right.$ |
| default | $-2a\left(\frac{1}{\sqrt{ex+d}}-\frac{d}{3(ex+d)^{\frac{3}{2}}}\right)-2b\left(\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}-\frac{\operatorname{arccsc}(cx)d}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\ \mathrm{Ellip}\right.}{}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(1/(e*x+d)^(1/2)-1/3*d/(e*x+d)^(3/2))-b*(1/(e*x+d)^(1/2)*arccsc(c
*x)-1/3*arccsc(c*x)*d/(e*x+d)^(3/2)-2/3/c*(2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(
1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e)
)^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*(e*x+d)^
(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)
*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2
*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+
e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2)
```

```
)*c^2*d^2*(e*x+d)^(1/2)+(c/(c*d-e))^(1/2)*c^2*d*(e*x+d)^2-((-c*(e*x+d)+c*d-
e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2
)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e*(e*x+d)^(1/2)+((-c*(e*x+
d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+
d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e*(e*x+d)^(1/2)-2*(
c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)-2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*
(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/
c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*e^2*(e*x+d)^(1/2)+(c/(c*d-
e))^(1/2)*c^2*d^3-(c/(c*d-e))^(1/2)*d*e^2)/(c*d-e)/(e*x+d)^(1/2)/(c*d+e)/(c
/(c*d-e))^(1/2)/d/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^
2)^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x + d)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2),x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2), x)
```

# 3.72 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

**Optimal**. Leaf size=298

$$\frac{4be(1-c^2x^2)}{3cd\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}-\frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}}-\frac{4b\sqrt{d+ex}\,\sqrt{1-c^2x^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right.\right.}{3d\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex}{cd+e}}}$$

```
[Out] -2/3*(a+b*arccsc(c*x))/e/(e*x+d)^(3/2)+4/3*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e^
2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2
^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d
^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/3*b*EllipticPi(1/
2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(
1/2)*(-c^2*x^2+1)^(1/2)/c/d/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]
time = 0.30, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5335, 1588, 972, 759, 21, 733, 435, 947, 174, 552, 551}

$$-\frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}}-\frac{4b\sqrt{1-c^2x^2}\,\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3dx\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{4b\sqrt{1-c^2x^2}\,\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3cdex\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{d+ex}}+\frac{4be(1-c^2x^2)}{3cdx\sqrt{1-\frac{1}{c^2x^2}}\,(c^2d^2-e^2)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*e*(1 - c^2*x^2))/(3*c*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d
 + e*x]) - (2*(a + b*ArcCsc[c*x]))/(3*e*(d + e*x)^(3/2)) - (4*b*Sqrt[d + e*
x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
e)])/(3*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d +
 e)]) + (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2,
ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*d*e*Sqrt[1 - 1/(c^2*x
^2)]*x*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
```

```
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{(2b)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)^{3/2}}\,dx}{3ce}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}} + \frac{}{dx\sqrt{d+ex}\ldots}\right.}{3ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{3cd\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\left(2b\sqrt{-\frac{1}{c^2}\ldots}\right.}{}$$

$$= \frac{4be(1-c^2x^2)}{3cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{\left(4b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\ldots}{3cd\left(d^2-\frac{e}{c}\ldots\right.}$$

$$= \frac{4be(1-c^2x^2)}{3cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int\ldots}{3cd\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\ldots}}$$

$$= \frac{4be(1-c^2x^2)}{3cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{\left(4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1\ldots}\right.}{}$$

$$= \frac{4be(1-c^2x^2)}{3cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{4b\sqrt{d+ex}\,\sqrt{1-c^2x\ldots}}{3d\left(c^2d^2-e^2\right)\ldots}$$

**Mathematica** [B] Leaf count is larger than twice the leaf count of optimal. 608 vs. 2(298) = 596.

time = 18.79, size = 608, normalized size = 2.04

$$\frac{2\left(-\frac{a}{e}+\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(d+ex)^2}{c^2d^3-de^2}-\frac{bex^2\csc^{-1}(cx)}{d^2}-\frac{b(d+ex)^2\csc^{-1}(cx)}{d^2e}+\frac{2bx(d+ex)\left(-cde\sqrt{1-\frac{1}{c^2x^2}}+(c^2d^2-e^2)\csc^{-1}(cx)\right)}{c^2d^4-d^2e^2}+\frac{2bd\left(\frac{c(d+ex)}{cd+e}\right)^{3/2}\sqrt{1-c^2x^2}\,\Pi\left(2;\mathrm{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{(cd-e)e\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{2bc(d+ex)\cos(2\csc^{-1}(cx))\left((d+ex)(-1+c^2x^2)+cdx\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)-\frac{x(1+cx)\sqrt{\frac{c(d+ex)}{cd-e}}\sqrt{\frac{e-cex}{cd+e}}\left((cd+e)E\left(\mathrm{ArcSin}\left(\sqrt{\frac{c(d+ex)}{cd-e}}\right)\middle|\frac{cd-e}{cd+e}\right)-eF\left(\mathrm{ArcSin}\left(\sqrt{\frac{c(d+ex)}{cd-e}}\right)\middle|\frac{cd-e}{cd+e}\right)\right)}{\sqrt{\frac{e(1+cx)}{-cd+e}}}+ex\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\Pi\left(2;\mathrm{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)\right)}{d(cd-e)(cd+e)\sqrt{1-\frac{1}{c^2x^2}}(-2+c^2x^2)}\right)}{3(d+ex)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (2*(-(a/e) + (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*(d + e*x)^2)/(c^2*d^3 - d*e^2) -
(b*e*x^2*ArcCsc[c*x])/d^2 - (b*(d + e*x)^2*ArcCsc[c*x])/(d^2*e) + (2*b*x*(d
 + e*x)*(-(c*d*e*Sqrt[1 - 1/(c^2*x^2)]) + (c^2*d^2 - e^2)*ArcCsc[c*x]))/(c^
2*d^4 - d^2*e^2) + (2*b*d*((c*(d + e*x))/(c*d + e))^(3/2)*Sqrt[1 - c^2*x^2]
*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/((c*d - e)*
e*Sqrt[1 - 1/(c^2*x^2)]*x) - (2*b*c*(d + e*x)*Cos[2*ArcCsc[c*x]]*((d + e*x)
*(-1 + c^2*x^2) + c*d*x*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ell
ipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (x*(1 + c*x)*Sqrt[
(c*(d + e*x))/(c*d - e)]*Sqrt[(e - c*e*x)/(c*d + e)]*((c*d + e)*EllipticE[A
rcSin[Sqrt[(c*(d + e*x))/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[Ar
cSin[Sqrt[(c*(d + e*x))/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*
x))/(-(c*d) + e)] + e*x*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ell
ipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(d*(c*d - e)*(
c*d + e)*Sqrt[1 - 1/(c^2*x^2)]*(-2 + c^2*x^2))))/(3*(d + e*x)^(3/2))
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 874 vs. 2(270) = 540.

time = 0.90, size = 875, normalized size = 2.94

| method | result |
|---|---|
| derivativedivides | $-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\mathrm{arccsc}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\ \mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd}{cd}}\right.}{3}\right.$ |
| default | $-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\mathrm{arccsc}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\ \mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd}{cd}}\right.}{3}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arccsc(c*x)+2/3/c*(((-c*(e*
x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*
x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)
-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elli
pticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*(e*x
+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(
1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(
1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2*d*(e*x+
d)^2+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*
EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e*(e
*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))
^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c
*d*e*(e*x+d)^(1/2)+2*(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)-((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*e^2*(e
*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2*d^3+(c/(c*d-e))^(1/2)*d*e^2)/(c*d-e)/(c/(
c*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/d^2/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c
^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

**Maxima** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*sqrt(x*e + d)/(x^3*e^3 + 3*d*x^2*e^2 + 3*d^2*x
*e + d^3), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x + d)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^(5/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^(5/2), x)
```

# 3.73 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$

**Optimal**. Leaf size=24

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x)
```

**Rubi** [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

**Mathematica** [A]
time = 44.76, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)), x]
```

**Maple** [A]
time = 8.01, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x\,(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x)
```

**Maxima** [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(b*d^2*x^2*e^2 + 2*b*d^3*x*e + b*d^4)*sqrt(d)*integrate(arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1))/((x^3*e^2 + 2*d*x^2*e + d^2*x)*sqrt(x*e + d)),
x) + 2*(3*a*x*e + 4*a*d)*sqrt(x*e + d)*sqrt(d) + 3*(a*x^2*e^2 + 2*a*d*x*e +
 a*d^2)*log(x*e/(x*e + 2*sqrt(x*e + d)*sqrt(d) + 2*d)))/((d^2*x^2*e^2 + 2*d
^3*x*e + d^4)*sqrt(d))
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*sqrt(x*e + d)/(x^4*e^3 + 3*d*x^3*e^2 + 3*d^2*x
^2*e + d^3*x), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x + d)^(5/2)*x), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(5/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(5/2)), x)
```

# 3.74 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$

**Optimal**. Leaf size=24

$$\mathrm{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x)
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

**Mathematica** [A]
time = 43.61, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

**Maple** [A]
time = 7.74, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x^2\,(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*(b*d^3*x^2*e + b*d^4*x)*sqrt(d)*integrate(arctan2(1, sqrt(c*x + 1
)*sqrt(c*x - 1))/((x^4*e^2 + 2*d*x^3*e + d^2*x^2)*sqrt(x*e + d)), x) - 5*(a
*x^2*e^2 + a*d*x*e)*log(x*e/(x*e + 2*sqrt(x*e + d)*sqrt(d) + 2*d)))*sqrt(x*
e + d) - 2*(15*a*x^2*e^2 + 20*a*d*x*e + 3*a*d^2)*sqrt(d))/((d^3*x^2*e + d^4
*x)*sqrt(x*e + d)*sqrt(d))
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*sqrt(x*e + d)/(x^5*e^3 + 3*d*x^4*e^2 + 3*d^2*x
^3*e + d^3*x^2), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x + d)^(5/2)*x^2), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(5/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(5/2)), x)
```

# 3.75 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

**Optimal**. Leaf size=540

$$\frac{4be(1-c^2x^2)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}}+\frac{16bce(1-c^2x^2)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}+\frac{4be(1-c^2x}{5cd^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{}{c^2}}}$$

```
[Out] -2/5*(a+b*arccsc(c*x))/e/(e*x+d)^(5/2)+4/15*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e
^2)/x/(e*x+d)^(3/2)/(1-1/c^2/x^2)^(1/2)+16/15*b*c*e*(-c^2*x^2+1)/(c^2*d^2-e
^2)^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/5*b*e*(-c^2*x^2+1)/c/d^2/(c^2*d
^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*(7*c^2*d^2-3*e^2)*Ellipt
icE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c
^2*x^2+1)^(1/2)/(c^2*d^3-d*e^2)^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))
^(1/2)+4/15*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2
))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e^2)/x/(1-1/c^2/
x^2)^(1/2)/(e*x+d)^(1/2)+4/5*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1
/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d^2/e
/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi** [A]
time = 0.54, antiderivative size = 637, normalized size of antiderivative = 1.18, number of steps used = 19, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {5335, 1588, 972, 759, 849, 858, 733, 435, 430, 21, 947, 174, 552, 551}

$$-\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}}+\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15dx\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{16bc^2\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15x\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)^2\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{4b\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5d^2x\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}+\frac{4b\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5cd^2ex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}+\frac{16bce(1-c^2x^2)}{15x\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)^2\sqrt{d+ex}}+\frac{4be(1-c^2x^2)}{5cd^2x\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)\sqrt{d+ex}}+\frac{4be(1-c^2x^2)}{15cdx\sqrt{1-\frac{1}{c^2x^2}}\left(c^2d^2-e^2\right)(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (4*b*e*(1 - c^2*x^2))/(15*c*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
e*x)^(3/2)) + (16*b*c*e*(1 - c^2*x^2))/(15*(c^2*d^2 - e^2)^2*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[d + e*x]) + (4*b*e*(1 - c^2*x^2))/(5*c*d^2*(c^2*d^2 - e^2)*S
qrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (2*(a + b*ArcCsc[c*x]))/(5*e*(d + e
*x)^(5/2)) - (16*b*c^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqr
t[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*(c^2*d^2 - e^2)^2*Sqrt[1 - 1/(c^
2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*Sqrt[d + e*x]*Sqrt[1 - c^2*
x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*d^2*(c^2
*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*S
qrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*d*(c^2*d^2 - e^2)*Sqrt[1 - 1/(c^2*x^2)]*
x*Sqrt[d + e*x]) + (4*b*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ell
```

```
ipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*c*d^2*e*Sqrt
[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 5335

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}} - \frac{(2b)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2(d+ex)^{5/2}}\,dx}{5ce}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{5/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}} - \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{5/2}\sqrt{-\frac{1}{c^2}+x^2}} - \frac{}{d^2(d+ex)^{3/2}\sqrt{}}\right.}{5ce\sqrt{1-\frac{1}{c^2x^2}}\,}$$

$$= -\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}} + \frac{\left(2b\sqrt{-\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{-\frac{1}{c^2}+x^2}}\,dx}{5cd^2\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{\left(2b\sqrt{-\frac{1}{c^2}\right.}{}$$

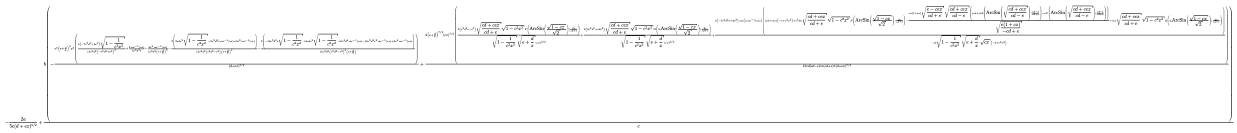
$$= \frac{4be(1-c^2x^2)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} + \frac{4be(1-c^2x^2)}{5cd^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= \frac{4be(1-c^2x^2)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} + \frac{16bce(1-c^2x^2)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= \frac{4be(1-c^2x^2)}{15cd\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} + \frac{16bce(1-c^2x^2)}{15\left(c^2d^2-e^2\right)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= \frac{4be(1-c^2x^2)}{} + \frac{16bce(1-c^2x^2)}{}$$

**Mathematica** [A]
time = 30.85, size = 1002, normalized size = 1.86



Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (-2*a)/(5*e*(d + e*x)^(5/2)) + (b*(-((c^4*(e + d/x)^4*x^4*((4*(-7*c^2*d^2 +
 3*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(15*c^2*d^2*(-(c^2*d^2) + e^2)^2) + (2*ArcCs
c[c*x])/(5*c^3*d^3*e) - (2*e^2*ArcCsc[c*x])/(5*c^3*d^3*(e + d/x)^3) - (2*(2
*c*d*e^2*Sqrt[1 - 1/(c^2*x^2)] - 9*c^2*d^2*e*ArcCsc[c*x] + 9*e^3*ArcCsc[c*x
]))/(15*c^3*d^3*(c^2*d^2 - e^2)*(e + d/x)^2) - (2*(-16*c^3*d^3*e*Sqrt[1 - 1
/(c^2*x^2)] + 8*c*d*e^3*Sqrt[1 - 1/(c^2*x^2)] + 9*c^4*d^4*ArcCsc[c*x] - 18*
c^2*d^2*e^2*ArcCsc[c*x] + 9*e^4*ArcCsc[c*x]))/(15*c^3*d^3*(c^2*d^2 - e^2)^2
*(e + d/x))))/(d + e*x)^(7/2)) + (2*(e + d/x)^(7/2)*(c*x)^(7/2)*((2*(c^2*d^
2*e - e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin
[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e +
d/x]*(c*x)^(3/2)) + (2*(3*c^3*d^3 + c*d*e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*
Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
 e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-7*c^2*d^2*e
+ 3*e^3)*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c
*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqr
t[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(
c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d
 - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d
- e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqr
t[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 -
c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*
Sqrt[c*x]*(-2 + c^2*x^2))))/(15*c*d*(c*d - e)^2*e*(c*d + e)^2*(d + e*x)^(7/
2))))/c
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1619 vs. $2(491) = 982$.
time = 0.93, size = 1620, normalized size = 3.00

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1620 |
| default | Expression too large to display | 1620 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arccsc(c*x)+2/15/c*(6*((-c*
(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3
/2)-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)
*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^4*d^4
*(e*x+d)^(3/2)+3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*
d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*
d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)-7*(c/(c*d-e))^(1/2)*c^
4*d^3*(e*x+d)^3+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c
*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1
/2))*c^3*d^3*e*(e*x+d)^(3/2)-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x
+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e
)/(c*d+e))^(1/2))*c^3*d^3*e*(e*x+d)^(3/2)+13*(c/(c*d-e))^(1/2)*c^4*d^4*(e*x
+d)^2-2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/
2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d
^2*e^2*(e*x+d)^(3/2)+3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+
e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e
))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2)-6*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((
-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2)
,1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/
2)-5*(c/(c*d-e))^(1/2)*c^4*d^5*(e*x+d)+3*(c/(c*d-e))^(1/2)*c^2*d*e^2*(e*x+d
)^3-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)
*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^3
*(e*x+d)^(3/2)+3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*
d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/
2))*c*d*e^3*(e*x+d)^(3/2)-(c/(c*d-e))^(1/2)*c^4*d^6-5*(c/(c*d-e))^(1/2)*c^2
*d^2*e^2*(e*x+d)^2+3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)
/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c
/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*e^4*(e*x+d)^(3/2)+8*(c/(c*d-e))^(1/2)*c^
2*d^3*e^2*(e*x+d)+2*(c/(c*d-e))^(1/2)*c^2*d^4*e^2-3*(c/(c*d-e))^(1/2)*d*e^4
*(e*x+d)-(c/(c*d-e))^(1/2)*d^2*e^4)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(3/2)
/(c*d+e)/(c^2*d^2-e^2)/d^3/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c
^2/e^2/x^2)^(1/2)))
```

**Maxima [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(%e-c*d>0)', see `assume?` for more
details
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
[Out] integral((b*arccsc(c*x) + a)*sqrt(x*e + d)/(x^4*e^4 + 4*d*x^3*e^3 + 6*d^2*x
^2*e^2 + 4*d^3*x*e + d^4), x)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x+d)**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/(e*x + d)^(7/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{{\left(d+e\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x)^(7/2),x)
[Out] int((a + b*asin(1/(c*x)))/(d + e*x)^(7/2), x)
```

# 3.76 $\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=206

$$\frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}}+\frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)$$

```
[Out] 1/5*d*x^5*(a+b*arccsc(c*x))+1/7*e*x^7*(a+b*arccsc(c*x))+1/560*b*(42*c^2*d+2
5*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^6/(c^2*x^2)^(1/2)+1/560*b*(42*c^2*d
+25*e)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)+1/840*b*(42*c^2*d+25*e)*x^
4*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)+1/42*b*e*x^6*(c^2*x^2-1)^(1/2)/c/(c
^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.368$, Rules used = {14, 5347, 12, 470, 327, 223, 212}

$$\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)+\frac{bex^6\sqrt{c^2x^2-1}}{42c\sqrt{c^2x^2}}+\frac{bx(42c^2d+25e)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{560c^6\sqrt{c^2x^2}}+\frac{bx^2\sqrt{c^2x^2-1}\,(42c^2d+25e)}{560c^5\sqrt{c^2x^2}}+\frac{bx^4\sqrt{c^2x^2-1}\,(42c^2d+25e)}{840c^3\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(42*c^2*d + 25*e)*x^2*Sqrt[-1 + c^2*x^2])/(560*c^5*Sqrt[c^2*x^2]) + (b*(
42*c^2*d + 25*e)*x^4*Sqrt[-1 + c^2*x^2])/(840*c^3*Sqrt[c^2*x^2]) + (b*e*x^6
*Sqrt[-1 + c^2*x^2])/(42*c*Sqrt[c^2*x^2]) + (d*x^5*(a + b*ArcCsc[c*x]))/5 +
 (e*x^7*(a + b*ArcCsc[c*x]))/7 + (b*(42*c^2*d + 25*e)*x*ArcTanh[(c*x)/Sqrt[
-1 + c^2*x^2]])/(560*c^6*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int \frac{x^4(7d+5ex^2}{35\sqrt{-1+c^2}}}{\sqrt{c^2x^2}}$$

$$=\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int \frac{x^4(7d+5ex^2)}{\sqrt{-1+c^2x}}}{35\sqrt{c^2x^2}}$$

$$=\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)$$

$$=\frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc\right.$$

$$=\frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}}+\frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{be}{}$$

$$=\frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}}+\frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{be}{}$$

$$=\frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}}+\frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{be}{}$$

**Mathematica** [A]
time = 0.17, size = 140, normalized size = 0.68

$$\frac{48ac^7x^5(7d+5ex^2)+bc^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2(75e+2c^2(63d+25ex^2)+c^4(84dx^2+40ex^4))+48bc^7x^5(7d+5ex^2)\csc^{-1}(cx)+3b(42c^2d+25e)\log\left(\left(1+\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{1680c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (48*a*c^7*x^5*(7*d + 5*e*x^2) + b*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*(75*e + 2*c
^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5*e
*x^2)*ArcCsc[c*x] + 3*b*(42*c^2*d + 25*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x
])/(1680*c^7)
```

**Maple** [A]
time = 0.41, size = 341, normalized size = 1.66

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\,\mathrm{arccsc}(cx)\,d\,c^5x^5}{5}+\frac{b\,c^5\mathrm{arccsc}(cx)\,e\,x^7}{7}+\frac{b\left(c^2x^2-1\right)c^2x^2d}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\,c^2\left(c^2x^2-1\right)x^4e}{42\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\left(c^2x^2-1\right)d}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{5b\left(c^2\right.}{168\sqrt{}}}{}$ |
| default | $\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\,\mathrm{arccsc}(cx)\,d\,c^5x^5}{5}+\frac{b\,c^5\mathrm{arccsc}(cx)\,e\,x^7}{7}+\frac{b\left(c^2x^2-1\right)c^2x^2d}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\,c^2\left(c^2x^2-1\right)x^4e}{42\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\left(c^2x^2-1\right)d}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{5b\left(c^2\right.}{168\sqrt{}}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(a/c^2*(1/5*d*c^7*x^5+1/7*e*c^7*x^7)+1/5*b*arccsc(c*x)*d*c^5*x^5+1/7*
b*c^5*arccsc(c*x)*e*x^7+1/20*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*c^2*
x^2*d+1/42*b*c^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^4*e+3/40*b*(c^2*
x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d+5/168*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x
^2)^(1/2)*x^2*e+3/40*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*d*
ln(c*x+(c^2*x^2-1)^(1/2))+5/112*b/c^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/
2)*e+5/112*b/c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e*ln(c*x+(
c^2*x^2-1)^(1/2)))
```

**Maxima** [A]

time = 0.26, size = 298, normalized size = 1.45

$$\frac{1}{7}ax^7e+\frac{1}{5}adx^5+\frac{1}{80}\left(16x^5\,\mathrm{arccsc}(cx)-\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bd+\frac{1}{672}\left(96x^7\,\mathrm{arccsc}(cx)+\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3c^6\left(\frac{1}{c^2x^2}-1\right)^2+3c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7*e + 1/5*a*d*x^5 + 1/80*(16*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2)
+ 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/
(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(
-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d + 1/672*(96*x^7*arccsc(c*x) + (2*(15*(-1
/(c^2*x^2) + 1)^(5/2) - 40*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2)
+ 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*
x^2) - 1) + c^6) + 15*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/
(c^2*x^2) + 1) - 1)/c^6)/c)*b*e
```

**Fricas** [A]

time = 0.46, size = 196, normalized size = 0.95

$$\frac{240\,ac^7x^7e+336\,ac^7dx^5+48\left(7\,bc^7dx^5-7\,bc^7d+5\left(bc^7x^7-bc^7\right)e\right)\mathrm{arccsc}\left(cx\right)-96\left(7\,bc^7d+5\,bc^7e\right)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-3\left(42\,bc^2d+25\,be\right)\log\left(-cx+\sqrt{c^2x^2-1}\right)+\left(84\,bc^5dx^3+126\,bc^3dx+5\left(8\,bc^5x^5+10\,bc^3x^3+15\,bcx\right)e\right)\sqrt{c^2x^2-1}}{1680\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*x^7*e + 336*a*c^7*d*x^5 + 48*(7*b*c^7*d*x^5 - 7*b*c^7*d +
 5*(b*c^7*x^7 - b*c^7)*e)*arccsc(c*x) - 96*(7*b*c^7*d + 5*b*c^7*e)*arctan(-
c*x + sqrt(c^2*x^2 - 1)) - 3*(42*b*c^2*d + 25*b*e)*log(-c*x + sqrt(c^2*x^2
- 1)) + (84*b*c^5*d*x^3 + 126*b*c^3*d*x + 5*(8*b*c^5*x^5 + 10*b*c^3*x^3 + 1
5*b*c*x)*e)*sqrt(c^2*x^2 - 1))/c^7
```

**Sympy** [A]
time = 14.91, size = 408, normalized size = 1.98

$$\frac{adx^5}{5}+\frac{aex^7}{7}+\frac{bdx^5\operatorname{acsc}(cx)}{5}+\frac{bex^7\operatorname{acsc}(cx)}{7}+\frac{bd\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for}\ |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}+\frac{be\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\operatorname{acosh}(cx)}{16c^6} & \text{for}\ |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\operatorname{asin}(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*acsc(c*x)/5 + b*e*x**7*acsc(c*x)/7 + b*d
*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1))
 - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2)
 > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 +
1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/
(5*c) + b*e*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1)) + x**5/(24*c*sqrt(c**
2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1)) - 5*x/(16*c**5*sqrt(c**
2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2) > 1), (-I*c*x**7/(6*s
qrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 + 1)) - 5*I*x**3/(48*c*
*3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*x**2 + 1)) - 5*I*asin(
c*x)/(16*c**6), True))/(7*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1166 vs. $2(178) = 356$.
time = 1.77, size = 1166, normalized size = 5.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*b*e*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*arcsin(1/(c*x))/c + 15*a
*e*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7/c + 5*b*e*x^6*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^6/c^2 + 84*b*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c +
 84*a*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 105*b*e*x^5*(sqrt(-1/(c^2*x^
2) + 1) + 1)^5*arcsin(1/(c*x))/c^3 + 105*a*e*x^5*(sqrt(-1/(c^2*x^2) + 1) +
1)^5/c^3 + 42*b*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^2 + 45*b*e*x^4*(sqrt
(-1/(c^2*x^2) + 1) + 1)^4/c^4 + 420*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*
arcsin(1/(c*x))/c^3 + 420*a*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^3 + 315*
```

```
b*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^5 + 315*a*e*x^3*(s
qrt(-1/(c^2*x^2) + 1) + 1)^3/c^5 + 336*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)
^2/c^4 + 225*b*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^6 + 840*b*d*x*(sqrt(-
1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 840*a*d*x*(sqrt(-1/(c^2*x^2) +
1) + 1)/c^5 + 525*b*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^7 +
525*a*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^7 + 1008*b*d*log(sqrt(-1/(c^2*x^2)
 + 1) + 1)/c^6 - 1008*b*d*log(1/(abs(c)*abs(x)))/c^6 + 600*b*e*log(sqrt(-1/
(c^2*x^2) + 1) + 1)/c^8 - 600*b*e*log(1/(abs(c)*abs(x)))/c^8 + 840*b*d*arcs
in(1/(c*x))/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 840*a*d/(c^7*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) + 525*b*e*arcsin(1/(c*x))/(c^9*x*(sqrt(-1/(c^2*x^2) +
1) + 1)) + 525*a*e/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 336*b*d/(c^8*x^2*
(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 225*b*e/(c^10*x^2*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^2) + 420*b*d*arcsin(1/(c*x))/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 + 420*a*d/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 315*b*e*arcsin(1/(c*x
))/(c^11*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 315*a*e/(c^11*x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3) - 42*b*d/(c^10*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) -
45*b*e/(c^12*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 84*b*d*arcsin(1/(c*x))/(
c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 84*a*d/(c^11*x^5*(sqrt(-1/(c^2*x
^2) + 1) + 1)^5) + 105*b*e*arcsin(1/(c*x))/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1
) + 1)^5) + 105*a*e/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - 5*b*e/(c^14
*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 15*b*e*arcsin(1/(c*x))/(c^15*x^7*(sq
rt(-1/(c^2*x^2) + 1) + 1)^7) + 15*a*e/(c^15*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1
)^7))*c
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^4*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

# 3.77 $\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=161

$$\frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)+\frac{b(20c^2d\cdots}{}$$

```
[Out] 1/3*d*x^3*(a+b*arccsc(c*x))+1/5*e*x^5*(a+b*arccsc(c*x))+1/120*b*(20*c^2*d+9
*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^4/(c^2*x^2)^(1/2)+1/120*b*(20*c^2*d+
9*e)*x^2*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)+1/20*b*e*x^4*(c^2*x^2-1)^(1/
2)/c/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {14, 5347, 12, 470, 327, 223, 212}

$$\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)+\frac{bex^4\sqrt{c^2x^2-1}}{20c\sqrt{c^2x^2}}+\frac{bx(20c^2d+9e)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{120c^4\sqrt{c^2x^2}}+\frac{bx^2\sqrt{c^2x^2-1}\,(20c^2d+9e)}{120c^3\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(20*c^2*d + 9*e)*x^2*Sqrt[-1 + c^2*x^2])/(120*c^3*Sqrt[c^2*x^2]) + (b*e*
x^4*Sqrt[-1 + c^2*x^2])/(20*c*Sqrt[c^2*x^2]) + (d*x^3*(a + b*ArcCsc[c*x]))/
3 + (e*x^5*(a + b*ArcCsc[c*x]))/5 + (b*(20*c^2*d + 9*e)*x*ArcTanh[(c*x)/Sqr
t[-1 + c^2*x^2]])/(120*c^4*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^2(5d+3ex^2)}{15\sqrt{-1+c^2}}}{\sqrt{c^2x^2}}$$

$$=\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^2(5d+3ex^2)}{\sqrt{-1+c^2x}}}{15\sqrt{c^2x^2}}$$

$$=\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)$$

$$=\frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-}\right.$$

$$=\frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-}\right.$$

$$=\frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-}\right.$$

**Mathematica [A]**
time = 0.12, size = 121, normalized size = 0.75

$$\frac{c^2x^2\left(8ac^3x(5d+3ex^2)+b\sqrt{1-\frac{1}{c^2x^2}}\,(9e+c^2(20d+6ex^2))\right)+8bc^5x^3(5d+3ex^2)\csc^{-1}(cx)+b(20c^2d+9e)\log\left(\left(1+\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
[Out] (c^2*x^2*(8*a*c^3*x*(5*d + 3*e*x^2) + b*Sqrt[1 - 1/(c^2*x^2)]*(9*e + c^2*(2
0*d + 6*e*x^2))) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcCsc[c*x] + b*(20*c^2*d +
9*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(120*c^5)
```

**Maple [A]**
time = 0.42, size = 267, normalized size = 1.66

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\,\mathrm{arccsc}(cx)d\,c^3x^3}{3}+\frac{b\,c^3\mathrm{arccsc}(cx)e\,x^5}{5}+\frac{b\left(c^2x^2-1\right)d}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\left(c^2x^2-1\right)x^2e}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{\right.}}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c^3}$ |

| | |
|---|---|
| default | $\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\,\mathrm{arccsc}(cx)d\,c^3x^3}{3}+\frac{b\,c^3\mathrm{arccsc}(cx)e\,x^5}{5}+\frac{b\left(c^2x^2-1\right)d}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\left(c^2x^2-1\right)x^2e}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\right.}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{c^3}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/c^2*(1/3*d*c^5*x^3+1/5*e*c^5*x^5)+1/3*b*arccsc(c*x)*d*c^3*x^3+1/5*
b*c^3*arccsc(c*x)*e*x^5+1/6*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d+1/2
0*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e+1/6*b*(c^2*x^2-1)^(1/2)/(
(c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*d*ln(c*x+(c^2*x^2-1)^(1/2))+3/40*b/c^2*(c^2*
x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e+3/40*b/c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2
-1)/c^2/x^2)^(1/2)/x*e*ln(c*x+(c^2*x^2-1)^(1/2)))
```

**Maxima** [A]
time = 0.26, size = 234, normalized size = 1.45

$$\frac{1}{5}ax^5e+\frac{1}{3}adx^3+\frac{1}{12}\left(4x^3\,\mathrm{arccsc}\,(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd+\frac{1}{80}\left(16x^5\,\mathrm{arccsc}\,(cx)-\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5*e + 1/3*a*d*x^3 + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2)
+ 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 -
log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d + 1/80*(16*x^5*arccsc(c*x) - (2
*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2)
- 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)
/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e
```

**Fricas** [A]
time = 0.41, size = 177, normalized size = 1.10

$$\frac{24\,ac^5x^5e+40\,ac^5dx^3+8\left(5\,bc^5dx^3-5\,bc^5d+3\left(bc^5x^5-bc^5\right)e\right)\mathrm{arccsc}\,(cx)-16\left(5\,bc^5d+3\,bc^5e\right)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-\left(20\,bc^2d+9\,be\right)\log\left(-cx+\sqrt{c^2x^2-1}\right)+\left(20\,bc^3dx+3\left(2\,bc^3x^3+3\,bcx\right)e\right)\sqrt{c^2x^2-1}}{120\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*x^5*e + 40*a*c^5*d*x^3 + 8*(5*b*c^5*d*x^3 - 5*b*c^5*d + 3*(
b*c^5*x^5 - b*c^5)*e)*arccsc(c*x) - 16*(5*b*c^5*d + 3*b*c^5*e)*arctan(-c*x
+ sqrt(c^2*x^2 - 1)) - (20*b*c^2*d + 9*b*e)*log(-c*x + sqrt(c^2*x^2 - 1)) +
 (20*b*c^3*d*x + 3*(2*b*c^3*x^3 + 3*b*c*x)*e)*sqrt(c^2*x^2 - 1))/c^5
```

**Sympy** [A]
time = 6.30, size = 294, normalized size = 1.83

$$\frac{adx^3}{3} + \frac{aex^5}{5} + \frac{bdx^3\operatorname{acsc}(cx)}{3} + \frac{bex^5\operatorname{acsc}(cx)}{5} + \frac{bd\left(\begin{cases} \frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1 \\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise} \end{cases}\right)}{3c} + \frac{be\left(\begin{cases} \frac{cx^5}{4\sqrt{c^2x^2-1}} + \frac{x^3}{8c\sqrt{c^2x^2-1}} - \frac{3x}{8c^3\sqrt{c^2x^2-1}} + \frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1 \\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}} - \frac{ix^3}{8c\sqrt{-c^2x^2+1}} + \frac{3ix}{8c^3\sqrt{-c^2x^2+1}} - \frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise} \end{cases}\right)}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*acsc(c*x)/3 + b*e*x**5*acsc(c*x)/5 + b*d
*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**
2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1
)) - I*asin(c*x)/(2*c**2), True))/(3*c) + b*e*Piecewise((c*x**5/(4*sqrt(c**
2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2
- 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**
2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2
*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 822 vs. 2(139) = 278.
time = 1.28, size = 822, normalized size = 5.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/960*(6*b*e*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c + 6*a*e*x
^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 3*b*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1
)^4/c^2 + 40*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + 40*
a*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c + 30*b*e*x^3*(sqrt(-1/(c^2*x^2) +
1) + 1)^3*arcsin(1/(c*x))/c^3 + 30*a*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c
^3 + 40*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 24*b*e*x^2*(sqrt(-1/(c
^2*x^2) + 1) + 1)^2/c^4 + 120*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(
c*x))/c^3 + 120*a*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 60*b*e*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 60*a*e*x*(sqrt(-1/(c^2*x^2) + 1)
+ 1)/c^5 + 160*b*d*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 160*b*d*log(1/(abs
(c)*abs(x)))/c^4 + 72*b*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 72*b*e*log(
1/(abs(c)*abs(x)))/c^6 + 120*b*d*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2)
+ 1) + 1)) + 120*a*d/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 60*b*e*arcsin(1
/(c*x))/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 60*a*e/(c^7*x*(sqrt(-1/(c^2*
x^2) + 1) + 1)) - 40*b*d/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 24*b*e/
(c^8*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 40*b*d*arcsin(1/(c*x))/(c^7*x^3*
(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 40*a*d/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) +
 1)^3) + 30*b*e*arcsin(1/(c*x))/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) +
```

```
30*a*e/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 3*b*e/(c^10*x^4*(sqrt(-1/
(c^2*x^2) + 1) + 1)^4) + 6*b*e*arcsin(1/(c*x))/(c^11*x^5*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^5) + 6*a*e/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))*c
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

# 3.78 $\int (d+ex^2)(a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=109

$$\frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}+dx(a+b\csc^{-1}(cx))+\frac{1}{3}ex^3(a+b\csc^{-1}(cx))+\frac{b(6c^2d+e)\,x\tanh^{-1}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

```
[Out] d*x*(a+b*arccsc(c*x))+1/3*e*x^3*(a+b*arccsc(c*x))+1/6*b*(6*c^2*d+e)*x*arcta
nh(c*x/(c^2*x^2-1)^(1/2))/c^2/(c^2*x^2)^(1/2)+1/6*b*e*x^2*(c^2*x^2-1)^(1/2)
/c/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5337, 12, 396, 223, 212}

$$dx(a+b\csc^{-1}(cx))+\frac{1}{3}ex^3(a+b\csc^{-1}(cx))+\frac{bx(6c^2d+e)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{6c^2\sqrt{c^2x^2}}+\frac{bex^2\sqrt{c^2x^2-1}}{6c\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*x^2*Sqrt[-1 + c^2*x^2])/(6*c*Sqrt[c^2*x^2]) + d*x*(a + b*ArcCsc[c*x])
+ (e*x^3*(a + b*ArcCsc[c*x]))/3 + (b*(6*c^2*d + e)*x*ArcTanh[(c*x)/Sqrt[-1
+ c^2*x^2]])/(6*c^2*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5337

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\int (d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{3d+ex^2}{3\sqrt{-1+c^2x^2}}}{\sqrt{c^2x^2}}$$

$$= dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{3d+ex^2}{\sqrt{-1+c^2x^2}}\,d}{3\sqrt{c^2x^2}}$$

$$= \frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)-$$

$$= \frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)-$$

$$= \frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+$$

**Mathematica** [A]
time = 0.19, size = 149, normalized size = 1.37

$$adx+\frac{1}{3}aex^3+\frac{bex^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{6c}+bdx\csc^{-1}(cx)+\frac{1}{3}bex^3\csc^{-1}(cx)+\frac{bd\sqrt{1-\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}+\frac{be\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (b*e*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + b*d*
x*ArcCsc[c*x] + (b*e*x^3*ArcCsc[c*x])/3 + (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*ArcT
anh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2] + (b*e*Log[x*(1 + Sqrt[(-
1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

**Maple [A]**
time = 0.27, size = 191, normalized size = 1.75

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+b\,\mathrm{arccsc}(cx)dcx+\frac{bc\,\mathrm{arccsc}(cx)e\,x^3}{3}+\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{c^2x^2-1}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{b\left(c^2x^2-1\right)e}{6c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{}}{}}{c}$ |
| default | $\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+b\,\mathrm{arccsc}(cx)dcx+\frac{bc\,\mathrm{arccsc}(cx)e\,x^3}{3}+\frac{b\sqrt{c^2x^2-1}\,d\ln\left(cx+\sqrt{c^2x^2-1}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}+\frac{b\left(c^2x^2-1\right)e}{6c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{}}{}}{c}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^2*(d*c^3*x+1/3*e*c^3*x^3)+b*arccsc(c*x)*d*c*x+1/3*b*c*arccsc(c*x)*
e*x^3+b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*d*ln(c*x+(c^2*x^2
-1)^(1/2))+1/6*b/c^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e+1/6*b/c^3*(c
^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e*ln(c*x+(c^2*x^2-1)^(1/2)))
```

**Maxima [A]**
time = 0.27, size = 155, normalized size = 1.42

$$\frac{1}{3}\,ax^3e+adx+\frac{1}{12}\left(4\,x^3\,\mathrm{arccsc}\,(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be+\frac{\left(2\,cx\,\mathrm{arccsc}\,(cx)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e + a*d*x + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(
c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sq
rt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*e + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-
1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d/c
```

**Fricas [A]**
time = 0.43, size = 147, normalized size = 1.35

$$\frac{2\,ac^3x^3e+6\,ac^3dx+\sqrt{c^2x^2-1}\,bcxe+2\left(3\,bc^3dx-3\,bc^3d+\left(bc^3x^3-bc^3\right)e\right)\mathrm{arccsc}\,(cx)-4\left(3\,bc^3d+bc^3e\right)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-\left(6\,bc^2d+be\right)\log\left(-cx+\sqrt{c^2x^2-1}\right)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3*e + 6*a*c^3*d*x + sqrt(c^2*x^2 - 1)*b*c*x*e + 2*(3*b*c^3*d
*x - 3*b*c^3*d + (b*c^3*x^3 - b*c^3)*e)*arccsc(c*x) - 4*(3*b*c^3*d + b*c^3*
```

```
e)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (6*b*c^2*d + b*e)*log(-c*x + sqrt(c^2
*x^2 - 1)))/c^3
```

**Sympy [A]**
time = 4.56, size = 153, normalized size = 1.40

$$adx + \frac{aex^3}{3} + bdx\operatorname{acsc}(cx) + \frac{bex^3\operatorname{acsc}(cx)}{3} + \frac{bd\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} + \frac{be\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2| > 1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x + a*e*x**3/3 + b*d*x*acsc(c*x) + b*e*x**3*acsc(c*x)/3 + b*d*Piecewise
((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c + b*e*Piecewise(
(x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-
I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin
(c*x)/(2*c**2), True))/(3*c)
```

**Giac [B]** Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(95) = 190.
time = 1.09, size = 473, normalized size = 4.34

$$\frac{1}{24}\left(\frac{bex^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{aex^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c} + \frac{bex^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c^2} + \frac{12bdx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{12adx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c} + \frac{3bex\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{cx}\right)}{c^3} + \frac{3aex\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^3} + \frac{24bd\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{24bd\log\left(\frac{1}{|c||x|}\right)}{c^2} + \frac{4be\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4} - \frac{4be\log\left(\frac{1}{|c||x|}\right)}{c^4} + \frac{12bd\arcsin\left(\frac{1}{cx}\right)}{c^3x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} + \frac{12ad}{c^3x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} + \frac{3be\arcsin\left(\frac{1}{cx}\right)}{c^5x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} + \frac{3ae}{c^5x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)} - \frac{be}{c^6x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2} + \frac{be\arcsin\left(\frac{1}{cx}\right)}{c^7x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3} + \frac{ae}{c^7x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/24*(b*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + a*e*x^3*(s
qrt(-1/(c^2*x^2) + 1) + 1)^3/c + b*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2
 + 12*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c + 12*a*d*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)/c + 3*b*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(
c*x))/c^3 + 3*a*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 24*b*d*log(sqrt(-1/(
c^2*x^2) + 1) + 1)/c^2 - 24*b*d*log(1/(abs(c)*abs(x)))/c^2 + 4*b*e*log(sqrt
(-1/(c^2*x^2) + 1) + 1)/c^4 - 4*b*e*log(1/(abs(c)*abs(x)))/c^4 + 12*b*d*arc
sin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 12*a*d/(c^3*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) + 3*b*e*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1)
 + 1)) + 3*a*e/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - b*e/(c^6*x^2*(sqrt(-1
/(c^2*x^2) + 1) + 1)^2) + b*e*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3) + a*e/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)*(a + b*asin(1/(c*x))),x)
[Out] int((d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

# 3.79 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^2}\,dx$

**Optimal**. Leaf size=87

$$-\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{x}+ex(a+b\csc^{-1}(cx))+\frac{bex\tanh^{-1}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

```
[Out] -d*(a+b*arccsc(c*x))/x+e*x*(a+b*arccsc(c*x))+b*e*x*arctanh(c*x/(c^2*x^2-1)^
(1/2))/(c^2*x^2)^(1/2)-b*c*d*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 5347, 462, 223, 212}

$$-\frac{d(a+b\csc^{-1}(cx))}{x}+ex(a+b\csc^{-1}(cx))-\frac{bcd\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}}+\frac{bex\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] -((b*c*d*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*x^2]) - (d*(a + b*ArcCsc[c*x]))/x + e
*x*(a + b*ArcCsc[c*x]) + (b*e*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[c^2
*x^2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx = -\frac{d(a+b\csc^{-1}(cx))}{x} + ex\left(a+b\csc^{-1}(cx)\right) + \frac{(bcx)\int \frac{-d+ex^2}{x^2\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{x} + ex\left(a+b\csc^{-1}(cx)\right) + \frac{(bcex)}{}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{x} + ex\left(a+b\csc^{-1}(cx)\right) + \frac{(bcex)S}{}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{x} + ex\left(a+b\csc^{-1}(cx)\right) + \frac{bex\tan}{}$$

**Mathematica [A]**
time = 0.09, size = 104, normalized size = 1.20

$$-\frac{ad}{x} + aex - bcd\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{bd\csc^{-1}(cx)}{x} + bex\csc^{-1}(cx) + \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*e*x - b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*d*ArcCsc[c*x
])/x + b*e*x*ArcCsc[c*x] + (b*e*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[
-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

**Maple [A]**
time = 0.27, size = 137, normalized size = 1.57

| method | result |
|---|---|
| derivativedivides | $c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b\,\mathrm{arccsc}(cx)ex}{c}-\frac{b\,\mathrm{arccsc}(cx)d}{cx}-\frac{b(c^2x^2-1)d}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\,e\ln\left(cx+\sqrt{c^2x^2-\right.}}{c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}\right.$ |
| default | $c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b\,\mathrm{arccsc}(cx)ex}{c}-\frac{b\,\mathrm{arccsc}(cx)d}{cx}-\frac{b(c^2x^2-1)d}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\sqrt{c^2x^2-1}\,e\ln\left(cx+\sqrt{c^2x^2-\right.}}{c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a/c^2*(e*c*x-d*c/x)+b/c*arccsc(c*x)*e*x-b*arccsc(c*x)*d/c/x-b*(c^2*x^2-1
)/c^2/x^2/((c^2*x^2-1)/c^2/x^2)^(1/2)*d+b/c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1
)/c^2/x^2)^(1/2)/x*e*ln(c*x+(c^2*x^2-1)^(1/2)))
```

**Maxima [A]**
time = 0.26, size = 91, normalized size = 1.05

$$-\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\mathrm{arccsc}\,(cx)}{x}\right)bd+axe+\frac{\left(2\,cx\,\mathrm{arccsc}\,(cx)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be}{2\,c}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*d + a*x*e + 1/2*(2*c*x*arccsc
(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))
*b*e/c - a*d/x
```

**Fricas [A]**
time = 0.40, size = 130, normalized size = 1.49

$$-\frac{bc^2dx-acx^2e+bxe\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}\,bcd+acd-(bcdx-bcd+(bcx^2-bcx)e)\,\mathrm{arccsc}\,(cx)-2\,(bcdx-bcxe)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -(b*c^2*d*x - a*c*x^2*e + b*x*e*log(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^
2 - 1)*b*c*d + a*c*d - (b*c*d*x - b*c*d + (b*c*x^2 - b*c*x)*e)*arccsc(c*x)
- 2*(b*c*d*x - b*c*x*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c*x)
```

**Sympy** [A]
time = 3.43, size = 73, normalized size = 0.84

$$-\frac{ad}{x}+aex-bcd\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd\operatorname{acsc}(cx)}{x}+bex\operatorname{acsc}(cx)+\frac{be\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**2,x)
```

```
[Out] -a*d/x + a*e*x - b*c*d*sqrt(1 - 1/(c**2*x**2)) - b*d*acsc(c*x)/x + b*e*x*ac
sc(c*x) + b*e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), Tr
ue))/c
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. 2(79) = 158.
time = 0.62, size = 1052, normalized size = 12.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*e*arcsin(1/(c*x))/(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sq
rt(-1/(c^2*x^2) + 1) + 1)^3)) + a*e/(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1
/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)) - 2*b*c*d/(x*(sqrt(-1/(c^2*x^2) +
1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3))) + 2*b*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c*x*(sqrt(-1/(c^2*x^
2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x
^2) + 1) + 1)^3))) - 2*b*e*log(1/(abs(c)*abs(x)))/(c*x*(sqrt(-1/(c^2*x^2) +
 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^3))) - 4*b*d*arcsin(1/(c*x))/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*
(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)
^3))) - 4*a*d/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2)
+ 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))) + 2*b*e*arcsin(1/(c
*x))/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1)
+ 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))) + 2*a*e/(c^2*x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqr
t(-1/(c^2*x^2) + 1) + 1)^3))) + 2*b*d/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3
*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1
)^3))) + 2*b*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c^3*x^3*(sqrt(-1/(c^2*x^2)
```

```
+ 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^
2) + 1) + 1)^3))) - 2*b*e*log(1/(abs(c)*abs(x)))/(c^3*x^3*(sqrt(-1/(c^2*x^2
) + 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*
x^2) + 1) + 1)^3))) + b*e*arcsin(1/(c*x))/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) +
1) + 1)^3))) + a*e/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x*(sqrt(-1/(
c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))))*c
```

**Mupad [B]**
time = 0.83, size = 75, normalized size = 0.86

$$a\,e\,x-\frac{a\,d}{x}+\frac{b\,e\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}-b\,c\,d\,\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{b\,d\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x}+b\,e\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^2,x)
```

```
[Out] a*e*x - (a*d)/x + (b*e*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c - b*c*d*(1 - 1/(
c^2*x^2))^(1/2) - (b*d*asin(1/(c*x)))/x + b*e*x*asin(1/(c*x))
```

# 3.80 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4}\,dx$

**Optimal**. Leaf size=105

$$-\frac{bc(2c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}-\frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{3x^3}-\frac{e(a+b\csc^{-1}(cx))}{x}$$

```
[Out] -1/3*d*(a+b*arccsc(c*x))/x^3-e*(a+b*arccsc(c*x))/x-1/9*b*c*(2*c^2*d+9*e)*(c
^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/9*b*c*d*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(
1/2)
```

**Rubi** [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 5347, 12, 464, 270}

$$-\frac{d(a+b\csc^{-1}(cx))}{3x^3}-\frac{e(a+b\csc^{-1}(cx))}{x}-\frac{bc\sqrt{c^2x^2-1}\,(2c^2d+9e)}{9\sqrt{c^2x^2}}-\frac{bcd\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] -1/9*(b*c*(2*c^2*d + 9*e)*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*x^2] - (b*c*d*Sqrt[-
1 + c^2*x^2])/(9*x^2*Sqrt[c^2*x^2]) - (d*(a + b*ArcCsc[c*x]))/(3*x^3) - (e*
(a + b*ArcCsc[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{d(a+b\csc^{-1}(cx))}{3x^3}-\frac{e(a+b\csc^{-1}(cx))}{x}+\frac{(bcx)\int \frac{-d-3ex^2}{3x^4\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{3x^3}-\frac{e(a+b\csc^{-1}(cx))}{x}+\frac{(bcx)\int \frac{-d-3ex^2}{x^4\sqrt{-1+c^2x^2}}\,dx}{3\sqrt{c^2x^2}}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{3x^3}-\frac{e(a+b\csc^{-1}(cx))}{x}+\frac{(bc(-2$$

$$= -\frac{bc(2c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}-\frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{3x^3}$$

**Mathematica** [**A**]
time = 0.08, size = 69, normalized size = 0.66

$$-\frac{3a(d+3ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}\,x(d+2c^2dx^2+9ex^2)+3b(d+3ex^2)\csc^{-1}(cx)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^4,x]
[Out] -1/9*(3*a*(d + 3*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 9*
e*x^2) + 3*b*(d + 3*e*x^2)*ArcCsc[c*x])/x^3
```

**Maple** [**A**]
time = 0.27, size = 121, normalized size = 1.15

| method | result | size |
|---|---|---|
| derivativedivides | $c^3\left(\frac{a\left(-\frac{d}{3c\,x^3}-\frac{e}{cx}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{3c\,x^3}-\frac{\operatorname{arccsc}(cx)e}{cx}-\frac{(c^2x^2-1)(2c^4d\,x^2+9c^2e\,x^2+c^2d)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$ | 121 |
| default | $c^3\left(\frac{a\left(-\frac{d}{3c\,x^3}-\frac{e}{cx}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{3c\,x^3}-\frac{\operatorname{arccsc}(cx)e}{cx}-\frac{(c^2x^2-1)(2c^4d\,x^2+9c^2e\,x^2+c^2d)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$ | 121 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/c^2*(-1/3*d/c/x^3-e/c/x)+b/c^2*(-1/3*arccsc(c*x)*d/c/x^3-arccsc(c*x)
*e/c/x-1/9*(c^2*x^2-1)*(2*c^4*d*x^2+9*c^2*e*x^2+c^2*d)/((c^2*x^2-1)/c^2/x^2
)^(1/2)/c^4/x^4))
```

**Maxima** [A]
time = 0.27, size = 96, normalized size = 0.91

$$\frac{1}{9}\,bd\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\operatorname{arccsc}\left(cx\right)}{x^3}\right)-\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}\left(cx\right)}{x}\right)be-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b*d*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c -
3*arccsc(c*x)/x^3) - (c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*e - a*e/x
 - 1/3*a*d/x^3
```

**Fricas** [A]
time = 0.36, size = 70, normalized size = 0.67

$$-\frac{9\,ax^2e+3\,ad+3\,(3\,bx^2e+bd)\operatorname{arccsc}\left(cx\right)+(2\,bc^2dx^2+9\,bx^2e+bd)\sqrt{c^2x^2-1}}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(9*a*x^2*e + 3*a*d + 3*(3*b*x^2*e + b*d)*arccsc(c*x) + (2*b*c^2*d*x^2
+ 9*b*x^2*e + b*d)*sqrt(c^2*x^2 - 1))/x^3
```

**Sympy [A]**
time = 2.89, size = 151, normalized size = 1.44

$$-\frac{ad}{3x^3}-\frac{ae}{x}-bce\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd\operatorname{acsc}(cx)}{3x^3}-\frac{be\operatorname{acsc}(cx)}{x}-\frac{bd\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**4,x)
```

```
[Out] -a*d/(3*x**3) - a*e/x - b*c*e*sqrt(1 - 1/(c**2*x**2)) - b*d*acsc(c*x)/(3*x*
*3) - b*e*acsc(c*x)/x - b*d*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c
*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x*
*2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

**Giac [A]**
time = 0.46, size = 136, normalized size = 1.30

$$\frac{1}{9}\left(bc^2d\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,bc^2d\sqrt{-\frac{1}{c^2x^2}+1}-\frac{3\,bcd\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}-\frac{3\,bcd\arcsin\left(\frac{1}{cx}\right)}{x}-9\,be\sqrt{-\frac{1}{c^2x^2}+1}-\frac{9\,be\arcsin\left(\frac{1}{cx}\right)}{cx}-\frac{9\,ae}{cx}-\frac{3\,ad}{cx^3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/9*(b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2) - 3*b*c^2*d*sqrt(-1/(c^2*x^2) + 1) -
3*b*c*d*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 3*b*c*d*arcsin(1/(c*x))/x - 9
*b*e*sqrt(-1/(c^2*x^2) + 1) - 9*b*e*arcsin(1/(c*x))/(c*x) - 9*a*e/(c*x) - 3
*a*d/(c*x^3))*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^4, x)
```

# 3.81 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx$

**Optimal**. Leaf size=152

$$-\frac{2bc^3(12c^2d+25e)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}}-\frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}}-\frac{bc(12c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{5x^5}-e($$

```
[Out] -1/5*d*(a+b*arccsc(c*x))/x^5-1/3*e*(a+b*arccsc(c*x))/x^3-2/225*b*c^3*(12*c^
2*d+25*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/25*b*c*d*(c^2*x^2-1)^(1/2)/x^
4/(c^2*x^2)^(1/2)-1/225*b*c*(12*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)
^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 5347, 12, 464, 277, 270}

$$-\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}-\frac{bc\sqrt{c^2x^2-1}(12c^2d+25e)}{225x^2\sqrt{c^2x^2}}-\frac{bcd\sqrt{c^2x^2-1}}{25x^4\sqrt{c^2x^2}}-\frac{2bc^3\sqrt{c^2x^2-1}(12c^2d+25e)}{225\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] (-2*b*c^3*(12*c^2*d + 25*e)*Sqrt[-1 + c^2*x^2])/(225*Sqrt[c^2*x^2]) - (b*c*
d*Sqrt[-1 + c^2*x^2])/(25*x^4*Sqrt[c^2*x^2]) - (b*c*(12*c^2*d + 25*e)*Sqrt[
-1 + c^2*x^2])/(225*x^2*Sqrt[c^2*x^2]) - (d*(a + b*ArcCsc[c*x]))/(5*x^5) -
(e*(a + b*ArcCsc[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}+\frac{(bcx)\int \frac{-3d-5ex^2}{15x^6\sqrt{-1+c^2x^2}}\,d}{\sqrt{c^2x^2}}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}+\frac{(bcx)\int \frac{-3d-5ex^2}{x^6\sqrt{-1+c^2x^2}}\,dx}{15\sqrt{c^2x^2}}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}+\frac{(bc(-1}{}$$

$$= -\frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}}-\frac{bc(12c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx}{5x^5}$$

$$= -\frac{2bc^3(12c^2d+25e)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}}-\frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}}-\frac{bc(12c^2d+25e}{225x^2}$$

**Mathematica** [A]

time = 0.12, size = 94, normalized size = 0.62

$$-\frac{15a(3d+5ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}\,x(25ex^2(1+2c^2x^2)+3d(3+4c^2x^2+8c^4x^4))+15b(3d+5ex^2)\csc^{-1}(cx)}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(15*a*(3*d + 5*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(25*e*x^2*(1 + 2
*c^2*x^2) + 3*d*(3 + 4*c^2*x^2 + 8*c^4*x^4)) + 15*b*(3*d + 5*e*x^2)*ArcCsc[
c*x])/x^5
```

**Maple [A]**
time = 0.29, size = 140, normalized size = 0.92

| method | result |
|---|---|
| derivativedivides | $c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)e}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)d}{5c^3x^5}-\frac{(c^2x^2-1)(24c^6dx^4+50c^4ex^4+12c^4dx^2+25c^2ex^2+9c^2d)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$ |
| default | $c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)e}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)d}{5c^3x^5}-\frac{(c^2x^2-1)(24c^6dx^4+50c^4ex^4+12c^4dx^2+25c^2ex^2+9c^2d)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(a/c^2*(-1/3*e/c^3/x^3-1/5*d/c^3/x^5)+b/c^2*(-1/3*arccsc(c*x)*e/c^3/x^3
-1/5*arccsc(c*x)*d/c^3/x^5-1/225*(c^2*x^2-1)*(24*c^6*d*x^4+50*c^4*e*x^4+12*
c^4*d*x^2+25*c^2*e*x^2+9*c^2*d)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6))
```

**Maxima [A]**
time = 0.27, size = 139, normalized size = 0.91

$$-\frac{1}{75}bd\left(\frac{3c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\operatorname{arccsc}(cx)}{x^5}\right)+\frac{1}{9}b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\operatorname{arccsc}(cx)}{x^3}\right)e-\frac{ae}{3x^3}-\frac{ad}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/75*b*d*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2
) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) + 1/9*b*((c^4*(-
1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*x)/x^
3)*e - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

**Fricas** [**A**]
time = 0.35, size = 93, normalized size = 0.61

$$-\frac{75\,ax^2e+45\,ad+15\,(5\,bx^2e+3\,bd)\,\mathrm{arccsc}\,(cx)+\left(24\,bc^4dx^4+12\,bc^2dx^2+9\,bd+25\,(2\,bc^2x^4+bx^2)e\right)\sqrt{c^2x^2-1}}{225\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(75*a*x^2*e + 45*a*d + 15*(5*b*x^2*e + 3*b*d)*arccsc(c*x) + (24*b*c^
4*d*x^4 + 12*b*c^2*d*x^2 + 9*b*d + 25*(2*b*c^2*x^4 + b*x^2)*e)*sqrt(c^2*x^2
 - 1))/x^5
```

**Sympy** [**A**]
time = 6.25, size = 280, normalized size = 1.84

$$-\frac{ad}{5x^5}-\frac{ae}{3x^3}-\frac{bd\,\mathrm{acsc}\,(cx)}{5x^5}-\frac{be\,\mathrm{acsc}\,(cx)}{3x^3}-\frac{bd\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for}\ |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}-\frac{be\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for}\ |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**6,x)
```

```
[Out] -a*d/(5*x**5) - a*e/(3*x**3) - b*d*acsc(c*x)/(5*x**5) - b*e*acsc(c*x)/(3*x*
*3) - b*d*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x
**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (
8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x*
*3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) - b*e*Piecewise((2*c*
*3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**
2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3
*x**3), True))/(3*c)
```

**Giac** [**A**]
time = 0.45, size = 245, normalized size = 1.61

$$-\frac{1}{225}\left(9\,bc^4d\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}-30\,bc^4d\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{45\,bc^3d\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x}+45\,bc^4d\sqrt{-\frac{1}{c^2x^2}+1}+\frac{90\,bc^3d\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}-25\,bc^2e\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{45\,bc^3d\arcsin\left(\frac{1}{cx}\right)}{x}+75\,bc^2e\sqrt{-\frac{1}{c^2x^2}+1}+\frac{75\,bce\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{75\,bce\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{75\,ae}{cx^3}+\frac{45\,ad}{cx^5}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/225*(9*b*c^4*d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) - 30*b*c^4*d*(
-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x
```

```
+ 45*b*c^4*d*sqrt(-1/(c^2*x^2) + 1) + 90*b*c^3*d*(1/(c^2*x^2) - 1)*arcsin(1
/(c*x))/x - 25*b*c^2*e*(-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d*arcsin(1/(c*x)
)/x + 75*b*c^2*e*sqrt(-1/(c^2*x^2) + 1) + 75*b*c*e*(1/(c^2*x^2) - 1)*arcsin
(1/(c*x))/x + 75*b*c*e*arcsin(1/(c*x))/x + 75*a*e/(c*x^3) + 45*a*d/(c*x^5))
*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^6, x)
```

# 3.82 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx$

**Optimal**. Leaf size=197

$$-\frac{8bc^5(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675\sqrt{c^2x^2}}-\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}}-\frac{4bc^3(30c^2d+49e)\sqrt{\cdots}}{3675x^2\sqrt{c^2x\cdots}}$$

```
[Out] -1/7*d*(a+b*arccsc(c*x))/x^7-1/5*e*(a+b*arccsc(c*x))/x^5-8/3675*b*c^5*(30*c
^2*d+49*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/49*b*c*d*(c^2*x^2-1)^(1/2)/x
^6/(c^2*x^2)^(1/2)-1/1225*b*c*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^
2)^(1/2)-4/3675*b*c^3*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 5347, 12, 464, 277, 270}

$$-\frac{d(a+b\csc^{-1}(cx))}{7x^7}-\frac{e(a+b\csc^{-1}(cx))}{5x^5}-\frac{bc\sqrt{c^2x^2-1}\,(30c^2d+49e)}{1225x^4\sqrt{c^2x^2}}-\frac{bcd\sqrt{c^2x^2-1}}{49x^6\sqrt{c^2x^2}}-\frac{8bc^5\sqrt{c^2x^2-1}\,(30c^2d+49e)}{3675\sqrt{c^2x^2}}-\frac{4bc^3\sqrt{c^2x^2-1}\,(30c^2d+49e)}{3675x^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] (-8*b*c^5*(30*c^2*d + 49*e)*Sqrt[-1 + c^2*x^2])/(3675*Sqrt[c^2*x^2]) - (b*c
*d*Sqrt[-1 + c^2*x^2])/(49*x^6*Sqrt[c^2*x^2]) - (b*c*(30*c^2*d + 49*e)*Sqrt
[-1 + c^2*x^2])/(1225*x^4*Sqrt[c^2*x^2]) - (4*b*c^3*(30*c^2*d + 49*e)*Sqrt[
-1 + c^2*x^2])/(3675*x^2*Sqrt[c^2*x^2]) - (d*(a + b*ArcCsc[c*x]))/(7*x^7) -
 (e*(a + b*ArcCsc[c*x]))/(5*x^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^8}\,dx &= -\frac{d(a+b\csc^{-1}(cx))}{7x^7}-\frac{e(a+b\csc^{-1}(cx))}{5x^5}+\frac{(bcx)\int\frac{-5d-7ex^2}{35x^8\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}} \\
&= -\frac{d(a+b\csc^{-1}(cx))}{7x^7}-\frac{e(a+b\csc^{-1}(cx))}{5x^5}+\frac{(bcx)\int\frac{-5d-7ex^2}{x^8\sqrt{-1+c^2x^2}}\,dx}{35\sqrt{c^2x^2}} \\
&= -\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{7x^7}-\frac{e(a+b\csc^{-1}(cx))}{5x^5}+\frac{(bc(-30 \\
&= -\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{7x^7} \\
&= -\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}}-\frac{4bc^3(30c^2d+49e)}{3675x^2\sqrt{} \\
&= -\frac{8bc^5(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675\sqrt{c^2x^2}}-\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\frac{bc(30c^2d+49e)}{1225x^4\sqrt{}}
\end{aligned}$$

**Mathematica [A]**
time = 0.12, size = 110, normalized size = 0.56

$$-\frac{105a(5d+7ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}\,x(49ex^2(3+4c^2x^2+8c^4x^4)+15d(5+6c^2x^2+8c^4x^4+16c^6x^6))+105b(5d+7ex^2)\csc^{-1}(cx)}{3675x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] -1/3675*(105*a*(5*d + 7*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(49*e*x^2*(3 +
 4*c^2*x^2 + 8*c^4*x^4) + 15*d*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) +
105*b*(5*d + 7*e*x^2)*ArcCsc[c*x])/x^7
```

**Maple [A]**
time = 0.30, size = 158, normalized size = 0.80

| method | result |
|---|---|
| derivativedivides | $c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\mathrm{arccsc}(cx)d}{7c^5x^7}-\frac{\mathrm{arccsc}(cx)e}{5c^5x^5}-\frac{\left(c^2x^2-1\right)\left(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x^4+90c^4d\,x\right.}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;c^8x^8}\right.}{c^2}\right.$ |
| default | $c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\mathrm{arccsc}(cx)d}{7c^5x^7}-\frac{\mathrm{arccsc}(cx)e}{5c^5x^5}-\frac{\left(c^2x^2-1\right)\left(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x^4+90c^4d\,x\right.}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;c^8x^8}\right.}{c^2}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] c^7*(a/c^2*(-1/7*d/c^5/x^7-1/5*e/c^5/x^5)+b/c^2*(-1/7*arccsc(c*x)*d/c^5/x^7
-1/5*arccsc(c*x)*e/c^5/x^5-1/3675*(c^2*x^2-1)*(240*c^8*d*x^6+392*c^6*e*x^6+
120*c^6*d*x^4+196*c^4*e*x^4+90*c^4*d*x^2+147*c^2*e*x^2+75*c^2*d)/((c^2*x^2-
1)/c^2/x^2)^(1/2)/c^8/x^8))
```

**Maxima [A]**
time = 0.27, size = 174, normalized size = 0.88

$$\frac{1}{245}\,bd\left(\frac{5\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{35\,\mathrm{arccsc}\,(cx)}{x^7}\right)-\frac{1}{75}\,b\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\mathrm{arccsc}\,(cx)}{x^5}\right)e-\frac{ae}{5\,x^5}-\frac{ad}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^(5/2
) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c - 35
*arccsc(c*x)/x^7) - 1/75*b*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c
^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^5)
*e - 1/5*a*e/x^5 - 1/7*a*d/x^7
```

**Fricas** [A]
time = 0.36, size = 113, normalized size = 0.57

$$-\frac{735\,ax^{2}e+525\,ad+105\,(7\,bx^{2}e+5\,bd)\,\mathrm{arccsc}\,(cx)+(240\,bc^{6}dx^{6}+120\,bc^{4}dx^{4}+90\,bc^{2}dx^{2}+75\,bd+49\,(8\,bc^{4}x^{6}+4\,bc^{2}x^{4}+3\,bx^{2})e)\sqrt{c^{2}x^{2}-1}}{3675\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/3675*(735*a*x^2*e + 525*a*d + 105*(7*b*x^2*e + 5*b*d)*arccsc(c*x) + (240
*b*c^6*d*x^6 + 120*b*c^4*d*x^4 + 90*b*c^2*d*x^2 + 75*b*d + 49*(8*b*c^4*x^6
+ 4*b*c^2*x^4 + 3*b*x^2)*e)*sqrt(c^2*x^2 - 1))/x^7
```

**Sympy** [A]
time = 34.49, size = 372, normalized size = 1.89

$$-\frac{ad}{7x^{7}}-\frac{ae}{5x^{5}}-\frac{bd\,\mathrm{acsc}\,(cx)}{7x^{7}}-\frac{be\,\mathrm{acsc}\,(cx)}{5x^{5}}-\frac{bd\left(\begin{cases}\frac{16c^{7}\sqrt{c^{2}x^{2}-1}}{35x}+\frac{8c^{5}\sqrt{c^{2}x^{2}-1}}{35x^{3}}+\frac{6c^{3}\sqrt{c^{2}x^{2}-1}}{35x^{5}}+\frac{c\sqrt{c^{2}x^{2}-1}}{7x^{7}} & \text{for}\ |c^{2}x^{2}|>1\\ \frac{16ic^{7}\sqrt{-c^{2}x^{2}+1}}{35x}+\frac{8ic^{5}\sqrt{-c^{2}x^{2}+1}}{35x^{3}}+\frac{6ic^{3}\sqrt{-c^{2}x^{2}+1}}{35x^{5}}+\frac{ic\sqrt{-c^{2}x^{2}+1}}{7x^{7}} & \text{otherwise}\end{cases}\right)}{7c}-\frac{be\left(\begin{cases}\frac{8c^{5}\sqrt{c^{2}x^{2}-1}}{15x}+\frac{4c^{3}\sqrt{c^{2}x^{2}-1}}{15x^{3}}+\frac{c\sqrt{c^{2}x^{2}-1}}{5x^{5}} & \text{for}\ |c^{2}x^{2}|>1\\ \frac{8ic^{5}\sqrt{-c^{2}x^{2}+1}}{15x}+\frac{4ic^{3}\sqrt{-c^{2}x^{2}+1}}{15x^{3}}+\frac{ic\sqrt{-c^{2}x^{2}+1}}{5x^{5}} & \text{otherwise}\end{cases}\right)}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**8,x)
```

```
[Out] -a*d/(7*x**7) - a*e/(5*x**5) - b*d*acsc(c*x)/(7*x**7) - b*e*acsc(c*x)/(5*x*
*5) - b*d*Piecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**2*
x**2 - 1)/(35*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2*x*
*2 - 1)/(7*x**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/(35*
x) + 8*I*c**5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**2 + 1
)/(35*x**5) + I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) - b*e*Piecewi
se((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3
) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c*
*2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c
**2*x**2 + 1)/(5*x**5), True))/(5*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(169) = 338.
time = 0.44, size = 367, normalized size = 1.86

$$-\frac{1}{3675}\left(75\,bc^{6}d\left(\frac{1}{c^{2}x^{2}}-1\right)^{3}\sqrt{-\frac{1}{c^{2}x^{2}}+1}+315\,bc^{6}d\left(\frac{1}{c^{2}x^{2}}-1\right)^{2}\sqrt{-\frac{1}{c^{2}x^{2}}+1}+\frac{525\,bc^{5}d\left(\frac{1}{c^{2}x^{2}}-1\right)^{3}\arcsin\left(\frac{1}{cx}\right)}{x}-525\,bc^{6}d\left(-\frac{1}{c^{2}x^{2}}+1\right)^{\frac{3}{2}}+\frac{1575\,bc^{5}d\left(\frac{1}{c^{2}x^{2}}-1\right)^{2}\arcsin\left(\frac{1}{cx}\right)}{x}+525\,bc^{6}d\sqrt{-\frac{1}{c^{2}x^{2}}+1}+147\,bc^{4}e\left(\frac{1}{c^{2}x^{2}}-1\right)^{2}\sqrt{-\frac{1}{c^{2}x^{2}}+1}+\frac{1575\,bc^{5}d\left(\frac{1}{c^{2}x^{2}}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}-490\,bc^{4}e\left(-\frac{1}{c^{2}x^{2}}+1\right)^{\frac{3}{2}}+\frac{525\,bc^{5}d\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{735\,bc^{3}e\left(\frac{1}{c^{2}x^{2}}-1\right)^{2}\arcsin\left(\frac{1}{cx}\right)}{x}+735\,bc^{4}e\sqrt{-\frac{1}{c^{2}x^{2}}+1}+\frac{1470\,bc^{3}e\left(\frac{1}{c^{2}x^{2}}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{735\,bc^{3}e\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{735\,ae}{cx^{5}}+\frac{525\,ad}{cx^{7}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

```
[Out] -1/3675*(75*b*c^6*d*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1) + 315*b*c^6*
d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) + 525*b*c^5*d*(1/(c^2*x^2) - 1
)^3*arcsin(1/(c*x))/x - 525*b*c^6*d*(-1/(c^2*x^2) + 1)^(3/2) + 1575*b*c^5*d
*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 525*b*c^6*d*sqrt(-1/(c^2*x^2) + 1)
 + 147*b*c^4*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) + 1575*b*c^5*d*(1
/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 490*b*c^4*e*(-1/(c^2*x^2) + 1)^(3/2) +
525*b*c^5*d*arcsin(1/(c*x))/x + 735*b*c^3*e*(1/(c^2*x^2) - 1)^2*arcsin(1/(c
*x))/x + 735*b*c^4*e*sqrt(-1/(c^2*x^2) + 1) + 1470*b*c^3*e*(1/(c^2*x^2) - 1
)*arcsin(1/(c*x))/x + 735*b*c^3*e*arcsin(1/(c*x))/x + 735*a*e/(c*x^5) + 525
*a*d/(c*x^7))*c
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^8,x)
```

```
[Out] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^8, x)
```

# 3.83 $\int x^5(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=196

$$\frac{b(4c^2d+3e)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}+\frac{b(8c^2d+9e)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}}+\frac{b(4c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}}+\frac{bex(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}}$$

```
[Out] 1/6*d*x^6*(a+b*arccsc(c*x))+1/8*e*x^8*(a+b*arccsc(c*x))+1/72*b*(8*c^2*d+9*e
)*x*(c^2*x^2-1)^(3/2)/c^7/(c^2*x^2)^(1/2)+1/120*b*(4*c^2*d+9*e)*x*(c^2*x^2-
1)^(5/2)/c^7/(c^2*x^2)^(1/2)+1/56*b*e*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^(1/
2)+1/24*b*(4*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 5347, 12, 457, 78}

$$\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{bx(c^2x^2-1)^{5/2}(4c^2d+9e)}{120c^7\sqrt{c^2x^2}}+\frac{bx(c^2x^2-1)^{3/2}(8c^2d+9e)}{72c^7\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,(4c^2d+3e)}{24c^7\sqrt{c^2x^2}}+\frac{bex(c^2x^2-1)^{7/2}}{56c^7\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(4*c^2*d + 3*e)*x*Sqrt[-1 + c^2*x^2])/(24*c^7*Sqrt[c^2*x^2]) + (b*(8*c^2
*d + 9*e)*x*(-1 + c^2*x^2)^(3/2))/(72*c^7*Sqrt[c^2*x^2]) + (b*(4*c^2*d + 9*
e)*x*(-1 + c^2*x^2)^(5/2))/(120*c^7*Sqrt[c^2*x^2]) + (b*e*x*(-1 + c^2*x^2)^
(7/2))/(56*c^7*Sqrt[c^2*x^2]) + (d*x^6*(a + b*ArcCsc[c*x]))/6 + (e*x^8*(a +
 b*ArcCsc[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^5\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^5(4d+3e}{24\sqrt{-1+}}}{\sqrt{c^2x^2}}$$

$$=\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^5(4d+3ex}{\sqrt{-1+c^2}}}{24\sqrt{c^2x^2}}$$

$$=\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\text{Subst}\left(\int\frac{x}{\sqrt{}}\right.}{48\sqrt{}}$$

$$=\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\text{Subst}\left(\int\left(\frac{}{c}\right.\right.}{}$$

$$=\frac{b(4c^2d+3e)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}+\frac{b(8c^2d+9e)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}}+\frac{b(4c^2}{}$$

**Mathematica [A]**
time = 0.17, size = 115, normalized size = 0.59

$$\frac{x\left(105ax^5(4d+3ex^2)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,(144e+8c^2(28d+9ex^2)+2c^4(56dx^2+27ex^4)+c^6(84dx^4+45ex^6))}{c^7}+105bx^5(4d+3ex^2)\csc^{-1}(cx)\right)}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(105*a*x^5*(4*d + 3*e*x^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*(144*e + 8*c^2*(28
*d + 9*e*x^2) + 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45*e*x^6)))/c
^7 + 105*b*x^5*(4*d + 3*e*x^2)*ArcCsc[c*x]))/2520
```

**Maple [A]**
time = 0.42, size = 152, normalized size = 0.78

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{1}{6}c^8d\,x^6+\frac{1}{8}e\,c^8x^8\right)}{c^2}+\frac{b\left(\frac{\mathrm{arccsc}(cx)d\,c^8x^6}{6}+\frac{\mathrm{arccsc}(cx)e\,c^8x^8}{8}+\frac{\left(c^2x^2-1\right)\left(45c^6e\,x^6+84c^6d\,x^4+54c^4e\,x^4+112c^4d\,x^2+72c^2e\,x^2+224\right.}{2520\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;cx}\right.}{c^2}}{c^6}$ |
| default | $\frac{\frac{a\left(\frac{1}{6}c^8d\,x^6+\frac{1}{8}e\,c^8x^8\right)}{c^2}+\frac{b\left(\frac{\mathrm{arccsc}(cx)d\,c^8x^6}{6}+\frac{\mathrm{arccsc}(cx)e\,c^8x^8}{8}+\frac{\left(c^2x^2-1\right)\left(45c^6e\,x^6+84c^6d\,x^4+54c^4e\,x^4+112c^4d\,x^2+72c^2e\,x^2+224\right.}{2520\sqrt{\frac{c^2x^2-1}{c^2x^2}}\;cx}\right.}{c^2}}{c^6}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(a/c^2*(1/6*c^8*d*x^6+1/8*e*c^8*x^8)+b/c^2*(1/6*arccsc(c*x)*d*c^8*x^6
+1/8*arccsc(c*x)*e*c^8*x^8+1/2520*(c^2*x^2-1)*(45*c^6*e*x^6+84*c^6*d*x^4+54
*c^4*e*x^4+112*c^4*d*x^2+72*c^2*e*x^2+224*c^2*d+144*e)/((c^2*x^2-1)/c^2/x^2
)^(1/2)/c/x))
```

**Maxima [A]**
time = 0.27, size = 185, normalized size = 0.94

$$\frac{1}{8}\,ax^8e+\frac{1}{6}\,adx^6+\frac{1}{90}\left(15\,x^6\,\mathrm{arccsc}\left(cx\right)+\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)bd+\frac{1}{280}\left(35\,x^8\,\mathrm{arccsc}\left(cx\right)+\frac{5\,c^6x^7\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}+21\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+35\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^7}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8*e + 1/6*a*d*x^6 + 1/90*(15*x^6*arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*
x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x
^2) + 1))/c^5)*b*d + 1/280*(35*x^8*arccsc(c*x) + (5*c^6*x^7*(-1/(c^2*x^2) +
 1)^(7/2) + 21*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x^2)
+ 1)^(3/2) + 35*x*sqrt(-1/(c^2*x^2) + 1))/c^7)*b*e
```

**Fricas [A]**
time = 0.38, size = 130, normalized size = 0.66

$$\frac{315\,ac^8x^8e+420\,ac^8dx^6+105\left(3\,bc^8x^8e+4\,bc^8dx^6\right)\mathrm{arccsc}\left(cx\right)+\left(84\,bc^6dx^4+112\,bc^4dx^2+224\,bc^2d+9\left(5\,bc^6x^6+6\,bc^4x^4+8\,bc^2x^2+16\,b\right)e\right)\sqrt{c^2x^2-1}}{2520\,c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^8*x^8*e + 420*a*c^8*d*x^6 + 105*(3*b*c^8*x^8*e + 4*b*c^8*d*
x^6)*arccsc(c*x) + (84*b*c^6*d*x^4 + 112*b*c^4*d*x^2 + 224*b*c^2*d + 9*(5*b
*c^6*x^6 + 6*b*c^4*x^4 + 8*b*c^2*x^2 + 16*b)*e)*sqrt(c^2*x^2 - 1))/c^8
```

**Sympy** [**A**]
time = 5.36, size = 364, normalized size = 1.86

$$\frac{adx^6}{6}+\frac{aex^8}{8}+\frac{bdx^6\operatorname{acsc}(cx)}{6}+\frac{bex^8\operatorname{acsc}(cx)}{8}+\frac{bd\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}+\frac{be\left(\begin{cases}\frac{x^6\sqrt{c^2x^2-1}}{7c}+\frac{6x^4\sqrt{c^2x^2-1}}{35c^3}+\frac{8x^2\sqrt{c^2x^2-1}}{35c^5}+\frac{16\sqrt{c^2x^2-1}}{35c^7} & \text{for } |c^2x^2|>1\\ \frac{ix^6\sqrt{-c^2x^2+1}}{7c}+\frac{6ix^4\sqrt{-c^2x^2+1}}{35c^3}+\frac{8ix^2\sqrt{-c^2x^2+1}}{35c^5}+\frac{16i\sqrt{-c^2x^2+1}}{35c^7} & \text{otherwise}\end{cases}\right)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**6/6 + a*e*x**8/8 + b*d*x**6*acsc(c*x)/6 + b*e*x**8*acsc(c*x)/8 + b*d
*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15
*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt
(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt
(-c**2*x**2 + 1)/(15*c**5), True))/(6*c) + b*e*Piecewise((x**6*sqrt(c**2*x*
*2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3) + 8*x**2*sqrt(c**2*x**
2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7), Abs(c**2*x**2) > 1), (
I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c**2*x**2 + 1)/(35*c**3)
 + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sqrt(-c**2*x**2 + 1)/(35*
c**7), True))/(8*c)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(168) = 336$.
time = 0.54, size = 1244, normalized size = 6.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/645120*(315*b*e*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*arcsin(1/(c*x))/c + 31
5*a*e*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8/c + 90*b*e*x^7*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^7/c^2 + 1680*b*d*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x
))/c + 1680*a*d*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 2520*b*e*x^6*(sqrt(-
1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c^3 + 2520*a*e*x^6*(sqrt(-1/(c^2*x^
2) + 1) + 1)^6/c^3 + 672*b*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c^2 + 882*b
*e*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c^4 + 10080*b*d*x^4*(sqrt(-1/(c^2*x^2
) + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 10080*a*d*x^4*(sqrt(-1/(c^2*x^2) + 1) +
 1)^4/c^3 + 8820*b*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^5
```

```
 + 8820*a*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^5 + 5600*b*d*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3/c^4 + 4410*b*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^6
 + 25200*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^5 + 25200
*a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 17640*b*e*x^2*(sqrt(-1/(c^2*x
^2) + 1) + 1)^2*arcsin(1/(c*x))/c^7 + 17640*a*e*x^2*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^2/c^7 + 33600*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 + 22050*b*e*x*(s
qrt(-1/(c^2*x^2) + 1) + 1)/c^8 + 33600*b*d*arcsin(1/(c*x))/c^7 + 33600*a*d/
c^7 + 22050*b*e*arcsin(1/(c*x))/c^9 + 22050*a*e/c^9 - 33600*b*d/(c^8*x*(sqr
t(-1/(c^2*x^2) + 1) + 1)) - 22050*b*e/(c^10*x*(sqrt(-1/(c^2*x^2) + 1) + 1))
 + 25200*b*d*arcsin(1/(c*x))/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 252
00*a*d/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 17640*b*e*arcsin(1/(c*x))
/(c^11*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 17640*a*e/(c^11*x^2*(sqrt(-1/(
c^2*x^2) + 1) + 1)^2) - 5600*b*d/(c^10*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
- 4410*b*e/(c^12*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 10080*b*d*arcsin(1/(
c*x))/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 10080*a*d/(c^11*x^4*(sqrt
(-1/(c^2*x^2) + 1) + 1)^4) + 8820*b*e*arcsin(1/(c*x))/(c^13*x^4*(sqrt(-1/(c
^2*x^2) + 1) + 1)^4) + 8820*a*e/(c^13*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) -
 672*b*d/(c^12*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - 882*b*e/(c^14*x^5*(sqr
t(-1/(c^2*x^2) + 1) + 1)^5) + 1680*b*d*arcsin(1/(c*x))/(c^13*x^6*(sqrt(-1/(
c^2*x^2) + 1) + 1)^6) + 1680*a*d/(c^13*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6)
+ 2520*b*e*arcsin(1/(c*x))/(c^15*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 2520
*a*e/(c^15*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) - 90*b*e/(c^16*x^7*(sqrt(-1/
(c^2*x^2) + 1) + 1)^7) + 315*b*e*arcsin(1/(c*x))/(c^17*x^8*(sqrt(-1/(c^2*x^
2) + 1) + 1)^8) + 315*a*e/(c^17*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8))*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^5*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

# 3.84 $\int x^3(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=153

$$\frac{b(3c^2d+2e)\,x\sqrt{-1+c^2x^2}}{12c^5\sqrt{c^2x^2}}+\frac{b(3c^2d+4e)\,x(-1+c^2x^2)^{3/2}}{36c^5\sqrt{c^2x^2}}+\frac{bex(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}$$

```
[Out] 1/4*d*x^4*(a+b*arccsc(c*x))+1/6*e*x^6*(a+b*arccsc(c*x))+1/36*b*(3*c^2*d+4*e
)*x*(c^2*x^2-1)^(3/2)/c^5/(c^2*x^2)^(1/2)+1/30*b*e*x*(c^2*x^2-1)^(5/2)/c^5/
(c^2*x^2)^(1/2)+1/12*b*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2
)
```

**Rubi** [A]
time = 0.09, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 5347, 12, 457, 78}

$$\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{bx(c^2x^2-1)^{3/2}(3c^2d+4e)}{36c^5\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,(3c^2d+2e)}{12c^5\sqrt{c^2x^2}}+\frac{bex(c^2x^2-1)^{5/2}}{30c^5\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(3*c^2*d + 2*e)*x*Sqrt[-1 + c^2*x^2])/(12*c^5*Sqrt[c^2*x^2]) + (b*(3*c^2
*d + 4*e)*x*(-1 + c^2*x^2)^(3/2))/(36*c^5*Sqrt[c^2*x^2]) + (b*e*x*(-1 + c^2
*x^2)^(5/2))/(30*c^5*Sqrt[c^2*x^2]) + (d*x^4*(a + b*ArcCsc[c*x]))/4 + (e*x^
6*(a + b*ArcCsc[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^3\left(3d+2ex^2\right)}{12\sqrt{-1+c^2 \ldots}}}{\sqrt{c^2x^2}}$$

$$=\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\int\frac{x^3\left(3d+2ex^2\right)}{\sqrt{-1+c^2x \ldots}}}{12\sqrt{c^2x^2}}$$

$$=\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\text{Subst}\left(\int\frac{x(3 \ldots}{\sqrt{- \ldots}}\right.}{24\sqrt{c \ldots}}$$

$$=\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)+\frac{(bcx)\text{Subst}\left(\int\left(\frac{\ldots}{c^4 \ldots}\right.\right.}{\ldots}$$

$$=\frac{b\left(3c^2d+2e\right)x\sqrt{-1+c^2x^2}}{12c^5\sqrt{c^2x^2}}+\frac{b\left(3c^2d+4e\right)x\left(-1+c^2x^2\right)^{3/2}}{36c^5\sqrt{c^2x^2}}+\frac{bex(- \ldots}{30c \ldots}$$

**Mathematica [A]**
time = 0.15, size = 97, normalized size = 0.63

$$\frac{1}{180}x\left(15ax^3\left(3d+2ex^2\right)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\left(16e+c^2\left(30d+8ex^2\right)+3c^4\left(5dx^2+2ex^4\right)\right)}{c^5}+15bx^3\left(3d+2ex^2\right)\csc^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(15*a*x^3*(3*d + 2*e*x^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*(16*e + c^2*(30*d +
 8*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^5 + 15*b*x^3*(3*d + 2*e*x^2)*ArcC
sc[c*x]))/180
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(131) = 262$.
time = 0.52, size = 307, normalized size = 2.01

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{1}{4}c^6 d x^4+\frac{1}{6}c^6 e x^6\right)}{c^2}-\frac{b c^4 \operatorname{arccsc}(cx) d^3}{12e^2}+\frac{b \operatorname{arccsc}(cx) d c^4 x^4}{4}+\frac{b c^4 e \operatorname{arccsc}(cx) x^6}{6}+\frac{b c^3 \sqrt{c^2x^2-1}\, d^3 \arctan\left(\frac{1}{\sqrt{c^2x^2-}}\right.}{12e^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x}}{c^4}$ |
| default | $\frac{\frac{a\left(\frac{1}{4}c^6 d x^4+\frac{1}{6}c^6 e x^6\right)}{c^2}-\frac{b c^4 \operatorname{arccsc}(cx) d^3}{12e^2}+\frac{b \operatorname{arccsc}(cx) d c^4 x^4}{4}+\frac{b c^4 e \operatorname{arccsc}(cx) x^6}{6}+\frac{b c^3 \sqrt{c^2x^2-1}\, d^3 \arctan\left(\frac{1}{\sqrt{c^2x^2-}}\right.}{12e^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x}}{c^4}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a/c^2*(1/4*c^6*d*x^4+1/6*c^6*e*x^6)-1/12*b*c^4/e^2*arccsc(c*x)*d^3+1
/4*b*arccsc(c*x)*d*c^4*x^4+1/6*b*c^4*e*arccsc(c*x)*x^6+1/12*b*c^3/e^2*(c^2*
x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*arctan(1/(c^2*x^2-1)^(1/2))+
1/12*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x*d+1/30*b*c*e*(c^2*x^2-1)
/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^3+1/6*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1
/2)/c/x*d+2/45*b/c*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x+4/45*b/c^3*e
*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

**Maxima [A]**
time = 0.28, size = 144, normalized size = 0.94

$$\frac{1}{6}ax^6e+\frac{1}{4}adx^4+\frac{1}{12}\left(3x^4\operatorname{arccsc}(cx)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd+\frac{1}{90}\left(15x^6\operatorname{arccsc}(cx)+\frac{3c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6*e + 1/4*a*d*x^4 + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^2
) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d + 1/90*(15*x^6*arccsc(c
*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(
3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e
```

**Fricas** [A]
time = 0.38, size = 111, normalized size = 0.73

$$\frac{30\,ac^{6}x^{6}e+45\,ac^{6}dx^{4}+15\,(2\,bc^{6}x^{6}e+3\,bc^{6}dx^{4})\operatorname{arccsc}(cx)+(15\,bc^{4}dx^{2}+30\,bc^{2}d+2\,(3\,bc^{4}x^{4}+4\,bc^{2}x^{2}+8\,b)e)\sqrt{c^{2}x^{2}-1}}{180\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*x^6*e + 45*a*c^6*d*x^4 + 15*(2*b*c^6*x^6*e + 3*b*c^6*d*x^4)
*arccsc(c*x) + (15*b*c^4*d*x^2 + 30*b*c^2*d + 2*(3*b*c^4*x^4 + 4*b*c^2*x^2
+ 8*b)*e)*sqrt(c^2*x^2 - 1))/c^6
```

**Sympy** [A]
time = 3.80, size = 272, normalized size = 1.78

$$\frac{adx^4}{4}+\frac{aex^6}{6}+\frac{bdx^4\operatorname{acsc}(cx)}{4}+\frac{bex^6\operatorname{acsc}(cx)}{6}+\frac{bd\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}+\frac{be\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*acsc(c*x)/4 + b*e*x**6*acsc(c*x)/6 + b*d
*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3)
, Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*
x**2 + 1)/(3*c**3), True))/(4*c) + b*e*Piecewise((x**4*sqrt(c**2*x**2 - 1)/
(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c*
*5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqr
t(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6
*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 900 vs. 2(131) = 262.
time = 0.47, size = 900, normalized size = 5.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/5760*(15*b*e*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c + 15*a*
e*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 6*b*e*x^5*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^5/c^2 + 90*b*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c +
90*a*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 90*b*e*x^4*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 90*a*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^
4/c^3 + 60*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 50*b*e*x^3*(sqrt(-1
```

```
/(c^2*x^2) + 1) + 1)^3/c^4 + 360*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arc
sin(1/(c*x))/c^3 + 360*a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 225*b*e
*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^5 + 225*a*e*x^2*(sqrt
(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 540*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4
+ 300*b*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 + 540*b*d*arcsin(1/(c*x))/c^5
+ 540*a*d/c^5 + 300*b*e*arcsin(1/(c*x))/c^7 + 300*a*e/c^7 - 540*b*d/(c^6*x*
(sqrt(-1/(c^2*x^2) + 1) + 1)) - 300*b*e/(c^8*x*(sqrt(-1/(c^2*x^2) + 1) + 1)
) + 360*b*d*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 360*
a*d/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 225*b*e*arcsin(1/(c*x))/(c^9
*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 225*a*e/(c^9*x^2*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^2) - 60*b*d/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 50*b*e/(c^
10*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 90*b*d*arcsin(1/(c*x))/(c^9*x^4*(s
qrt(-1/(c^2*x^2) + 1) + 1)^4) + 90*a*d/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1
)^4) + 90*b*e*arcsin(1/(c*x))/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 9
0*a*e/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 6*b*e/(c^12*x^5*(sqrt(-1/
(c^2*x^2) + 1) + 1)^5) + 15*b*e*arcsin(1/(c*x))/(c^13*x^6*(sqrt(-1/(c^2*x^2
) + 1) + 1)^6) + 15*a*e/(c^13*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6))*c
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

# 3.85 $\int x(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=138

$$\frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}}+\frac{bex(-1+c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}}+\frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{4e}+\frac{bcd^2x\text{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{4e\sqrt{c^2x^2}}$$

```
[Out] 1/4*(e*x^2+d)^2*(a+b*arccsc(c*x))/e+1/12*b*e*x*(c^2*x^2-1)^(3/2)/c^3/(c^2*x
^2)^(1/2)+1/4*b*c*d^2*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/4*b*(
2*c^2*d+e)*x*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5345, 457, 90, 65, 211}

$$\frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{4e}+\frac{bcd^2x\text{ArcTan}\left(\sqrt{c^2x^2-1}\right)}{4e\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,(2c^2d+e)}{4c^3\sqrt{c^2x^2}}+\frac{bex(c^2x^2-1)^{3/2}}{12c^3\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(2*c^2*d + e)*x*Sqrt[-1 + c^2*x^2])/(4*c^3*Sqrt[c^2*x^2]) + (b*e*x*(-1 +
 c^2*x^2)^(3/2))/(12*c^3*Sqrt[c^2*x^2]) + ((d + e*x^2)^2*(a + b*ArcCsc[c*x]
))/(4*e) + (b*c*d^2*x*ArcTan[Sqrt[-1 + c^2*x^2]])/(4*e*Sqrt[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{(bcx)\int\frac{\left(d+ex^2\right)^2}{x\sqrt{-1+c^2x^2}}\,dx}{4e\sqrt{c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{(bcx)\text{Subst}\left(\int\frac{(d+ex)^2}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{8e\sqrt{c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{(bcx)\text{Subst}\left(\int\left(\frac{e(2c^2d+e)}{c^2\sqrt{-1+c^2x}}+\frac{\ldots}{x\sqrt{-\ldots}}\right.\right.}{8e\sqrt{\ldots}}$$

$$=\frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}}+\frac{bex\left(-1+c^2x^2\right)^{3/2}}{12c^3\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\,\mathrm{c}\ldots\right.}{4e}$$

$$=\frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}}+\frac{bex\left(-1+c^2x^2\right)^{3/2}}{12c^3\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\,\mathrm{c}\ldots\right.}{4e}$$

$$=\frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}}+\frac{bex\left(-1+c^2x^2\right)^{3/2}}{12c^3\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\,\mathrm{c}\ldots\right.}{4e}$$

**Mathematica [A]**
time = 0.08, size = 78, normalized size = 0.57

$$\frac{x\left(3ac^3x(2d+ex^2)+b\sqrt{1-\frac{1}{c^2x^2}}\,(2e+c^2(6d+ex^2))+3bc^3x(2d+ex^2)\csc^{-1}(cx)\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(3*a*c^3*x*(2*d + e*x^2) + b*Sqrt[1 - 1/(c^2*x^2)]*(2*e + c^2*(6*d + e*x
^2)) + 3*b*c^3*x*(2*d + e*x^2)*ArcCsc[c*x]))/(12*c^3)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(118) = 236$.
time = 0.50, size = 238, normalized size = 1.72

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{(c^2 e x^2+c^2 d)^2 a}{4c^2 e}+\frac{b c^2 \operatorname{arccsc}(cx) d^2}{4e}+\frac{b \operatorname{arccsc}(cx) d c^2 x^2}{2}+\frac{b c^2 e \operatorname{arccsc}(cx) x^4}{4}-\frac{bc\sqrt{c^2x^2-1}\, d^2 \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x}+ \cdots}{c^2}$ |
| default | $\frac{\frac{(c^2 e x^2+c^2 d)^2 a}{4c^2 e}+\frac{b c^2 \operatorname{arccsc}(cx) d^2}{4e}+\frac{b \operatorname{arccsc}(cx) d c^2 x^2}{2}+\frac{b c^2 e \operatorname{arccsc}(cx) x^4}{4}-\frac{bc\sqrt{c^2x^2-1}\, d^2 \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x}+ \cdots}{c^2}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/4*(c^2*e*x^2+c^2*d)^2*a/c^2/e+1/4*b*c^2/e*arccsc(c*x)*d^2+1/2*b*ar
ccsc(c*x)*d*c^2*x^2+1/4*b*c^2*e*arccsc(c*x)*x^4-1/4*b*c/e*(c^2*x^2-1)^(1/2)
/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2*arctan(1/(c^2*x^2-1)^(1/2))+1/2*b*(c^2*x
^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*d+1/12*b/c*e*(c^2*x^2-1)/((c^2*x^2-1)
/c^2/x^2)^(1/2)*x+1/6*b/c^3*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

**Maxima** [A]
time = 0.26, size = 100, normalized size = 0.72

$$\frac{1}{4}\,ax^4e+\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\operatorname{arccsc}\left(cx\right)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd+\frac{1}{12}\left(3\,x^4\operatorname{arccsc}\left(cx\right)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e + 1/2*a*d*x^2 + 1/2*(x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)
/c)*b*d + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x
*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*e
```

**Fricas** [A]
time = 0.37, size = 89, normalized size = 0.64

$$\frac{3\,ac^4x^4e+6\,ac^4dx^2+3\left(bc^4x^4e+2\,bc^4dx^2\right)\operatorname{arccsc}\left(cx\right)+\left(6\,bc^2d+\left(bc^2x^2+2\,b\right)e\right)\sqrt{c^2x^2-1}}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4*e + 6*a*c^4*d*x^2 + 3*(b*c^4*x^4*e + 2*b*c^4*d*x^2)*arccs
c(c*x) + (6*b*c^2*d + (b*c^2*x^2 + 2*b)*e)*sqrt(c^2*x^2 - 1))/c^4
```

**Sympy** [A]
time = 2.36, size = 177, normalized size = 1.28

$$\frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\operatorname{acsc}(cx)}{2}+\frac{bex^4\operatorname{acsc}(cx)}{4}+\frac{bd\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}+\frac{be\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*acsc(c*x)/2 + b*e*x**4*acsc(c*x)/4 + b*d
*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2
+ 1)/c, True))/(2*c) + b*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sq
rt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 +
1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(118) = 236.
time = 0.45, size = 556, normalized size = 4.03

$$\frac{1}{192}\left(\frac{3bex^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{3aex^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c}+\frac{2bex^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c^2}+\frac{24bdx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{24adx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c}+\frac{12bex^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)}{c^3}+\frac{12aex^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c^3}+\frac{48bdx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{18bex\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{48bd\arcsin\left(\frac{1}{cx}\right)}{c^3}+\frac{48ad}{c^3}+\frac{18be\arcsin\left(\frac{1}{cx}\right)}{c^5}+\frac{18ae}{c^5}-\frac{48bd}{c^4x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}-\frac{18be}{c^6x\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}+\frac{24bd\arcsin\left(\frac{1}{cx}\right)}{c^5x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}+\frac{24ad}{c^5x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}+\frac{12be\arcsin\left(\frac{1}{cx}\right)}{c^7x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}+\frac{12ae}{c^7x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}-\frac{2be}{c^8x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}+\frac{3be\arcsin\left(\frac{1}{cx}\right)}{c^9x^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}+\frac{3ae}{c^9x^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a*e*x
^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 2*b*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1
)^3/c^2 + 24*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + 24*
a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c + 12*b*e*x^2*(sqrt(-1/(c^2*x^2) +
1) + 1)^2*arcsin(1/(c*x))/c^3 + 12*a*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c
^3 + 48*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 18*b*e*x*(sqrt(-1/(c^2*x^2
) + 1) + 1)/c^4 + 48*b*d*arcsin(1/(c*x))/c^3 + 48*a*d/c^3 + 18*b*e*arcsin(1
/(c*x))/c^5 + 18*a*e/c^5 - 48*b*d/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 18
*b*e/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 24*b*d*arcsin(1/(c*x))/(c^5*x^2
*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 24*a*d/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^2) + 12*b*e*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) +
 12*a*e/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 2*b*e/(c^8*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3) + 3*b*e*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^4) + 3*a*e/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
[Out] int(x*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

# 3.86 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x} \, dx$

**Optimal**. Leaf size=124

$$\frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{2}ibd\csc^{-1}(cx)^2+\frac{1}{2}ex^2\left(a+b\csc^{-1}(cx)\right)-bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+bd\csc^{-1}(cx)$$

```
[Out] 1/2*I*b*d*arccsc(c*x)^2+1/2*e*x^2*(a+b*arccsc(c*x))-b*d*arccsc(c*x)*ln(1-(I
/c/x+(1-1/c^2/x^2)^(1/2))^2)+b*d*arccsc(c*x)*ln(1/x)-d*(a+b*arccsc(c*x))*ln
(1/x)+1/2*I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+1/2*b*e*x*(1-1/c^2
/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.21, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {5349, 14, 4815, 6874, 270, 2363, 4721, 3798, 2221, 2317, 2438}

$$-d\log\left(\frac{1}{x}\right)\left(a+b\csc^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\csc^{-1}(cx)\right)+\frac{bex\sqrt{1-\frac{1}{c^2x^2}}}{2c}+\frac{1}{2}ibd\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)+\frac{1}{2}ibd\csc^{-1}(cx)^2-bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+bd\log\left(\frac{1}{x}\right)\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] (b*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c) + (I/2)*b*d*ArcCsc[c*x]^2 + (e*x^2*(a +
 b*ArcCsc[c*x]))/2 - b*d*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + b*d*A
rcCsc[c*x]*Log[x^(-1)] - d*(a + b*ArcCsc[c*x])*Log[x^(-1)] + (I/2)*b*d*Poly
Log[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4815

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
```

```
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x}\,dx = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}ex^2\big(a+b\csc^{-1}(cx)\big) - d\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{-\frac{e}{2x^2}+}{\sqrt{1-}} \right.}{c}$$

$$= \frac{1}{2}ex^2\big(a+b\csc^{-1}(cx)\big) - d\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \left(-\frac{}{2x} \right.\right.}{}$$

$$= \frac{1}{2}ex^2\big(a+b\csc^{-1}(cx)\big) - d\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{(bd)\text{Subst}\left(\int \frac{}{\sqrt{}} \right.}{}$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ex^2\big(a+b\csc^{-1}(cx)\big) + bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d\big(a$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ex^2\big(a+b\csc^{-1}(cx)\big) + bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - d\big(a$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2\big(a+b\csc^{-1}(cx)\big) + bd\csc^{-1}(c$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2\big(a+b\csc^{-1}(cx)\big) - bd\csc^{-1}(c$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2\big(a+b\csc^{-1}(cx)\big) - bd\csc^{-1}(c$$

$$= \frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ibd\csc^{-1}(cx)^2 + \frac{1}{2}ex^2\big(a+b\csc^{-1}(cx)\big) - bd\csc^{-1}(c$$

**Mathematica [A]**

time = 0.06, size = 108, normalized size = 0.87

$$\frac{1}{2}aex^2+\frac{bex\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c}+\frac{1}{2}bex^2\csc^{-1}(cx)-bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+ad\log(x)+\frac{1}{2}ibd\left(\csc^{-1}(cx)^2+\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] (a*e*x^2)/2 + (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*e*x^2*ArcCs
c[c*x])/2 - b*d*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + a*d*Log[x] + (
I/2)*b*d*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])])
```

**Maple** [A]
time = 4.88, size = 198, normalized size = 1.60

| method | result |
|---|---|
| derivativedivides | $\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{ibd\operatorname{arccsc}(cx)^2}{2}+\frac{b\operatorname{arccsc}(cx)e\,x^2}{2}+\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,ex}{2c}-\frac{ibe}{2c^2}-bd\operatorname{arccsc}(cx)\ln\Big($ |
| default | $\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{ibd\operatorname{arccsc}(cx)^2}{2}+\frac{b\operatorname{arccsc}(cx)e\,x^2}{2}+\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,ex}{2c}-\frac{ibe}{2c^2}-bd\operatorname{arccsc}(cx)\ln\Big($ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*e*x^2+a*d*ln(c*x)+1/2*I*b*d*arccsc(c*x)^2+1/2*b*arccsc(c*x)*e*x^2+1/2
*b/c*((c^2*x^2-1)/c^2/x^2)^(1/2)*e*x-1/2*I*b/c^2*e-b*d*arccsc(c*x)*ln(1-I/c
/x-(1-1/c^2/x^2)^(1/2))-b*d*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I*b
*d*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+I*b*d*polylog(2,I/c/x+(1-1/c^2/x^2
)^(1/2))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2*e + a*d*log(x) + 1/4*(2*I*b*c^2*d*log(-c*x + 1)*log(x) + 2*I*b*c^
2*d*log(x)^2 + 2*I*b*c^2*d*dilog(c*x) + 2*I*b*c^2*d*dilog(-c*x) - I*(b*(log
(c*x + 1)/c^2 + log(c*x - 1)/c^2)*e + 8*b*d*integrate(1/2*log(x)/(c^2*x^3 -
 x), x))*c^2 + 2*(b*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e + I*b*c^2
*e*log(c))*x^2 + 4*c^2*integrate(1/2*(b*x^2*e + 2*b*d*log(x))*sqrt(c*x + 1)
*sqrt(c*x - 1)/(c^2*x^3 - x), x) + I*b*e*log(c*x - 1) + (-I*b*c^2*x^2*e - 2
```

```
*I*b*c^2*d*log(x))*log(c^2*x^2) + (2*I*b*c^2*d*log(x) + I*b*e)*log(c*x + 1)
 - 2*(-I*b*c^2*x^2*e - 2*(b*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I
*b*c^2*log(c))*d)*log(x))/c^2
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsc(c*x))/x, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsc}(cx))\left(d + ex^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)/x, x)
```

**Giac [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]Undef/
Unsigned
```

**Mupad [B]**
time = 0.92, size = 111, normalized size = 0.90

$$\frac{a\,e\,x^2}{2}-a\,d\,\ln\left(\frac{1}{x}\right)-b\,d\,\ln\left(1-{\mathrm{e}}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2{}\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{c\,x}\right)+\frac{b\,e\,x\,\left(\sqrt{1-\frac{1}{c^2\,x^2}}+c\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{2\,c}+\frac{b\,d\,\mathrm{polylog}\left(2,{\mathrm{e}}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{2}+\frac{b\,d\,{\mathrm{asin}\left(\frac{1}{c\,x}\right)}^2\,1{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x,x)
```

```
[Out] (b*d*polylog(2, exp(asin(1/(c*x))*2i))*1i)/2 - a*d*log(1/x) + (b*d*asin(1/(
c*x))^2*1i)/2 + (a*e*x^2)/2 - b*d*log(1 - exp(asin(1/(c*x))*2i))*asin(1/(c*
x)) + (b*e*x*((1 - 1/(c^2*x^2))^(1/2) + c*x*asin(1/(c*x))))/(2*c)
```

# 3.87 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^3}\,dx$

**Optimal**. Leaf size=137

$$-\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2d\csc^{-1}(cx)+\frac{1}{2}ibe\csc^{-1}(cx)^2-\frac{d(a+b\csc^{-1}(cx))}{2x^2}-be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)$$

```
[Out] 1/4*b*c^2*d*arccsc(c*x)+1/2*I*b*e*arccsc(c*x)^2-1/2*d*(a+b*arccsc(c*x))/x^2
-b*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+b*e*arccsc(c*x)*ln(1/x
)-e*(a+b*arccsc(c*x))*ln(1/x)+1/2*I*b*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2
))^2)-1/4*b*c*d*(1-1/c^2/x^2)^(1/2)/x
```

**Rubi** [A]
time = 0.21, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {5349, 14, 4815, 12, 6874, 327, 222, 2363, 4721, 3798, 2221, 2317, 2438}

$$-\frac{d(a+b\csc^{-1}(cx))}{2x^2}-e\log\left(\frac{1}{x}\right)(a+b\csc^{-1}(cx))-\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2d\csc^{-1}(cx)+\frac{1}{2}ibe\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)+\frac{1}{2}ibe\csc^{-1}(cx)^2-be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+be\log\left(\frac{1}{x}\right)\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] -1/4*(b*c*d*Sqrt[1 - 1/(c^2*x^2)])/x + (b*c^2*d*ArcCsc[c*x])/4 + (I/2)*b*e*
ArcCsc[c*x]^2 - (d*(a + b*ArcCsc[c*x]))/(2*x^2) - b*e*ArcCsc[c*x]*Log[1 - E
^((2*I)*ArcCsc[c*x])] + b*e*ArcCsc[c*x]*Log[x^(-1)] - e*(a + b*ArcCsc[c*x])
*Log[x^(-1)] + (I/2)*b*e*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4815

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^3}\,dx = -\text{Subst}\left(\int \frac{(e+dx^2)\,\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{2x^2} - e\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{dx^2+2e\,\text{lo}}{2\sqrt{1-}}\right.}{c}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{2x^2} - e\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{dx^2+2e\,\text{log}}{\sqrt{1-}}\right.}{2c}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{2x^2} - e\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \left(\frac{dx}{\sqrt{1-}}\right.\right.}{}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{2x^2} - e\big(a+b\csc^{-1}(cx)\big)\log\left(\frac{1}{x}\right) + \frac{(bd)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{2c}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} - \frac{d(a+b\csc^{-1}(cx))}{2x^2} + be\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - e\big(a+$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) - \frac{d(a+b\csc^{-1}(cx))}{2x^2} + be\csc^{-1}(cx)$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\csc^{-1}(c}{2x^2}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\csc^{-1}(c}{2x^2}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\csc^{-1}(c}{2x^2}$$

$$= -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{1}{4}bc^2d\csc^{-1}(cx) + \frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\csc^{-1}(c}{2x^2}$$

**Mathematica [A]**
time = 0.07, size = 125, normalized size = 0.91

$$-\frac{ad}{2x^2}-\frac{bcd\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{4x}-\frac{bd\csc^{-1}(cx)}{2x^2}+\frac{1}{4}bc^2d\text{ArcSin}\left(\frac{1}{cx}\right)-be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+ae\log(x)+\frac{1}{2}ibe\left(\csc^{-1}(cx)^2+\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] -1/2*(a*d)/x^2 - (b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*d*ArcCsc
[c*x])/(2*x^2) + (b*c^2*d*ArcSin[1/(c*x)])/4 - b*e*ArcCsc[c*x]*Log[1 - E^((
2*I)*ArcCsc[c*x])] + a*e*Log[x] + (I/2)*b*e*(ArcCsc[c*x]^2 + PolyLog[2, E^(
(2*I)*ArcCsc[c*x])])
```

**Maple [A]**
time = 1.11, size = 204, normalized size = 1.49

| method | result |
|---|---|
| derivativedivides | $c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{ib\text{arccsc}(cx)^2e}{2c^2}-\frac{be\,\text{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}-\frac{be\,\text{arccsc}(cx)\ln\left(1-\right.}{c}\right.$ |
| default | $c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{ib\text{arccsc}(cx)^2e}{2c^2}-\frac{be\,\text{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}-\frac{be\,\text{arccsc}(cx)\ln\left(1-\right.}{c}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a*d/c^2/x^2+a/c^2*e*ln(c*x)+1/2*I*b/c^2*arccsc(c*x)^2*e-b/c^2*e*a
rccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-b/c^2*e*arccsc(c*x)*ln(1-I/c/x-(
1-1/c^2/x^2)^(1/2))+I*b/c^2*e*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+I*b/c^2*
e*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+1/4*b*d*arccsc(c*x)*cos(2*arccsc(c*
x))-1/8*b*d*sin(2*arccsc(c*x)))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*d*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^
3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) + (c^2*integra
```

```
te(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) + arctan2(1, sq
rt(c*x + 1)*sqrt(c*x - 1))*log(x))*b*e + a*e*log(x) - 1/2*a*d/x^2
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsc(c*x))/x^3, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsc}(cx))(d + ex^2)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsc(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)/x**3, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsc(c*x) + a)/x^3, x)
```

**Mupad [B]**
time = 0.99, size = 124, normalized size = 0.91

$$-a\,e\,\ln\left(\frac{1}{x}\right)-\frac{a\,d}{2\,x^2}-b\,e\,\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2{}\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{c\,x}\right)-\frac{b\,c\,d\,\sqrt{1-\frac{1}{c^2\,x^2}}}{4\,x}-\frac{b\,c^2\,d\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\,\left(\frac{2}{c^2\,x^2}-1\right)}{4}+\frac{b\,e\,\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2{}\mathrm{i}}\right)\,1{}\mathrm{i}}{2}+\frac{b\,e\,{\mathrm{asin}\left(\frac{1}{c\,x}\right)}^2\,1{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^3,x)
```

```
[Out] (b*e*polylog(2, exp(asin(1/(c*x))*2i))*1i)/2 - a*e*log(1/x) + (b*e*asin(1/(
c*x))^2*1i)/2 - (a*d)/(2*x^2) - b*e*log(1 - exp(asin(1/(c*x))*2i))*asin(1/(
c*x)) - (b*c*d*(1 - 1/(c^2*x^2))^(1/2))/(4*x) - (b*c^2*d*asin(1/(c*x))*(2/(
c^2*x^2) - 1))/4
```

# 3.88 $\int x^2(d+ex^2)^2 \left(a+b\csc^{-1}(cx)\right) \, dx$

**Optimal**. Leaf size=252

$$\frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}}+\frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3$$

```
[Out] 1/3*d^2*x^3*(a+b*arccsc(c*x))+2/5*d*e*x^5*(a+b*arccsc(c*x))+1/7*e^2*x^7*(a+
b*arccsc(c*x))+1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*x*arctanh(c*x/(c^2
*x^2-1)^(1/2))/c^6/(c^2*x^2)^(1/2)+1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2
)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)+1/840*b*e*(84*c^2*d+25*e)*x^4*(
c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)+1/42*b*e^2*x^6*(c^2*x^2-1)^(1/2)/c/(c^
2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.17, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {276, 5347, 12, 1281, 470, 327, 223, 212}

$$\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)+\frac{be^2x^6\sqrt{c^2x^2-1}}{42c\sqrt{c^2x^2}}+\frac{bex^4\sqrt{c^2x^2-1}\,(84c^2d+25e)}{840c^3\sqrt{c^2x^2}}+\frac{bx(280c^4d^2+252c^2de+75e^2)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{1680c^6\sqrt{c^2x^2}}+\frac{bx^2\sqrt{c^2x^2-1}\,(280c^4d^2+252c^2de+75e^2)}{1680c^5\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x^2*Sqrt[-1 + c^2*x^2])/(1680*c^5*S
qrt[c^2*x^2]) + (b*e*(84*c^2*d + 25*e)*x^4*Sqrt[-1 + c^2*x^2])/(840*c^3*Sqr
t[c^2*x^2]) + (b*e^2*x^6*Sqrt[-1 + c^2*x^2])/(42*c*Sqrt[c^2*x^2]) + (d^2*x^
3*(a + b*ArcCsc[c*x]))/3 + (2*d*e*x^5*(a + b*ArcCsc[c*x]))/5 + (e^2*x^7*(a
+ b*ArcCsc[c*x]))/7 + (b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x*ArcTanh[(c*
x)/Sqrt[-1 + c^2*x^2]])/(1680*c^6*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^2 (d+ex^2)^2 \left(a+b\csc^{-1}(cx)\right) dx = \frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right) + \frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\,\mathrm{cs}\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right) + \frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\,\mathrm{cs}\right.$$

$$= \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right) + \frac{2}{5}dex^5\left(a+b\csc^{-1}(\right.$$

$$= \frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{3}d^2x^3(a+$$

$$= \frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}} + \frac{be(84c^2d+25e)\,x^4}{840c^3\sqrt{c^2}}$$

$$= \frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}} + \frac{be(84c^2d+25e)\,x^4}{840c^3\sqrt{c^2}}$$

$$= \frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}} + \frac{be(84c^2d+25e)\,x^4}{840c^3\sqrt{c^2}}$$

**Mathematica** [A]
time = 0.23, size = 184, normalized size = 0.73

$$\frac{c^2x^2\left(16ac^5x(35d^2+42dex^2+15e^2x^4)+b\sqrt{1-\frac{1}{c^2x^2}}\,(75e^2+2c^2e(126d+25ex^2)+8c^4(35d^2+21dex^2+5e^2x^4))\right)+16bc^7x^3(35d^2+42dex^2+15e^2x^4)\csc^{-1}(cx)+b(280c^4d^2+252c^2de+75e^2)\log\left(\left(1+\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{1680c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (c^2*x^2*(16*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) + b*Sqrt[1 - 1/(c^2
*x^2)]*(75*e^2 + 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21*d*e*x^2 +
5*e^2*x^4))) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCsc[c*x]
+ b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x]
)/(1680*c^7)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(222) = 444$.
time = 0.52, size = 475, normalized size = 1.88

| method | result |
|---|---|

| | |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\,\mathrm{arccsc}(cx)d^2c^3x^3}{3}+\frac{2b\,c^3\mathrm{arccsc}(cx)de\,x^5}{5}+\frac{b\,c^3\mathrm{arccsc}(cx)e^2x^7}{7}+\frac{b\left(c^2x^2-1\right)d^2}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\left(c^2x^2-1\right)\dots}{10\sqrt{\frac{c^2x^2\dots}{c^2x\dots}}}\dots}{\dots}$ |
| default | $\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\,\mathrm{arccsc}(cx)d^2c^3x^3}{3}+\frac{2b\,c^3\mathrm{arccsc}(cx)de\,x^5}{5}+\frac{b\,c^3\mathrm{arccsc}(cx)e^2x^7}{7}+\frac{b\left(c^2x^2-1\right)d^2}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\left(c^2x^2-1\right)\dots}{10\sqrt{\frac{c^2x^2\dots}{c^2x\dots}}}\dots}{\dots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/c^4*(1/3*d^2*c^7*x^3+2/5*d*c^7*e*x^5+1/7*e^2*c^7*x^7)+1/3*b*arccsc
(c*x)*d^2*c^3*x^3+2/5*b*c^3*arccsc(c*x)*d*e*x^5+1/7*b*c^3*arccsc(c*x)*e^2*x
^7+1/6*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2+1/10*b*(c^2*x^2-1)/((c
^2*x^2-1)/c^2/x^2)^(1/2)*x^2*d*e+1/42*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(
1/2)*x^4*e^2+1/6*b*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*d^2*ln
(c*x+(c^2*x^2-1)^(1/2))+3/20*b/c^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*
d*e+5/168*b/c^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e^2+3/20*b/c^3*
(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*e*ln(c*x+(c^2*x^2-1)^(1/2
))+5/112*b/c^4*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2+5/112*b/c^5*(c^2
*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2)))
```

**Maxima [A]**
time = 0.27, size = 404, normalized size = 1.60

$$\frac{1}{7}ax^7e^2+\frac{2}{5}adx^5e+\frac{1}{3}ad^2x^3+\frac{1}{12}\left(4x^3\,\mathrm{arccsc}\left(cx\right)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd^2+\frac{1}{40}\left(16x^5\,\mathrm{arccsc}\left(cx\right)-\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bde+\frac{1}{672}\left(96x^7\,\mathrm{arccsc}\left(cx\right)+\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3c^6\left(\frac{1}{c^2x^2}-1\right)^2+3c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7*e^2 + 2/5*a*d*x^5*e + 1/3*a*d^2*x^3 + 1/12*(4*x^3*arccsc(c*x) + (
2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x
^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d^2 + 1/40*(1
6*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) +
1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(
-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d*
e + 1/672*(96*x^7*arccsc(c*x) + (2*(15*(-1/(c^2*x^2) + 1)^(5/2) - 40*(-1/(c
^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) - 1)^3 +
3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*log(sqrt(-1
/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^6)/c)*b*e^2
```

**Fricas [A]**
time = 0.57, size = 272, normalized size = 1.08

$$\frac{240\,ac^7x^7e^2+672\,ac^7dx^5e+560\,ac^7d^2x^3+16\left(35\,bc^7d^2x^3-35\,bc^7d^2+15\left(bc^7x^7-bc^7\right)e^2+42\left(bc^7dx^5-bc^7d\right)e\right)\mathrm{arccsc}\left(cx\right)-32\left(35\,bc^7d^2+42\,bc^7de+15\,bc^7e^2\right)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-\left(280\,bc^4d^2+252\,bc^2de+75\,be^2\right)\log\left(-cx+\sqrt{c^2x^2-1}\right)+\left(280\,bc^5d^2x+5\left(8\,bc^5x^5+10\,bc^3x^3+15\,bcx\right)e^2+84\left(2\,bc^5dx^3+3\,bc^3dx\right)e\right)\sqrt{c^2x^2-1}}{1680\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*x^7*e^2 + 672*a*c^7*d*x^5*e + 560*a*c^7*d^2*x^3 + 16*(35*
b*c^7*d^2*x^3 - 35*b*c^7*d^2 + 15*(b*c^7*x^7 - b*c^7)*e^2 + 42*(b*c^7*d*x^5
 - b*c^7*d)*e)*arccsc(c*x) - 32*(35*b*c^7*d^2 + 42*b*c^7*d*e + 15*b*c^7*e^2
)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (280*b*c^4*d^2 + 252*b*c^2*d*e + 75*b*
e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) + (280*b*c^5*d^2*x + 5*(8*b*c^5*x^5 + 10
*b*c^3*x^3 + 15*b*c*x)*e^2 + 84*(2*b*c^5*d*x^3 + 3*b*c^3*d*x)*e)*sqrt(c^2*x
^2 - 1))/c^7
```

**Sympy** [A]
time = 15.16, size = 542, normalized size = 2.15

$$\frac{ad^2x^3}{3}+\frac{2adex^5}{5}+\frac{ae^2x^7}{7}+\frac{bd^2x^3\operatorname{acsc}(cx)}{3}+\frac{2bdex^5\operatorname{acsc}(cx)}{5}+\frac{be^2x^7\operatorname{acsc}(cx)}{7}+\frac{bd^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}+\frac{2bde\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}+\frac{be^2\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\operatorname{acosh}(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\operatorname{asin}(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*acsc(c*x)/3 +
2*b*d*e*x**5*acsc(c*x)/5 + b*e**2*x**7*acsc(c*x)/7 + b*d**2*Piecewise((x*sq
rt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x
**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)
/(2*c**2), True))/(3*c) + 2*b*d*e*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1))
 + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*ac
osh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1))
 - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1))
- 3*I*asin(c*x)/(8*c**4), True))/(5*c) + b*e**2*Piecewise((c*x**7/(6*sqrt(c
**2*x**2 - 1)) + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**
2*x**2 - 1)) - 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6),
Abs(c**2*x**2) > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqr
t(-c**2*x**2 + 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c*
*5*sqrt(-c**2*x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. $2(222) = 444$.
time = 5.30, size = 1579, normalized size = 6.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*b*e^2*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*arcsin(1/(c*x))/c + 15
*a*e^2*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7/c + 5*b*e^2*x^6*(sqrt(-1/(c^2*x^2
```

```
) + 1) + 1)^6/c^2 + 168*b*d*e*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(
c*x))/c + 168*a*d*e*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 105*b*e^2*x^5*(s
qrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c^3 + 105*a*e^2*x^5*(sqrt(-1/(
c^2*x^2) + 1) + 1)^5/c^3 + 84*b*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^2
+ 560*b*d^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + 560*a*d^
2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c + 45*b*e^2*x^4*(sqrt(-1/(c^2*x^2) +
1) + 1)^4/c^4 + 840*b*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x)
)/c^3 + 840*a*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^3 + 560*b*d^2*x^2*(s
qrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 315*b*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1) +
1)^3*arcsin(1/(c*x))/c^5 + 315*a*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^5
 + 672*b*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^4 + 1680*b*d^2*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 1680*a*d^2*x*(sqrt(-1/(c^2*x^2)
+ 1) + 1)/c^3 + 225*b*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^6 + 1680*b*d
*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 1680*a*d*e*x*(sqrt(
-1/(c^2*x^2) + 1) + 1)/c^5 + 2240*b*d^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4
 - 2240*b*d^2*log(1/(abs(c)*abs(x)))/c^4 + 525*b*e^2*x*(sqrt(-1/(c^2*x^2) +
 1) + 1)*arcsin(1/(c*x))/c^7 + 525*a*e^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^7
 + 2016*b*d*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 2016*b*d*e*log(1/(abs(c
)*abs(x)))/c^6 + 1680*b*d^2*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1)
+ 1)) + 1680*a*d^2/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 600*b*e^2*log(sqr
t(-1/(c^2*x^2) + 1) + 1)/c^8 - 600*b*e^2*log(1/(abs(c)*abs(x)))/c^8 + 1680*
b*d*e*arcsin(1/(c*x))/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1680*a*d*e/(c^
7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 560*b*d^2/(c^6*x^2*(sqrt(-1/(c^2*x^2) +
 1) + 1)^2) + 525*b*e^2*arcsin(1/(c*x))/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)
) + 525*a*e^2/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 672*b*d*e/(c^8*x^2*(sq
rt(-1/(c^2*x^2) + 1) + 1)^2) + 560*b*d^2*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3) + 560*a*d^2/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 - 225*b*e^2/(c^10*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 840*b*d*e*arcsin(1
/(c*x))/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 840*a*d*e/(c^9*x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3) + 315*b*e^2*arcsin(1/(c*x))/(c^11*x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3) + 315*a*e^2/(c^11*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 - 84*b*d*e/(c^10*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 45*b*e^2/(c^12*x^4*
(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 168*b*d*e*arcsin(1/(c*x))/(c^11*x^5*(sqrt
(-1/(c^2*x^2) + 1) + 1)^5) + 168*a*d*e/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) +
1)^5) + 105*b*e^2*arcsin(1/(c*x))/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)
 + 105*a*e^2/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - 5*b*e^2/(c^14*x^6*
(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 15*b*e^2*arcsin(1/(c*x))/(c^15*x^7*(sqrt(
-1/(c^2*x^2) + 1) + 1)^7) + 15*a*e^2/(c^15*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)
^7))*c
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

# 3.89 $\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=191

$$\frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5($$

```
[Out] d^2*x*(a+b*arccsc(c*x))+2/3*d*e*x^3*(a+b*arccsc(c*x))+1/5*e^2*x^5*(a+b*arcc
sc(c*x))+1/120*b*(120*c^4*d^2+40*c^2*d*e+9*e^2)*x*arctanh(c*x/(c^2*x^2-1)^(
1/2))/c^4/(c^2*x^2)^(1/2)+1/120*b*e*(40*c^2*d+9*e)*x^2*(c^2*x^2-1)^(1/2)/c^
3/(c^2*x^2)^(1/2)+1/20*b*e^2*x^4*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.08, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$,
Rules used = {200, 5337, 12, 1173, 396, 223, 212}

$$d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)+\frac{be^2x^4\sqrt{c^2x^2-1}}{20c\sqrt{c^2x^2}}+\frac{bx(120c^4d^2+40c^2de+9e^2)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{120c^4\sqrt{c^2x^2}}+\frac{bex^2\sqrt{c^2x^2-1}\,(40c^2d+9e)}{120c^3\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*(40*c^2*d + 9*e)*x^2*Sqrt[-1 + c^2*x^2])/(120*c^3*Sqrt[c^2*x^2]) + (b*
e^2*x^4*Sqrt[-1 + c^2*x^2])/(20*c*Sqrt[c^2*x^2]) + d^2*x*(a + b*ArcCsc[c*x]
) + (2*d*e*x^3*(a + b*ArcCsc[c*x]))/3 + (e^2*x^5*(a + b*ArcCsc[c*x]))/5 + (
b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*x*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(
120*c^4*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 5337

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx\right.$$

$$= d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx\right.$$

$$= \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+$$

$$= \frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}\right.$$

$$= \frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}\right.$$

$$= \frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}\right.$$

**Mathematica** [A]
time = 0.15, size = 151, normalized size = 0.79

$$\frac{c^2x\left(8ac^3(15d^2+10dex^2+3e^2x^4)+be\sqrt{1-\frac{1}{c^2x^2}}\,x(9e+c^2(40d+6ex^2))\right)+8bc^5x(15d^2+10dex^2+3e^2x^4)\csc^{-1}(cx)+b(120c^4d^2+40c^2de+9e^2)\log\left(\left(1+\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (c^2*x*(8*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 - 1/(c^2*x^2
)]*x*(9*e + c^2*(40*d + 6*e*x^2))) + 8*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2
*x^4)*ArcCsc[c*x] + b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Log[(1 + Sqrt[1 -
1/(c^2*x^2)])*x])/(120*c^5)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(169) = 338.
time = 0.37, size = 358, normalized size = 1.87

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(d^2c^5x+\frac{2}{3}dc^5ex^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\,\mathrm{arccsc}(cx)d^2cx+\frac{2bc\,\mathrm{arccsc}(cx)de\,x^3}{3}+\frac{bc\,\mathrm{arccsc}(cx)e^2x^5}{5}+\frac{b\sqrt{c^2x^2-1}\,d^2\ln\left(cx+\sqrt{c^2x\right.}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{}$ |

| | |
|---|---|
| default | $\frac{\frac{a\left(d^2c^5x+\frac{2}{3}dc^5ex^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\,\mathrm{arccsc}(cx)d^2cx+\frac{2bc\,\mathrm{arccsc}(cx)de\,x^3}{3}+\frac{bc\,\mathrm{arccsc}(cx)e^2x^5}{5}+\frac{b\sqrt{c^2x^2-1}\,d^2\ln\left(cx+\sqrt{c^2\right.}}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^4*(d^2*c^5*x+2/3*d*c^5*e*x^3+1/5*e^2*c^5*x^5)+b*arccsc(c*x)*d^2*c*
x+2/3*b*c*arccsc(c*x)*d*e*x^3+1/5*b*c*arccsc(c*x)*e^2*x^5+b*(c^2*x^2-1)^(1/
2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))+1/3*b/c^2*
(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e+1/20*b/c^2*(c^2*x^2-1)/((c^2*x^
2-1)/c^2/x^2)^(1/2)*x^2*e^2+1/3*b/c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^
2)^(1/2)/x*d*e*ln(c*x+(c^2*x^2-1)^(1/2))+3/40*b/c^4*(c^2*x^2-1)/((c^2*x^2-1
)/c^2/x^2)^(1/2)*e^2+3/40*b/c^5*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/
2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2)))
```

**Maxima** [A]
time = 0.26, size = 296, normalized size = 1.55

$$\frac{1}{5}ax^5e^2+\frac{2}{3}adx^3e+ad^2x+\frac{1}{6}\left(4x^3\,\mathrm{arccsc}(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde+\frac{\left(2cx\,\mathrm{arccsc}(cx)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^2}{2c}+\frac{1}{80}\left(16x^5\,\mathrm{arccsc}(cx)-\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5*e^2 + 2/3*a*d*x^3*e + a*d^2*x + 1/6*(4*x^3*arccsc(c*x) + (2*sqrt(
-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1
) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e + 1/2*(2*c*x*arc
csc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) +
1))*b*d^2/c + 1/80*(16*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5
*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1)
 + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) +
 1) - 1)/c^4)/c)*b*e^2
```

**Fricas** [A]
time = 0.49, size = 236, normalized size = 1.24

$$\frac{24ac^5x^5e^2+80ac^5dx^3e+120ac^5d^2x+8\left(15bc^5d^2x-15bc^5d^2+3\left(bc^5x^5-bc^5\right)e^2+10\left(bc^5dx^3-bc^5d\right)e\right)\mathrm{arccsc}(cx)-16\left(15bc^5d^2+10bc^5de+3bc^5e^2\right)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-\left(120bc^4d^2+40bc^2de+9be^2\right)\log\left(-cx+\sqrt{c^2x^2-1}\right)+\left(40bc^3dxe+3\left(2bc^3x^3+3bcx\right)e^2\right)\sqrt{c^2x^2-1}}{120c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*x^5*e^2 + 80*a*c^5*d*x^3*e + 120*a*c^5*d^2*x + 8*(15*b*c^5*
d^2*x - 15*b*c^5*d^2 + 3*(b*c^5*x^5 - b*c^5)*e^2 + 10*(b*c^5*d*x^3 - b*c^5*
```

```
d)*e)*arccsc(c*x) - 16*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*arctan(-
c*x + sqrt(c^2*x^2 - 1)) - (120*b*c^4*d^2 + 40*b*c^2*d*e + 9*b*e^2)*log(-c*
x + sqrt(c^2*x^2 - 1)) + (40*b*c^3*d*x*e + 3*(2*b*c^3*x^3 + 3*b*c*x)*e^2)*s
qrt(c^2*x^2 - 1))/c^5
```

**Sympy** [A]
time = 7.35, size = 355, normalized size = 1.86

$$ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x\operatorname{acsc}(cx) + \frac{2bdex^3\operatorname{acsc}(cx)}{3} + \frac{be^2x^5\operatorname{acsc}(cx)}{5} + \frac{bd^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} + \frac{2bde\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c} + \frac{be^2\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}} + \frac{x^3}{8c\sqrt{c^2x^2-1}} - \frac{3x}{8c^3\sqrt{c^2x^2-1}} + \frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}} - \frac{ix^3}{8c\sqrt{-c^2x^2+1}} + \frac{3ix}{8c^3\sqrt{-c^2x^2+1}} - \frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*acsc(c*x) + 2*b*d*e*x*
*3*acsc(c*x)/3 + b*e**2*x**5*acsc(c*x)/5 + b*d**2*Piecewise((acosh(c*x), Ab
s(c**2*x**2) > 1), (-I*asin(c*x), True))/c + 2*b*d*e*Piecewise((x*sqrt(c**2
*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*
sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**
2), True))/(3*c) + b*e**2*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/
(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)
/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**
3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*as
in(c*x)/(8*c**4), True))/(5*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. $2(169) = 338$.
time = 3.45, size = 1033, normalized size = 5.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/960*(6*b*e^2*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c + 6*a*e
^2*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 3*b*e^2*x^4*(sqrt(-1/(c^2*x^2) +
1) + 1)^4/c^2 + 80*b*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))
/c + 80*a*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c + 30*b*e^2*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^3 + 30*a*e^2*x^3*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^3/c^3 + 80*b*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 480*b*
d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c + 480*a*d^2*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)/c + 24*b*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^4 +
240*b*d*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 240*a*d*e*x*
(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 960*b*d^2*log(sqrt(-1/(c^2*x^2) + 1) + 1
)/c^2 - 960*b*d^2*log(1/(abs(c)*abs(x)))/c^2 + 60*b*e^2*x*(sqrt(-1/(c^2*x^2
) + 1) + 1)*arcsin(1/(c*x))/c^5 + 60*a*e^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c
```

```
^5 + 320*b*d*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 320*b*d*e*log(1/(abs(c
)*abs(x)))/c^4 + 480*b*d^2*arcsin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) +
 1)) + 480*a*d^2/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 72*b*e^2*log(sqrt(-
1/(c^2*x^2) + 1) + 1)/c^6 - 72*b*e^2*log(1/(abs(c)*abs(x)))/c^6 + 240*b*d*e
*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 240*a*d*e/(c^5*x*(s
qrt(-1/(c^2*x^2) + 1) + 1)) + 60*b*e^2*arcsin(1/(c*x))/(c^7*x*(sqrt(-1/(c^2
*x^2) + 1) + 1)) + 60*a*e^2/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 80*b*d*e
/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 24*b*e^2/(c^8*x^2*(sqrt(-1/(c^2
*x^2) + 1) + 1)^2) + 80*b*d*e*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3) + 80*a*d*e/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 30*b*e^2*
arcsin(1/(c*x))/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 30*a*e^2/(c^9*x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 3*b*e^2/(c^10*x^4*(sqrt(-1/(c^2*x^2) +
1) + 1)^4) + 6*b*e^2*arcsin(1/(c*x))/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)
^5) + 6*a*e^2/(c^11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))*c
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
[Out] int((d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

# 3.90 $\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2}\,dx$

**Optimal**. Leaf size=163

$$-\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}+\frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex(a+b\csc^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\csc^{-1}(cx$$

```
[Out] -d^2*(a+b*arccsc(c*x))/x+2*d*e*x*(a+b*arccsc(c*x))+1/3*e^2*x^3*(a+b*arccsc(
c*x))+1/6*b*e*(12*c^2*d+e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^2/(c^2*x^2)^(
1/2)-b*c*d^2*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/6*b*e^2*x^2*(c^2*x^2-1)^(1
/2)/c/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 5347, 12, 1279, 396, 223, 212}

$$-\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex(a+b\csc^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\csc^{-1}(cx))-\frac{bcd^2\sqrt{c^2x^2-1}}{\sqrt{c^2x^2}}+\frac{bex(12c^2d+e)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{6c^2\sqrt{c^2x^2}}+\frac{be^2x^2\sqrt{c^2x^2-1}}{6c\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] -((b*c*d^2*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*x^2]) + (b*e^2*x^2*Sqrt[-1 + c^2*x^
2])/(6*c*Sqrt[c^2*x^2]) - (d^2*(a + b*ArcCsc[c*x]))/x + 2*d*e*x*(a + b*ArcC
sc[c*x]) + (e^2*x^3*(a + b*ArcCsc[c*x]))/3 + (b*e*(12*c^2*d + e)*x*ArcTanh[
(c*x)/Sqrt[-1 + c^2*x^2]])/(6*c^2*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2} dx = -\frac{d^2(a+b\csc^{-1}(cx))}{x} + 2dex(a+b\csc^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\csc^{-1}(cx)$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{x} + 2dex(a+b\csc^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\csc^{-1}(cx)$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{x} + 2dex(a+b\csc^{-1}(cx)) + \frac{1}{3}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} + \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{x} + 2dex$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} + \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{x} + 2dex$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} + \frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{x} + 2dex$$

**Mathematica [A]**
time = 0.12, size = 134, normalized size = 0.82

$$\frac{c^2\left(b\sqrt{1-\frac{1}{c^2x^2}}\,x(-6c^2d^2+e^2x^2)+2ac(-3d^2+6dex^2+e^2x^4)\right)+2bc^3(-3d^2+6dex^2+e^2x^4)\csc^{-1}(cx)+be(12c^2d+e)\,x\log\left(\left(1+\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{6c^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^2,x]
```

```
[Out] (c^2*(b*Sqrt[1 - 1/(c^2*x^2)]*x*(-6*c^2*d^2 + e^2*x^2) + 2*a*c*(-3*d^2 + 6*
d*e*x^2 + e^2*x^4)) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcCsc[c*x] +
b*e*(12*c^2*d + e)*x*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3*x)
```

**Maple [A]**
time = 0.37, size = 273, normalized size = 1.67

| method | result |
|---|---|
| derivativedivides | $c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b\,\mathrm{arccsc}(cx)dex}{c}+\frac{b\,\mathrm{arccsc}(cx)e^2x^3}{3c}-\frac{b\,\mathrm{arccsc}(cx)d^2}{cx}-\frac{b(c^2x^2-1)d^2}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\right.$ |

| default | $c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b\,\mathrm{arccsc}(cx)dex}{c}+\frac{b\,\mathrm{arccsc}(cx)e^2x^3}{3c}-\frac{b\,\mathrm{arccsc}(cx)d^2}{cx}-\frac{b(c^2x^2-1)d^2}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\right.$ |
|---|---|

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a/c^4*(2*c^3*d*e*x+1/3*e^2*c^3*x^3-c^3*d^2/x)+2*b/c*arccsc(c*x)*d*e*x+1/
3*b/c*arccsc(c*x)*e^2*x^3-b*arccsc(c*x)*d^2/c/x-b*(c^2*x^2-1)/c^2/x^2/((c^2
*x^2-1)/c^2/x^2)^(1/2)*d^2+2*b/c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/x*d*e*ln(c*x+(c^2*x^2-1)^(1/2))+1/6*b/c^4*(c^2*x^2-1)/((c^2*x^2-1)/c^
2/x^2)^(1/2)*e^2+1/6*b/c^5*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*
e^2*ln(c*x+(c^2*x^2-1)^(1/2)))
```

**Maxima [A]**
time = 0.27, size = 197, normalized size = 1.21

$$\frac{1}{3}\,ax^3e^2-\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\mathrm{arccsc}\,(cx)}{x}\right)bd^2+2\,adxe+\frac{1}{12}\left(4\,x^3\,\mathrm{arccsc}\,(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2+\frac{\left(2\,cx\,\mathrm{arccsc}\,(cx)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bde}{c}-\frac{ad^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e^2 - (c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*d^2 + 2*a*d*x*
e + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) -
 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) +
1) - 1)/c^2)/c)*b*e^2 + (2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1
) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d*e/c - a*d^2/x
```

**Fricas [A]**
time = 0.46, size = 234, normalized size = 1.44

$$\frac{2\,ac^3x^4e^2-6\,bc^4d^2x+12\,ac^3dx^2e-6\,ac^3d^2+2\,(3\,bc^3d^2x-3\,bc^3d^2+(bc^3x^4-bc^3x)e^2+6\,(bc^3dx^2-bc^3dx)e)\,\mathrm{arccsc}\,(cx)+4\,(3\,bc^3d^2x-6\,bc^3dxe-bc^3xe^2)\,\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-(12\,bc^2dxe+bxe^2)\log\left(-cx+\sqrt{c^2x^2-1}\right)-(6\,bc^3d^2-bcx^2e^2)\sqrt{c^2x^2-1}}{6\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^4*e^2 - 6*b*c^4*d^2*x + 12*a*c^3*d*x^2*e - 6*a*c^3*d^2 + 2*(
3*b*c^3*d^2*x - 3*b*c^3*d^2 + (b*c^3*x^4 - b*c^3*x)*e^2 + 6*(b*c^3*d*x^2 -
b*c^3*d*x)*e)*arccsc(c*x) + 4*(3*b*c^3*d^2*x - 6*b*c^3*d*x*e - b*c^3*x*e^2)
*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (12*b*c^2*d*x*e + b*x*e^2)*log(-c*x + s
qrt(c^2*x^2 - 1)) - (6*b*c^3*d^2 - b*c*x^2*e^2)*sqrt(c^2*x^2 - 1))/(c^3*x)
```

**Sympy** [**A**]
time = 5.55, size = 207, normalized size = 1.27

$$-\frac{ad^2}{x}+2adex+\frac{ae^2x^3}{3}-bcd^2\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd^2\operatorname{acsc}(cx)}{x}+2bdex\operatorname{acsc}(cx)+\frac{be^2x^3\operatorname{acsc}(cx)}{3}+\frac{2bde\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}+\frac{be^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**2,x)
```

```
[Out] -a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 - b*c*d**2*sqrt(1 - 1/(c**2*x**2)) -
b*d**2*acsc(c*x)/x + 2*b*d*e*x*acsc(c*x) + b*e**2*x**3*acsc(c*x)/3 + 2*b*d*
e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c + b*e
**2*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*
x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2
+ 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2502 vs. 2(145) = 290.
time = 2.56, size = 2502, normalized size = 15.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

```
[Out] 1/24*(b*e^2*arcsin(1/(c*x))/(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*
x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)) + a*e^2/(c/(x^3*(sqrt(-1/(c^2*x^2) + 1
) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)) + b*e^2/(c*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sq
rt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*b*d*e*arcsin(1/(c*x))/(x^2*(sqrt(-1/(c^
2*x^2) + 1) + 1)^2*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt
(-1/(c^2*x^2) + 1) + 1)^5))) + 24*a*d*e/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2
*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^5))) - 24*b*c*d^2/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^3*(sqrt(-
1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*b
*e^2*arcsin(1/(c*x))/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x^3*(sqrt(
-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*
a*e^2/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x^3*(sqrt(-1/(c^2*x^2) +
1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 48*b*d*e*log(sqrt
(-1/(c^2*x^2) + 1) + 1)/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) - 4
8*b*d*e*log(1/(abs(c)*abs(x)))/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5
))) - 48*b*d^2*arcsin(1/(c*x))/(x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*
(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))
```

```
) - 48*a*d^2/(x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*b*e^2*log(sq
rt(-1/(c^2*x^2) + 1) + 1)/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^3*(
sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)))
 - 4*b*e^2*log(1/(abs(c)*abs(x)))/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(
c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) +
 1)^5))) + b*e^2/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 48*b*d
*e*arcsin(1/(c*x))/(c^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1
/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 48*a
*d*e/(c^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1
) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*b*d^2/(c*x^5*(s
qrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c
*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 6*b*e^2*arcsin(1/(c*x))/(c^4*x^4*(
sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(
c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 6*a*e^2/(c^4*x^4*(sqrt(-1/(c^2*x^
2) + 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/
(c^2*x^2) + 1) + 1)^5))) + 48*b*d*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c^3*x^
5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) +
1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) - 48*b*d*e*log(1/(abs(c)*abs(x))
)/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) +
 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*b*e^2*log(sqrt(-1/(
c^2*x^2) + 1) + 1)/(c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1
/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) - 4*b*
e^2*log(1/(abs(c)*abs(x)))/(c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*
(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))
) - b*e^2/(c^5*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x^3*(sqrt(-1/(c^2*x^2
) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*b*d*e*arcs
in(1/(c*x))/(c^4*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x^3*(sqrt(-1/(c^2*x
^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*a*d*e/(c
^4*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^
3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*b*e^2*arcsin(1/(c*x))/(
c^6*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)
^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 4*a*e^2/(c^6*x^6*(sqrt(-
1/(c^2*x^2) + 1) + 1)^6*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*
(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) - b*e^2/(c^7*x^7*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^7*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2
) + 1) + 1)^5))) + b*e^2*arcsin(1/(c*x))/(c^8*x^8*(sqrt(-1/(c^2*x^2) + 1) +
 1)^8*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^5))) + a*e^2/(c^8*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*(c/(x^3*(sq
rt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))))*
c
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^2, x)
```

## 3.91 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^4}\,dx$

**Optimal**. Leaf size=157

$$-\frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}}-\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}+e^2x(a+b\,\mathrm{c}$$

```
[Out] -1/3*d^2*(a+b*arccsc(c*x))/x^3-2*d*e*(a+b*arccsc(c*x))/x+e^2*x*(a+b*arccsc(
c*x))+b*e^2*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/(c^2*x^2)^(1/2)-2/9*b*c*d*(c^2
*d+9*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/9*b*c*d^2*(c^2*x^2-1)^(1/2)/x^2
/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 5347, 12, 1279, 462, 223, 212}

$$-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}+e^2x(a+b\csc^{-1}(cx))-\frac{bcd^2\sqrt{c^2x^2-1}}{9x^2\sqrt{c^2x^2}}-\frac{2bcd\sqrt{c^2x^2-1}\,(c^2d+9e)}{9\sqrt{c^2x^2}}+\frac{be^2x\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] (-2*b*c*d*(c^2*d + 9*e)*Sqrt[-1 + c^2*x^2])/(9*Sqrt[c^2*x^2]) - (b*c*d^2*Sq
rt[-1 + c^2*x^2])/(9*x^2*Sqrt[c^2*x^2]) - (d^2*(a + b*ArcCsc[c*x]))/(3*x^3)
 - (2*d*e*(a + b*ArcCsc[c*x]))/x + e^2*x*(a + b*ArcCsc[c*x]) + (b*e^2*x*Arc
Tanh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[c^2*x^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{d^2(a+b\csc^{-1}(cx))}{3x^3} - \frac{2de(a+b\csc^{-1}(cx))}{x} + e^2x(a+b\csc^{-1}(cx)) +$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{3x^3} - \frac{2de(a+b\csc^{-1}(cx))}{x} + e^2x(a+b\csc^{-1}(cx)) +$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3x^3} - \frac{2de(a+b\csc^{-1}(cx))}{x} + e$$

$$= -\frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(c}{3x^3}$$

$$= -\frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(c}{3x^3}$$

$$= -\frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(c}{3x^3}$$

**Mathematica** [A]
time = 0.18, size = 125, normalized size = 0.80

$$-\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}\,x(d+2c^2dx^2+18ex^2)+3a(d^2+6dex^2-3e^2x^4)}{9x^3} - \frac{b(d^2+6dex^2-3e^2x^4)\csc^{-1}(cx)}{3x^3} + \frac{be^2\log\left(\left(1+\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] -1/9*(b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 18*e*x^2) + 3*a*(d^2
 + 6*d*e*x^2 - 3*e^2*x^4))/x^3 - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCsc[c*
x])/(3*x^3) + (b*e^2*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/c
```

**Maple** [A]
time = 0.39, size = 251, normalized size = 1.60

| method | result |
|---|---|
| derivativedivides | $c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\,\mathrm{arccsc}(cx)e^2x}{c^3}-\frac{b\,\mathrm{arccsc}(cx)d^2}{3c^3x^3}-\frac{2b\,\mathrm{arccsc}(cx)de}{c^3x}-\frac{2b(c^2x^2-1)d^2}{9c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b(}{9\sqrt{}}\right.$ |

| | |
|---|---|
| default | $c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\,\mathrm{arccsc}(cx)e^2x}{c^3}-\frac{b\,\mathrm{arccsc}(cx)d^2}{3c^3x^3}-\frac{2b\,\mathrm{arccsc}(cx)de}{c^3x}-\frac{2b(c^2x^2-1)d^2}{9c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b(c^2}{9\sqrt{\frac{c^2}{c}}}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/c^4*(e^2*c*x-1/3*c*d^2/x^3-2*c*d*e/x)+b/c^3*arccsc(c*x)*e^2*x-1/3*b*
arccsc(c*x)*d^2/c^3/x^3-2*b/c^3*arccsc(c*x)*d*e/x-2/9*b*(c^2*x^2-1)/c^2/x^2
/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2-1/9*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1
/2)/c^4/x^4*d^2-2*b/c^4*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^2*d*e+b/c
^5*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(
1/2)))
```

**Maxima [A]**
time = 0.26, size = 158, normalized size = 1.01

$$\frac{1}{9}\,bd^2\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\mathrm{arccsc}\left(cx\right)}{x^3}\right)-2\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\mathrm{arccsc}\left(cx\right)}{x}\right)bde+axe^2+\frac{\left(2\,cx\,\mathrm{arccsc}\left(cx\right)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be^2}{2\,c}-\frac{2\,ade}{x}-\frac{ad^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b*d^2*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c
- 3*arccsc(c*x)/x^3) - 2*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*d*e +
 a*x*e^2 + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-
sqrt(-1/(c^2*x^2) + 1) + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

**Fricas [A]**
time = 0.41, size = 233, normalized size = 1.48

$$-\frac{2\,bc^4d^2x^3-9\,acx^4e^2+9\,bx^3e^2\log\left(-cx+\sqrt{c^2x^2-1}\right)+3\,acd^2-3\left(bcd^2x^3-bcd^2+3\left(bcx^4-bcx^3\right)e^2+6\left(bcdx^3-bcdx^2\right)e\right)\mathrm{arccsc}\left(cx\right)-6\left(bcd^2x^3+6\,bcdx^3e-3\,bcx^3e^2\right)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+18\left(bc^2dx^3+acdx^2\right)e+\left(2\,bc^3d^2x^2+18\,bcdx^2e+bcd^2\right)\sqrt{c^2x^2-1}}{9\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(2*b*c^4*d^2*x^3 - 9*a*c*x^4*e^2 + 9*b*x^3*e^2*log(-c*x + sqrt(c^2*x^2
 - 1)) + 3*a*c*d^2 - 3*(b*c*d^2*x^3 - b*c*d^2 + 3*(b*c*x^4 - b*c*x^3)*e^2 +
 6*(b*c*d*x^3 - b*c*d*x^2)*e)*arccsc(c*x) - 6*(b*c*d^2*x^3 + 6*b*c*d*x^3*e
- 3*b*c*x^3*e^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 18*(b*c^2*d*x^3 + a*c*d
*x^2)*e + (2*b*c^3*d^2*x^2 + 18*b*c*d*x^2*e + b*c*d^2)*sqrt(c^2*x^2 - 1))/(
c*x^3)
```

**Sympy [A]**
time = 5.53, size = 211, normalized size = 1.34

$$-\frac{ad^2}{3x^3}-\frac{2ade}{x}+ae^2x-2bcde\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd^2\,\mathrm{acsc}\,(cx)}{3x^3}-\frac{2bde\,\mathrm{acsc}\,(cx)}{x}+be^2x\,\mathrm{acsc}\,(cx)-\frac{bd^2\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for}\ |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}+\frac{be^2\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for}\ |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**4,x)
```

```
[Out] -a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x - 2*b*c*d*e*sqrt(1 - 1/(c**2*x**2))
 - b*d**2*acsc(c*x)/(3*x**3) - 2*b*d*e*acsc(c*x)/x + b*e**2*x*acsc(c*x) - b
*d**2*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(
3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sq
rt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c) + b*e**2*Piecewise((acosh(c*x), A
bs(c**2*x**2) > 1), (-I*asin(c*x), True))/c
```

**Giac** [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4288 vs. $2(139) = 278$.
time = 101.45, size = 4288, normalized size = 27.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")
```

```
[Out] -1/18*(4*b*c^3*d^2/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2)
 + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-
1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 9
*b*e^2*arcsin(1/(c*x))/(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1
/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7)) - 9*a*e^2/(c/(x*(sqrt(-1/(c^2*x^
2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt
(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7)) +
36*b*c*d*e/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) +
1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x
^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 18*b*e^2*
log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*(c/(x*(sq
rt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(
c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1
) + 1)^7))) + 18*b*e^2*log(1/(abs(c)*abs(x)))/(c*x*(sqrt(-1/(c^2*x^2) + 1)
+ 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1
/(c^2*x^2) + 1) + 1)^7))) + 72*b*d*e*arcsin(1/(c*x))/(x^2*(sqrt(-1/(c^2*x^2
) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*
x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7
*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 72*a*d*e/(x^2*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) +
1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(
-1/(c^2*x^2) + 1) + 1)^7))) + 12*b*c*d^2/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^
3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) +
1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c
```

```
^2*x^2) + 1) + 1)^7))) - 36*b*e^2*arcsin(1/(c*x))/(c^2*x^2*(sqrt(-1/(c^2*x^
2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2
*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^
7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) - 36*a*e^2/(c^2*x^2*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^
2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(
sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 36*b*d*e/(c*x^3*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) +
1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(
-1/(c^2*x^2) + 1) + 1)^7))) + 48*b*d^2*arcsin(1/(c*x))/(x^4*(sqrt(-1/(c^2*x
^2) + 1) + 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^
2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x
^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 48*a*d^2/(x^4*(sqrt(-1/(c^2*x^2) + 1
) + 1)^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqr
t(-1/(c^2*x^2) + 1) + 1)^7))) - 54*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c
^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) +
 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) +
 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 54*b*e^2*log(1
/(abs(c)*abs(x)))/(c^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x*(sqrt(-1/(c
^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*
(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7
))) + 144*b*d*e*arcsin(1/(c*x))/(c^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/
(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^
2) + 1) + 1)^7))) + 144*a*d*e/(c^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*(c/(x
*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) +
 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^7))) - 12*b*d^2/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/(x*(sqr
t(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c
^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^7))) - 54*b*e^2*arcsin(1/(c*x))/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)
^4*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) +
 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(
c^2*x^2) + 1) + 1)^7))) - 54*a*e^2/(c^4*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*
(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)
^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2
*x^2) + 1) + 1)^7))) - 36*b*d*e/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*(c/
(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)
 + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^
2) + 1) + 1)^7))) - 54*b*e^2*log(sqrt(-1/(c^2*x...
```

**Mupad [F]**

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^{2}+d\right)^{2}\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^4, x)
```

# 3.92 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^6}\,dx$

**Optimal**. Leaf size=183

$$-\frac{bc(24c^4d^2+100c^2de+225e^2)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}}-\frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}}-\frac{2bcd(6c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}}-\frac{d^2(a+b$$

```
[Out] -1/5*d^2*(a+b*arccsc(c*x))/x^5-2/3*d*e*(a+b*arccsc(c*x))/x^3-e^2*(a+b*arccs
c(c*x))/x-1/225*b*c*(24*c^4*d^2+100*c^2*d*e+225*e^2)*(c^2*x^2-1)^(1/2)/(c^2
*x^2)^(1/2)-1/25*b*c*d^2*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)-2/225*b*c*d*
(6*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.12, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {276, 5347, 12, 1279, 464, 270}

$$-\frac{d^2(a+b\csc^{-1}(cx))}{5x^5}-\frac{2de(a+b\csc^{-1}(cx))}{3x^3}-\frac{e^2(a+b\csc^{-1}(cx))}{x}-\frac{bcd^2\sqrt{c^2x^2-1}}{25x^4\sqrt{c^2x^2}}-\frac{2bcd\sqrt{c^2x^2-1}(6c^2d+25e)}{225x^2\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}(24c^4d^2+100c^2de+225e^2)}{225\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(b*c*(24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*Sqrt[-1 + c^2*x^2])/Sqrt[c
^2*x^2] - (b*c*d^2*Sqrt[-1 + c^2*x^2])/(25*x^4*Sqrt[c^2*x^2]) - (2*b*c*d*(6
*c^2*d + 25*e)*Sqrt[-1 + c^2*x^2])/(225*x^2*Sqrt[c^2*x^2]) - (d^2*(a + b*Ar
cCsc[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcCsc[c*x]))/(3*x^3) - (e^2*(a + b*Arc
Csc[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{d^2(a+b\csc^{-1}(cx))}{5x^5} - \frac{2de(a+b\csc^{-1}(cx))}{3x^3} - \frac{e^2(a+b\csc^{-1}(cx))}{x} +$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{5x^5} - \frac{2de(a+b\csc^{-1}(cx))}{3x^3} - \frac{e^2(a+b\csc^{-1}(cx))}{x} +$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{5x^5} - \frac{2de(a+b\csc^{-1}(cx))}{3x^3} - \frac{e}{}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{2bcd(6c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}}{5x^5}$$

$$= -\frac{bc(225e^2+4c^2d(6c^2d+25e))\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - 2$$

**Mathematica** [A]
time = 0.16, size = 127, normalized size = 0.69

$$-\frac{15a(3d^2+10dex^2+15e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}\,x(225e^2x^4+50dex^2(1+2c^2x^2)+3d^2(3+4c^2x^2+8c^4x^4))+15b(3d^2+10dex^2+15e^2x^4)\csc^{-1}(cx)}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^2)]*
x*(225*e^2*x^4 + 50*d*e*x^2*(1 + 2*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8*c^4*
x^4)) + 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcCsc[c*x])/x^5
```

**Maple** [A]
time = 0.38, size = 191, normalized size = 1.04

| method | result |
|---|---|
| derivativedivides | $c^5\left(\frac{a\left(-\frac{2de}{3c\,x^3}-\frac{d^2}{5c\,x^5}-\frac{e^2}{cx}\right)}{c^4}+\frac{b\left(-\frac{2\,\mathrm{arccsc}(cx)de}{3c\,x^3}-\frac{\mathrm{arccsc}(cx)d^2}{5c\,x^5}-\frac{\mathrm{arccsc}(cx)e^2}{cx}-\frac{(c^2x^2-1)(24c^8d^2x^4+100c^6de\,x^4+12c^6d^2x^2}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c^4}\right.$ |
| default | $c^5\left(\frac{a\left(-\frac{2de}{3c\,x^3}-\frac{d^2}{5c\,x^5}-\frac{e^2}{cx}\right)}{c^4}+\frac{b\left(-\frac{2\,\mathrm{arccsc}(cx)de}{3c\,x^3}-\frac{\mathrm{arccsc}(cx)d^2}{5c\,x^5}-\frac{\mathrm{arccsc}(cx)e^2}{cx}-\frac{(c^2x^2-1)(24c^8d^2x^4+100c^6de\,x^4+12c^6d^2x^2}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c^4}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(a/c^4*(-2/3/c*d*e/x^3-1/5/c*d^2/x^5-e^2/c/x)+b/c^4*(-2/3*arccsc(c*x)/c
*d*e/x^3-1/5*arccsc(c*x)/c*d^2/x^5-arccsc(c*x)*e^2/c/x-1/225*(c^2*x^2-1)*(2
4*c^8*d^2*x^4+100*c^6*d*e*x^4+12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*x^2
+9*c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6))
```

**Maxima** [A]
time = 0.26, size = 181, normalized size = 0.99

$$-\frac{1}{75}bd^2\left(\frac{3c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\mathrm{arccsc}(cx)}{x^5}\right)+\frac{2}{9}bd\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\mathrm{arccsc}(cx)}{x^3}\right)e-\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\mathrm{arccsc}(cx)}{x}\right)be^2-\frac{ae^2}{x}-\frac{2ade}{3x^3}-\frac{ad^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/75*b*d^2*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3
/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) + 2/9*b*d*((c^
4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*x
)/x^3)*e - (c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*e^2 - a*e^2/x - 2/3
*a*d*e/x^3 - 1/5*a*d^2/x^5
```

**Fricas [A]**
time = 0.39, size = 131, normalized size = 0.72

$$-\frac{225\,ax^4e^2+150\,adx^2e+45\,ad^2+15\,(15\,bx^4e^2+10\,bdx^2e+3\,bd^2)\operatorname{arccsc}(cx)+(24\,bc^4d^2x^4+12\,bc^2d^2x^2+225\,bx^4e^2+9\,bd^2+50\,(2\,bc^2dx^4+bdx^2)e)\sqrt{c^2x^2-1}}{225\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(225*a*x^4*e^2 + 150*a*d*x^2*e + 45*a*d^2 + 15*(15*b*x^4*e^2 + 10*b*
d*x^2*e + 3*b*d^2)*arccsc(c*x) + (24*b*c^4*d^2*x^4 + 12*b*c^2*d^2*x^2 + 225
*b*x^4*e^2 + 9*b*d^2 + 50*(2*b*c^2*d*x^4 + b*d*x^2)*e)*sqrt(c^2*x^2 - 1))/x
^5
```

**Sympy [A]**
time = 6.90, size = 335, normalized size = 1.83

$$-\frac{ad^2}{5x^5}-\frac{2ade}{3x^3}-\frac{ae^2}{x}-bce^2\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd^2\operatorname{acsc}(cx)}{5x^5}-\frac{2bde\operatorname{acsc}(cx)}{3x^3}-\frac{be^2\operatorname{acsc}(cx)}{x}-\frac{bd^2\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}-\frac{2bde\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**6,x)
```

```
[Out] -a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x - b*c*e**2*sqrt(1 - 1/(c**2*
x**2)) - b*d**2*acsc(c*x)/(5*x**5) - 2*b*d*e*acsc(c*x)/(3*x**3) - b*e**2*ac
sc(c*x)/x - b*d**2*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sq
rt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2
) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 +
1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) - 2*b*d*e*Pi
ecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3),
 Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2
*x**2 + 1)/(3*x**3), True))/(3*c)
```

**Giac [A]**
time = 0.44, size = 314, normalized size = 1.72

$$-\frac{1}{225}\left(9\,bc^4d^2\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}-30\,bc^4d^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{45\,bc^3d^2\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x}+45\,bc^4d^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{90\,bc^3d^2\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}-50\,bc^2de\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{45\,bc^3d^2\arcsin\left(\frac{1}{cx}\right)}{x}+150\,bc^2de\sqrt{-\frac{1}{c^2x^2}+1}+\frac{150\,bcde\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{150\,bcde\arcsin\left(\frac{1}{cx}\right)}{x}+225\,be^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{225\,be^2\arcsin\left(\frac{1}{cx}\right)}{cx}+\frac{225\,ae^2}{cx}+\frac{150\,ade}{cx^3}+\frac{45\,ad^2}{cx^5}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/225*(9*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) - 30*b*c^4*d
^2*(-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d^2*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*
x))/x + 45*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1) + 90*b*c^3*d^2*(1/(c^2*x^2) - 1
)*arcsin(1/(c*x))/x - 50*b*c^2*d*e*(-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d^2*
arcsin(1/(c*x))/x + 150*b*c^2*d*e*sqrt(-1/(c^2*x^2) + 1) + 150*b*c*d*e*(1/(
c^2*x^2) - 1)*arcsin(1/(c*x))/x + 150*b*c*d*e*arcsin(1/(c*x))/x + 225*b*e^2
*sqrt(-1/(c^2*x^2) + 1) + 225*b*e^2*arcsin(1/(c*x))/(c*x) + 225*a*e^2/(c*x)
 + 150*a*d*e/(c*x^3) + 45*a*d^2/(c*x^5))*c
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^6, x)
```

# 3.93 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^8}\,dx$

**Optimal**. Leaf size=241

$$-\frac{2bc^3(360c^4d^2+1176c^2de+1225e^2)\sqrt{-1+c^2x^2}}{11025\sqrt{c^2x^2}}-\frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}}-$$

```
[Out] -1/7*d^2*(a+b*arccsc(c*x))/x^7-2/5*d*e*(a+b*arccsc(c*x))/x^5-1/3*e^2*(a+b*a
rccsc(c*x))/x^3-2/11025*b*c^3*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-
1)^(1/2)/(c^2*x^2)^(1/2)-1/49*b*c*d^2*(c^2*x^2-1)^(1/2)/x^6/(c^2*x^2)^(1/2)
-2/1225*b*c*d*(15*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)-1/11025
*b*c*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1
/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 5347, 12, 1279, 464, 277, 270}

$$-\frac{d^2(a+b\csc^{-1}(cx))}{7x^7}-\frac{2de(a+b\csc^{-1}(cx))}{5x^5}-\frac{e^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{bcd^2\sqrt{c^2x^2-1}}{49x^6\sqrt{c^2x^2}}-\frac{2bcd\sqrt{c^2x^2-1}(15c^2d+49e)}{1225x^4\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}(360c^4d^2+1176c^2de+1225e^2)}{11025x^2\sqrt{c^2x^2}}-\frac{2bc^3\sqrt{c^2x^2-1}(360c^4d^2+1176c^2de+1225e^2)}{11025\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] (-2*b*c^3*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 + c^2*x^2])/(1102
5*Sqrt[c^2*x^2]) - (b*c*d^2*Sqrt[-1 + c^2*x^2])/(49*x^6*Sqrt[c^2*x^2]) - (2
*b*c*d*(15*c^2*d + 49*e)*Sqrt[-1 + c^2*x^2])/(1225*x^4*Sqrt[c^2*x^2]) - (b*
c*(360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 + c^2*x^2])/(11025*x^2*Sq
rt[c^2*x^2]) - (d^2*(a + b*ArcCsc[c*x]))/(7*x^7) - (2*d*e*(a + b*ArcCsc[c*x
]))/(5*x^5) - (e^2*(a + b*ArcCsc[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^8} dx = -\frac{d^2(a+b\csc^{-1}(cx))}{7x^7} - \frac{2de(a+b\csc^{-1}(cx))}{5x^5} - \frac{e^2(a+b\csc^{-1}(cx))}{3x^3} +$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{7x^7} - \frac{2de(a+b\csc^{-1}(cx))}{5x^5} - \frac{e^2(a+b\csc^{-1}(cx))}{3x^3} +$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{7x^7} - \frac{2de(a+b\csc^{-1}(cx))}{5x^5} - \frac{e}{}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{d^2(a+b\csc}{7x^7}$$

$$= -\frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{bc(1225e^2+}{}$$

$$= -\frac{2bc^3(1225e^2+24c^2d(15c^2d+49e))\sqrt{-1+c^2x^2}}{11025\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}$$

## Mathematica [A]

time = 0.16, size = 153, normalized size = 0.63

$$-\frac{105a(15d^2+42dex^2+35e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}\,x(1225e^2x^4(1+2c^2x^2)+294dex^2(3+4c^2x^2+8c^4x^4)+45d^2(5+6c^2x^2+8c^4x^4+16c^6x^6))+105b(15d^2+42dex^2+35e^2x^4)\csc^{-1}(cx)}{11025x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] -1/11025*(105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^
2)]*x*(1225*e^2*x^4*(1 + 2*c^2*x^2) + 294*d*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*x^
4) + 45*d^2*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) + 105*b*(15*d^2 + 42*
d*e*x^2 + 35*e^2*x^4)*ArcCsc[c*x])/x^7
```

## Maple [A]

time = 0.38, size = 223, normalized size = 0.93

| method | result |
|---|---|
| derivativedivides | $c^7 \left( \frac{a\left(-\frac{d^2}{7c^3x^7} - \frac{e^2}{3c^3x^3} - \frac{2de}{5c^3x^5}\right)}{c^4} + \frac{b\left(-\frac{\operatorname{arccsc}(cx)d^2}{7c^3x^7} - \frac{\operatorname{arccsc}(cx)e^2}{3c^3x^3} - \frac{2\operatorname{arccsc}(cx)de}{5c^3x^5} - \frac{(c^2x^2-1)(720c^{10}d^2x^6+2352c^8de\,x^6}{}\right.}{} \right.$ |

| | |
|---|---|
| default | $c^7\left(\frac{a\left(-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}-\frac{2de}{5c^3x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d^2}{7c^3x^7}-\frac{\operatorname{arccsc}(cx)e^2}{3c^3x^3}-\frac{2\operatorname{arccsc}(cx)de}{5c^3x^5}-\frac{(c^2x^2-1)(720c^{10}d^2x^6+2352c^8de\,x^6+\right.}{c^4}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] c^7*(a/c^4*(-1/7/c^3*d^2/x^7-1/3*e^2/c^3/x^3-2/5/c^3*d*e/x^5)+b/c^4*(-1/7*a
rccsc(c*x)/c^3*d^2/x^7-1/3*arccsc(c*x)*e^2/c^3/x^3-2/5*arccsc(c*x)/c^3*d*e/
x^5-1/11025*(c^2*x^2-1)*(720*c^10*d^2*x^6+2352*c^8*d*e*x^6+360*c^8*d^2*x^4+
2450*c^6*e^2*x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2+1225*c^4*e^2*x^4+882*c^4*
d*e*x^2+225*c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^8/x^8))
```

**Maxima [A]**
time = 0.28, size = 241, normalized size = 1.00

$$\frac{1}{245}\,bd^2\left(\frac{5\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{35\,\operatorname{arccsc}(cx)}{x^7}\right)-\frac{2}{75}\,bd\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\operatorname{arccsc}(cx)}{x^5}\right)e+\frac{1}{9}\,b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\operatorname{arccsc}(cx)}{x^3}\right)e^2-\frac{ae^2}{3\,x^3}-\frac{2\,ade}{5\,x^5}-\frac{ad^2}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d^2*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^(5
/2) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c -
35*arccsc(c*x)/x^7) - 2/75*b*d*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-
1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/
x^5)*e + 1/9*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1
))/c - 3*arccsc(c*x)/x^3)*e^2 - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x
^7
```

**Fricas [A]**
time = 0.35, size = 165, normalized size = 0.68

$$-\frac{3675\,ax^4e^2+4410\,adx^2e+1575\,ad^2+105\,(35\,bx^4e^2+42\,bdx^2e+15\,bd^2)\operatorname{arccsc}(cx)+(720\,bc^6d^2x^6+360\,bc^4d^2x^4+270\,bc^2d^2x^2+225\,bd^2+1225\,(2\,bc^2x^6+bx^4)e^2+294\,(8\,bc^4dx^6+4\,bc^2dx^4+3\,bdx^2)e)\sqrt{c^2x^2-1}}{11025\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/11025*(3675*a*x^4*e^2 + 4410*a*d*x^2*e + 1575*a*d^2 + 105*(35*b*x^4*e^2
+ 42*b*d*x^2*e + 15*b*d^2)*arccsc(c*x) + (720*b*c^6*d^2*x^6 + 360*b*c^4*d^2
*x^4 + 270*b*c^2*d^2*x^2 + 225*b*d^2 + 1225*(2*b*c^2*x^6 + b*x^4)*e^2 + 294
*(8*b*c^4*d*x^6 + 4*b*c^2*d*x^4 + 3*b*d*x^2)*e)*sqrt(c^2*x^2 - 1))/x^7
```

**Sympy** [A]
time = 35.89, size = 510, normalized size = 2.12

$$-\frac{ad^2}{7x^7}-\frac{2ade}{5x^5}-\frac{ae^2}{3x^3}-\frac{bd^2\operatorname{acsc}(cx)}{7x^7}-\frac{2bde\operatorname{acsc}(cx)}{5x^5}-\frac{be^2\operatorname{acsc}(cx)}{3x^3}-\frac{bd^2\left(\begin{cases}\frac{16c^7\sqrt{c^2x^2-1}}{35x}+\frac{8c^5\sqrt{c^2x^2-1}}{35x^3}+\frac{6c^3\sqrt{c^2x^2-1}}{35x^5}+\frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for } |c^2x^2|>1\\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x}+\frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3}+\frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5}+\frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise}\end{cases}\right)}{7c}-\frac{2bde\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}-\frac{be^2\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**8,x)
```

```
[Out] -a*d**2/(7*x**7) - 2*a*d*e/(5*x**5) - a*e**2/(3*x**3) - b*d**2*acsc(c*x)/(7
*x**7) - 2*b*d*e*acsc(c*x)/(5*x**5) - b*e**2*acsc(c*x)/(3*x**3) - b*d**2*Pi
ecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**2*x**2 - 1)/(3
5*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2*x**2 - 1)/(7*x
**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/(35*x) + 8*I*c**
5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**2 + 1)/(35*x**5)
+ I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) - 2*b*d*e*Piecewise((8*c*
*5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sq
rt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2
+ 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2
 + 1)/(5*x**5), True))/(5*c) - b*e**2*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)
/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqr
t(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. 2(211) = 422.
time = 0.48, size = 491, normalized size = 2.04

$$-\frac{1}{11025}\left(225\,bc^6d^2\left(\frac{1}{c^2x^2}-1\right)^3\sqrt{-\frac{1}{c^2x^2}+1}+945\,bc^6d^2\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{1575\,bc^5d^2\left(\frac{1}{c^2x^2}-1\right)^3\arcsin\left(\frac{1}{cx}\right)}{x}-1575\,bc^6d^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{4725\,bc^5d^2\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x}+1575\,bc^6d^2\sqrt{-\frac{1}{c^2x^2}+1}+882\,bc^4de\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{4725\,bc^5d^2\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}-2940\,bc^4de\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{1575\,bc^5d^2\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{4410\,bc^3de\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x}+4410\,bc^4de\sqrt{-\frac{1}{c^2x^2}+1}+\frac{8820\,bc^3de\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}-1225\,bc^2e^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{4410\,bc^3de\arcsin\left(\frac{1}{cx}\right)}{x}+3675\,bc^2e^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{3675\,bce^2\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{3675\,bce^2\arcsin\left(\frac{1}{cx}\right)}{x}+\frac{3675\,ae^2}{cx^3}+\frac{4410\,ade}{cx^5}+\frac{1575\,ad^2}{cx^7}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

```
[Out] -1/11025*(225*b*c^6*d^2*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1) + 945*b*
c^6*d^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) + 1575*b*c^5*d^2*(1/(c^2
*x^2) - 1)^3*arcsin(1/(c*x))/x - 1575*b*c^6*d^2*(-1/(c^2*x^2) + 1)^(3/2) +
4725*b*c^5*d^2*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 1575*b*c^6*d^2*sqrt(
-1/(c^2*x^2) + 1) + 882*b*c^4*d*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1
) + 4725*b*c^5*d^2*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 2940*b*c^4*d*e*(-1
/(c^2*x^2) + 1)^(3/2) + 1575*b*c^5*d^2*arcsin(1/(c*x))/x + 4410*b*c^3*d*e*(
1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 4410*b*c^4*d*e*sqrt(-1/(c^2*x^2) + 1
) + 8820*b*c^3*d*e*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 1225*b*c^2*e^2*(-1
/(c^2*x^2) + 1)^(3/2) + 4410*b*c^3*d*e*arcsin(1/(c*x))/x + 3675*b*c^2*e^2*s
qrt(-1/(c^2*x^2) + 1) + 3675*b*c*e^2*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x +
3675*b*c*e^2*arcsin(1/(c*x))/x + 3675*a*e^2/(c*x^3) + 4410*a*d*e/(c*x^5) +
1575*a*d^2/(c*x^7))*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^8} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^8,x)
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^8, x)
```

# 3.94 $\int x^3(d+ex^2)^2 (a+b\csc^{-1}(cx))\ dx$

**Optimal**. Leaf size=242

$$\frac{b(6c^4d^2+8c^2de+3e^2)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}+\frac{b(6c^4d^2+16c^2de+9e^2)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}}+\frac{be(8c^2d+9e)\,x(-1+c^2x^2)}{120c^7\sqrt{c^2x^2}}$$

```
[Out] 1/4*d^2*x^4*(a+b*arccsc(c*x))+1/3*d*e*x^6*(a+b*arccsc(c*x))+1/8*e^2*x^8*(a+
b*arccsc(c*x))+1/72*b*(6*c^4*d^2+16*c^2*d*e+9*e^2)*x*(c^2*x^2-1)^(3/2)/c^7/
(c^2*x^2)^(1/2)+1/120*b*e*(8*c^2*d+9*e)*x*(c^2*x^2-1)^(5/2)/c^7/(c^2*x^2)^(
1/2)+1/56*b*e^2*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^(1/2)+1/24*b*(6*c^4*d^2+8
*c^2*d*e+3*e^2)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.16, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {272, 45, 5347, 12, 1265, 785}

$$\frac{1}{4}d^2x^4(a+b\csc^{-1}(cx))+\frac{1}{3}dex^6(a+b\csc^{-1}(cx))+\frac{1}{8}e^2x^8(a+b\csc^{-1}(cx))+\frac{bex(c^2x^2-1)^{5/2}(8c^2d+9e)}{120c^7\sqrt{c^2x^2}}+\frac{be^2x(c^2x^2-1)^{7/2}}{56c^7\sqrt{c^2x^2}}+\frac{bx(c^2x^2-1)^{3/2}(6c^4d^2+16c^2de+9e^2)}{72c^7\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}(6c^4d^2+8c^2de+3e^2)}{24c^7\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*x*Sqrt[-1 + c^2*x^2])/(24*c^7*Sqrt[c^2*x
^2]) + (b*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*x*(-1 + c^2*x^2)^(3/2))/(72*c^7*
Sqrt[c^2*x^2]) + (b*e*(8*c^2*d + 9*e)*x*(-1 + c^2*x^2)^(5/2))/(120*c^7*Sqrt
[c^2*x^2]) + (b*e^2*x*(-1 + c^2*x^2)^(7/2))/(56*c^7*Sqrt[c^2*x^2]) + (d^2*x
^4*(a + b*ArcCsc[c*x]))/4 + (d*e*x^6*(a + b*ArcCsc[c*x]))/3 + (e^2*x^8*(a +
 b*ArcCsc[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right) \, dx = \frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc\right.$$

$$= \frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc\right.$$

$$= \frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc\right.$$

$$= \frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc\right.$$

$$= \frac{b(6c^4d^2+8c^2de+3e^2)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}+\frac{b(6c^4d^2+16c^2de+9e^2)\,x(-}{72c^7\sqrt{c^2x^2}}$$

**Mathematica [A]**
time = 0.20, size = 159, normalized size = 0.66

$$\frac{x\left(105ax^3(6d^2+8dex^2+3e^2x^4)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,(144e^2+8c^2e(56d+9ex^2)+c^4(420d^2+224dex^2+54e^2x^4)+3c^6(70d^2x^2+56dex^4+15e^2x^6))}{c^7}+105bx^3(6d^2+8dex^2+3e^2x^4)\csc^{-1}(cx)\right)}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(105*a*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + (b*Sqrt[1 - 1/(c^2*x^2)]*(1
44*e^2 + 8*c^2*e*(56*d + 9*e*x^2) + c^4*(420*d^2 + 224*d*e*x^2 + 54*e^2*x^4
) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/c^7 + 105*b*x^3*(6*d^2 +
 8*d*e*x^2 + 3*e^2*x^4)*ArcCsc[c*x]))/2520
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(212) = 424$.
time = 0.61, size = 499, normalized size = 2.06

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{1}{4}c^8d^2x^4+\frac{1}{3}c^8dex^6+\frac{1}{8}e^2c^8x^8\right)}{c^4}-\frac{bc^4\operatorname{arccsc}(cx)d^4}{24e^2}+\frac{b\operatorname{arccsc}(cx)d^2c^4x^4}{4}+\frac{bc^4e\operatorname{arccsc}(cx)dx^6}{3}+\frac{bc^4e^2\operatorname{arccsc}(cx)x^8}{8}+\frac{bc^3\sqrt{c^2}\cdots}{\cdots}}{\cdots}$ |
| default | $\frac{\frac{a\left(\frac{1}{4}c^8d^2x^4+\frac{1}{3}c^8dex^6+\frac{1}{8}e^2c^8x^8\right)}{c^4}-\frac{bc^4\operatorname{arccsc}(cx)d^4}{24e^2}+\frac{b\operatorname{arccsc}(cx)d^2c^4x^4}{4}+\frac{bc^4e\operatorname{arccsc}(cx)dx^6}{3}+\frac{bc^4e^2\operatorname{arccsc}(cx)x^8}{8}+\frac{bc^3\sqrt{c^2}\cdots}{\cdots}}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a/c^4*(1/4*c^8*d^2*x^4+1/3*c^8*d*e*x^6+1/8*e^2*c^8*x^8)-1/24*b*c^4/e
^2*arccsc(c*x)*d^4+1/4*b*arccsc(c*x)*d^2*c^4*x^4+1/3*b*c^4*e*arccsc(c*x)*d*
x^6+1/8*b*c^4*e^2*arccsc(c*x)*x^8+1/24*b*c^3/e^2*(c^2*x^2-1)^(1/2)/((c^2*x^
2-1)/c^2/x^2)^(1/2)/x*d^4*arctan(1/(c^2*x^2-1)^(1/2))+1/12*b*(c^2*x^2-1)/((
c^2*x^2-1)/c^2/x^2)^(1/2)*c*x*d^2+1/15*b*c*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x
^2)^(1/2)*x^3*d+1/56*b*c*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^5+1/
6*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*d^2+4/45*b/c*e*(c^2*x^2-1)/
((c^2*x^2-1)/c^2/x^2)^(1/2)*x*d+3/140*b/c*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/
x^2)^(1/2)*x^3+8/45*b/c^3*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d+1/3
5*b/c^3*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x+2/35*b/c^5*e^2*(c^2*x
^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

**Maxima** [A]
time = 0.26, size = 253, normalized size = 1.05

$$\frac{1}{8}\,ax^8e^2 + \frac{1}{3}\,adx^6e + \frac{1}{4}\,ad^2x^4 + \frac{1}{12}\left(3\,x^4\,\mathrm{arccsc}\left(cx\right) + \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd^2 + \frac{1}{45}\left(15\,x^6\,\mathrm{arccsc}\left(cx\right) + \frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)bde + \frac{1}{280}\left(35\,x^8\,\mathrm{arccsc}\left(cx\right) + \frac{5\,c^6x^7\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}} + 21\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 35\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 35\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^7}\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8*e^2 + 1/3*a*d*x^6*e + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arccsc(c*x) + (
c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d^2 +
 1/45*(15*x^6*arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^
3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*d*e + 1/28
0*(35*x^8*arccsc(c*x) + (5*c^6*x^7*(-1/(c^2*x^2) + 1)^(7/2) + 21*c^4*x^5*(-
1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 35*x*sqrt(-1
/(c^2*x^2) + 1))/c^7)*b*e^2
```

**Fricas** [A]
time = 0.39, size = 184, normalized size = 0.76

$$\frac{315\,ac^8x^8e^2 + 840\,ac^8dx^6e + 630\,ac^8d^2x^4 + 105\left(3\,bc^8x^8e^2 + 8\,bc^8dx^6e + 6\,bc^8d^2x^4\right)\mathrm{arccsc}\left(cx\right) + \left(210\,bc^6d^2x^2 + 420\,bc^4d^2 + 9\left(5\,bc^6x^6 + 6\,bc^4x^4 + 8\,bc^2x^2 + 16\,b\right)e^2 + 56\left(3\,bc^6dx^4 + 4\,bc^4dx^2 + 8\,bc^2d\right)e\right)\sqrt{c^2x^2-1}}{2520\,c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^8*x^8*e^2 + 840*a*c^8*d*x^6*e + 630*a*c^8*d^2*x^4 + 105*(3*
b*c^8*x^8*e^2 + 8*b*c^8*d*x^6*e + 6*b*c^8*d^2*x^4)*arccsc(c*x) + (210*b*c^6
*d^2*x^2 + 420*b*c^4*d^2 + 9*(5*b*c^6*x^6 + 6*b*c^4*x^4 + 8*b*c^2*x^2 + 16*
b)*e^2 + 56*(3*b*c^6*d*x^4 + 4*b*c^4*d*x^2 + 8*b*c^2*d)*e)*sqrt(c^2*x^2 - 1
))/c^8
```

**Sympy** [A]
time = 6.47, size = 493, normalized size = 2.04

$$\frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4\,\mathrm{acsc}\left(cx\right)}{4} + \frac{bdex^6\,\mathrm{acsc}\left(cx\right)}{3} + \frac{be^2x^8\,\mathrm{acsc}\left(cx\right)}{8} + \frac{bd^2\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c} + \frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for}\ \left|c^2x^2\right| > 1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c} + \frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c} + \frac{bde\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c} + \frac{4x^2\sqrt{c^2x^2-1}}{15c^3} + \frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for}\ \left|c^2x^2\right| > 1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c} + \frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3} + \frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{3c} + \frac{be^2\left(\begin{cases}\frac{x^6\sqrt{c^2x^2-1}}{7c} + \frac{6x^4\sqrt{c^2x^2-1}}{35c^3} + \frac{8x^2\sqrt{c^2x^2-1}}{35c^5} + \frac{16\sqrt{c^2x^2-1}}{35c^7} & \text{for}\ \left|c^2x^2\right| > 1\\ \frac{ix^6\sqrt{-c^2x^2+1}}{7c} + \frac{6ix^4\sqrt{-c^2x^2+1}}{35c^3} + \frac{8ix^2\sqrt{-c^2x^2+1}}{35c^5} + \frac{16i\sqrt{-c^2x^2+1}}{35c^7} & \text{otherwise}\end{cases}\right)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*acsc(c*x)/4 + b*
d*e*x**6*acsc(c*x)/3 + b*e**2*x**8*acsc(c*x)/8 + b*d**2*Piecewise((x**2*sqr
t(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1
), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3),
True))/(4*c) + b*d*e*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**2*sqr
t(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2
) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/
```

```
(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(3*c) + b*e**2*Piece
wise((x**6*sqrt(c**2*x**2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3)
 + 8*x**2*sqrt(c**2*x**2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7),
 Abs(c**2*x**2) > 1), (I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c
**2*x**2 + 1)/(35*c**3) + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sq
rt(-c**2*x**2 + 1)/(35*c**7), True))/(8*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1706 vs. $2(212) = 424$.
time = 0.58, size = 1706, normalized size = 7.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/645120*(315*b*e^2*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*arcsin(1/(c*x))/c +
315*a*e^2*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8/c + 90*b*e^2*x^7*(sqrt(-1/(c^2
*x^2) + 1) + 1)^7/c^2 + 3360*b*d*e*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsi
n(1/(c*x))/c + 3360*a*d*e*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 2520*b*e^2
*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c^3 + 2520*a*e^2*x^6*(s
qrt(-1/(c^2*x^2) + 1) + 1)^6/c^3 + 1344*b*d*e*x^5*(sqrt(-1/(c^2*x^2) + 1) +
 1)^5/c^2 + 10080*b*d^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/
c + 10080*a*d^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 882*b*e^2*x^5*(sqrt(
-1/(c^2*x^2) + 1) + 1)^5/c^4 + 20160*b*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)
^4*arcsin(1/(c*x))/c^3 + 20160*a*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^3
 + 6720*b*d^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 8820*b*e^2*x^4*(sqrt
(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^5 + 8820*a*e^2*x^4*(sqrt(-1/(c^
2*x^2) + 1) + 1)^4/c^5 + 11200*b*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^4
 + 40320*b*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 403
20*a*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 4410*b*e^2*x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3/c^6 + 50400*b*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*a
rcsin(1/(c*x))/c^5 + 50400*a*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 6
0480*b*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 17640*b*e^2*x^2*(sqrt(-1/(c
^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^7 + 17640*a*e^2*x^2*(sqrt(-1/(c^2*x^2
) + 1) + 1)^2/c^7 + 67200*b*d*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 + 60480*
b*d^2*arcsin(1/(c*x))/c^5 + 60480*a*d^2/c^5 + 22050*b*e^2*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)/c^8 + 67200*b*d*e*arcsin(1/(c*x))/c^7 + 67200*a*d*e/c^7 - 604
80*b*d^2/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 22050*b*e^2*arcsin(1/(c*x))
/c^9 + 22050*a*e^2/c^9 - 67200*b*d*e/(c^8*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) +
 40320*b*d^2*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 403
20*a*d^2/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 22050*b*e^2/(c^10*x*(sq
rt(-1/(c^2*x^2) + 1) + 1)) + 50400*b*d*e*arcsin(1/(c*x))/(c^9*x^2*(sqrt(-1/
(c^2*x^2) + 1) + 1)^2) + 50400*a*d*e/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^
2) - 6720*b*d^2/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 17640*b*e^2*arcs
```

```
in(1/(c*x))/(c^11*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 17640*a*e^2/(c^11*x
^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 11200*b*d*e/(c^10*x^3*(sqrt(-1/(c^2*x^
2) + 1) + 1)^3) + 10080*b*d^2*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) +
 1) + 1)^4) + 10080*a*d^2/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 4410*b
*e^2/(c^12*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 20160*b*d*e*arcsin(1/(c*x)
)/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 20160*a*d*e/(c^11*x^4*(sqrt(-
1/(c^2*x^2) + 1) + 1)^4) + 8820*b*e^2*arcsin(1/(c*x))/(c^13*x^4*(sqrt(-1/(c
^2*x^2) + 1) + 1)^4) + 8820*a*e^2/(c^13*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4)
 - 1344*b*d*e/(c^12*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - 882*b*e^2/(c^14*x
^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 3360*b*d*e*arcsin(1/(c*x))/(c^13*x^6*(
sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 3360*a*d*e/(c^13*x^6*(sqrt(-1/(c^2*x^2) +
1) + 1)^6) + 2520*b*e^2*arcsin(1/(c*x))/(c^15*x^6*(sqrt(-1/(c^2*x^2) + 1) +
 1)^6) + 2520*a*e^2/(c^15*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) - 90*b*e^2/(c
^16*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7) + 315*b*e^2*arcsin(1/(c*x))/(c^17*x
^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8) + 315*a*e^2/(c^17*x^8*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^8))*c
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

# 3.95 $\int x(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=195

$$\frac{b(3c^4d^2+3c^2de+e^2)\,x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}}+\frac{be(3c^2d+2e)\,x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}}+\frac{be^2x(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}}+\frac{(d+ex^2)^3\,(a+}{6e}$$

```
[Out] 1/6*(e*x^2+d)^3*(a+b*arccsc(c*x))/e+1/18*b*e*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(3
/2)/c^5/(c^2*x^2)^(1/2)+1/30*b*e^2*x*(c^2*x^2-1)^(5/2)/c^5/(c^2*x^2)^(1/2)+
1/6*b*c*d^3*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/6*b*(3*c^4*d^2+
3*c^2*d*e+e^2)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5345, 457, 90, 65, 211}

$$\frac{(d+ex^2)^3\,(a+b\csc^{-1}(cx))}{6e}+\frac{bcd^3x\text{ArcTan}\left(\sqrt{c^2x^2-1}\right)}{6e\sqrt{c^2x^2}}+\frac{bex(c^2x^2-1)^{3/2}\,(3c^2d+2e)}{18c^5\sqrt{c^2x^2}}+\frac{be^2x(c^2x^2-1)^{5/2}}{30c^5\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,(3c^4d^2+3c^2de+e^2)}{6c^5\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(3*c^4*d^2 + 3*c^2*d*e + e^2)*x*Sqrt[-1 + c^2*x^2])/(6*c^5*Sqrt[c^2*x^2]
) + (b*e*(3*c^2*d + 2*e)*x*(-1 + c^2*x^2)^(3/2))/(18*c^5*Sqrt[c^2*x^2]) + (
b*e^2*x*(-1 + c^2*x^2)^(5/2))/(30*c^5*Sqrt[c^2*x^2]) + ((d + e*x^2)^3*(a +
b*ArcCsc[c*x]))/(6*e) + (b*c*d^3*x*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*e*Sqrt[c^
2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)}{6e}+\frac{(bcx)\int\frac{\left(d+ex^2\right)^3}{x\sqrt{-1+c^2x^2}}\,dx}{6e\sqrt{c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)}{6e}+\frac{(bcx)\text{Subst}\left(\int\frac{(d+ex)^3}{x\sqrt{-1+c^2x}}\,dx,x,x^2\right)}{12e\sqrt{c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)}{6e}+\frac{(bcx)\text{Subst}\left(\int\left(\frac{e\left(3c^4d^2+3c^2de+e^2\right)}{c^4\sqrt{-1+c^2x}}+\frac{}{x\sqrt{}}\right.\right.}{}$$

$$=\frac{b\left(3c^4d^2+3c^2de+e^2\right)x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}}+\frac{be\left(3c^2d+2e\right)x\left(-1+c^2x^2\right)^{3/2}}{18c^5\sqrt{c^2x^2}}$$

$$=\frac{b\left(3c^4d^2+3c^2de+e^2\right)x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}}+\frac{be\left(3c^2d+2e\right)x\left(-1+c^2x^2\right)^{3/2}}{18c^5\sqrt{c^2x^2}}$$

$$=\frac{b\left(3c^4d^2+3c^2de+e^2\right)x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}}+\frac{be\left(3c^2d+2e\right)x\left(-1+c^2x^2\right)^{3/2}}{18c^5\sqrt{c^2x^2}}$$

**Mathematica [A]**

time = 0.18, size = 124, normalized size = 0.64

$$\frac{1}{90}x\left(15ax\left(3d^2+3dex^2+e^2x^4\right)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\left(8e^2+2c^2e(15d+2ex^2)+3c^4(15d^2+5dex^2+e^2x^4)\right)}{c^5}+15bx\left(3d^2+3dex^2+e^2x^4\right)\csc^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (x*(15*a*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + (b*Sqrt[1 - 1/(c^2*x^2)]*(8*e^2
+ 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)))/c^5 + 1
5*b*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCsc[c*x]))/90
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(169) = 338$.
time = 0.63, size = 376, normalized size = 1.93

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\left(c^2ex^2+c^2d\right)^3a}{6c^4e}+\frac{bc^2\operatorname{arccsc}(cx)d^3}{6e}+\frac{b\operatorname{arccsc}(cx)d^2c^2x^2}{2}+\frac{bc^2e\operatorname{arccsc}(cx)dx^4}{2}+\frac{bc^2e^2\operatorname{arccsc}(cx)x^6}{6}-\frac{bc\sqrt{c^2x^2-1}\,d^3\operatorname{arctan}}{6e\sqrt{\frac{c^2x^2}{c^2x}}}}{}$ |
| default | $\frac{\frac{\left(c^2ex^2+c^2d\right)^3a}{6c^4e}+\frac{bc^2\operatorname{arccsc}(cx)d^3}{6e}+\frac{b\operatorname{arccsc}(cx)d^2c^2x^2}{2}+\frac{bc^2e\operatorname{arccsc}(cx)dx^4}{2}+\frac{bc^2e^2\operatorname{arccsc}(cx)x^6}{6}-\frac{bc\sqrt{c^2x^2-1}\,d^3\operatorname{arctan}}{6e\sqrt{\frac{c^2x^2}{c^2x}}}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/6*(c^2*e*x^2+c^2*d)^3*a/c^4/e+1/6*b*c^2/e*arccsc(c*x)*d^3+1/2*b*ar
ccsc(c*x)*d^2*c^2*x^2+1/2*b*c^2*e*arccsc(c*x)*d*x^4+1/6*b*c^2*e^2*arccsc(c*
x)*x^6-1/6*b*c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*arctan
(1/(c^2*x^2-1)^(1/2))+1/2*b*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x*d^2
+1/6*b/c*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x*d+1/30*b/c*e^2*(c^2*x^
2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^3+1/3*b/c^3*e*(c^2*x^2-1)/((c^2*x^2-1)/c
^2/x^2)^(1/2)/x*d+2/45*b/c^3*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x+
4/45*b/c^5*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

**Maxima** [A]
time = 0.26, size = 189, normalized size = 0.97

$$\frac{1}{6}ax^6e^2+\frac{1}{2}adx^4e+\frac{1}{2}ad^2x^2+\frac{1}{2}\left(x^2\operatorname{arccsc}(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2+\frac{1}{6}\left(3x^4\operatorname{arccsc}(cx)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bde+\frac{1}{90}\left(15x^6\operatorname{arccsc}(cx)+\frac{3c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6*e^2 + 1/2*a*d*x^4*e + 1/2*a*d^2*x^2 + 1/2*(x^2*arccsc(c*x) + x*sq
rt(-1/(c^2*x^2) + 1)/c)*b*d^2 + 1/6*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*
x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d*e + 1/90*(15*x^6*arc
csc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) +
 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e^2
```

**Fricas** [A]
time = 0.37, size = 150, normalized size = 0.77

$$\frac{15\,ac^6x^6e^2+45\,ac^6dx^4e+45\,ac^6d^2x^2+15\,(bc^6x^6e^2+3\,bc^6dx^4e+3\,bc^6d^2x^2)\,\mathrm{arccsc}\,(cx)+(45\,bc^4d^2+(3\,bc^4x^4+4\,bc^2x^2+8\,b)e^2+15\,(bc^4dx^2+2\,bc^2d)e)\sqrt{c^2x^2-1}}{90\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^6*x^6*e^2 + 45*a*c^6*d*x^4*e + 45*a*c^6*d^2*x^2 + 15*(b*c^6*x^
6*e^2 + 3*b*c^6*d*x^4*e + 3*b*c^6*d^2*x^2)*arccsc(c*x) + (45*b*c^4*d^2 + (3
*b*c^4*x^4 + 4*b*c^2*x^2 + 8*b)*e^2 + 15*(b*c^4*d*x^2 + 2*b*c^2*d)*e)*sqrt(
c^2*x^2 - 1))/c^6
```

**Sympy** [A]
time = 4.09, size = 352, normalized size = 1.81

$$\frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\,\mathrm{acsc}\,(cx)}{2}+\frac{bdex^4\,\mathrm{acsc}\,(cx)}{2}+\frac{be^2x^6\,\mathrm{acsc}\,(cx)}{6}+\frac{bd^2\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}+\frac{bde\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{2c}+\frac{be^2\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*acsc(c*x)/2 + b*
d*e*x**4*acsc(c*x)/2 + b*e**2*x**6*acsc(c*x)/6 + b*d**2*Piecewise((sqrt(c**
2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c)
 + b*d*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/
(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqr
t(-c**2*x**2 + 1)/(3*c**3), True))/(2*c) + b*e**2*Piecewise((x**4*sqrt(c**2
*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2
- 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4
*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5)
, True))/(6*c)
```

**Giac** [B] Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(169) = 338$.
time = 0.52, size = 1160, normalized size = 5.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] 1/5760*(15*b*e^2*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c + 15*
a*e^2*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 6*b*e^2*x^5*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^5/c^2 + 180*b*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c
*x))/c + 180*a*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 90*b*e^2*x^4*(sqr
t(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 90*a*e^2*x^4*(sqrt(-1/(c^2
*x^2) + 1) + 1)^4/c^3 + 120*b*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 +
720*b*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + 720*a*d^2*
x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c + 50*b*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^3/c^4 + 720*b*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/
c^3 + 720*a*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 1440*b*d^2*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)/c^2 + 225*b*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*
arcsin(1/(c*x))/c^5 + 225*a*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 10
80*b*d*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 1440*b*d^2*arcsin(1/(c*x))/c^
3 + 1440*a*d^2/c^3 + 300*b*e^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 + 1080*b*
d*e*arcsin(1/(c*x))/c^5 + 1080*a*d*e/c^5 - 1440*b*d^2/(c^4*x*(sqrt(-1/(c^2*
x^2) + 1) + 1)) + 300*b*e^2*arcsin(1/(c*x))/c^7 + 300*a*e^2/c^7 - 1080*b*d*
e/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 720*b*d^2*arcsin(1/(c*x))/(c^5*x^2
*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 720*a*d^2/(c^5*x^2*(sqrt(-1/(c^2*x^2) +
1) + 1)^2) - 300*b*e^2/(c^8*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 720*b*d*e*arc
sin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 720*a*d*e/(c^7*x^2*
(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 225*b*e^2*arcsin(1/(c*x))/(c^9*x^2*(sqrt(
-1/(c^2*x^2) + 1) + 1)^2) + 225*a*e^2/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)
^2) - 120*b*d*e/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 50*b*e^2/(c^10*x
^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 180*b*d*e*arcsin(1/(c*x))/(c^9*x^4*(sq
rt(-1/(c^2*x^2) + 1) + 1)^4) + 180*a*d*e/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) +
 1)^4) + 90*b*e^2*arcsin(1/(c*x))/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4)
 + 90*a*e^2/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) - 6*b*e^2/(c^12*x^5*(
sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 15*b*e^2*arcsin(1/(c*x))/(c^13*x^6*(sqrt(-
1/(c^2*x^2) + 1) + 1)^6) + 15*a*e^2/(c^13*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^
6))*c
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(e\,x^2+d\right)^2\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

# 3.96 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x}\,dx$

**Optimal**. Leaf size=186

$$\frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}\right.$$

```
[Out] 1/2*I*b*d^2*arccsc(c*x)^2+d*e*x^2*(a+b*arccsc(c*x))+1/4*e^2*x^4*(a+b*arccsc
(c*x))-b*d^2*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+b*d^2*arccsc(c
*x)*ln(1/x)-d^2*(a+b*arccsc(c*x))*ln(1/x)+1/2*I*b*d^2*polylog(2,(I/c/x+(1-1
/c^2/x^2)^(1/2))^2)+1/6*b*e*(6*c^2*d+e)*x*(1-1/c^2/x^2)^(1/2)/c^3+1/12*b*e^
2*x^3*(1-1/c^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.30, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5349, 272, 45, 4815, 6874, 464, 270, 2363, 4721, 3798, 2221, 2317, 2438}

$$-d^2\log\left(\frac{1}{x}\right)\left(a+b\csc^{-1}(cx)\right)+dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{be^2x^3\sqrt{1-\frac{1}{c^2x^2}}}{12c}+\frac{bex\sqrt{1-\frac{1}{c^2x^2}}\,(6c^2d+e)}{6c^3}+\frac{1}{2}ibd^2\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)+\frac{1}{2}ibd^2\csc^{-1}(cx)^2-bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+bd^2\log\left(\frac{1}{x}\right)\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] (b*e*(6*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x)/(6*c^3) + (b*e^2*Sqrt[1 - 1/(c^
2*x^2)]*x^3)/(12*c) + (I/2)*b*d^2*ArcCsc[c*x]^2 + d*e*x^2*(a + b*ArcCsc[c*x
]) + (e^2*x^4*(a + b*ArcCsc[c*x]))/4 - b*d^2*ArcCsc[c*x]*Log[1 - E^((2*I)*A
rcCsc[c*x])] + b*d^2*ArcCsc[c*x]*Log[x^(-1)] - d^2*(a + b*ArcCsc[c*x])*Log[
x^(-1)] + (I/2)*b*d^2*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4815

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)}{x}\,dx = -\mathrm{Subst}\left(\int \frac{(e+dx^2)^2\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)-d^2\left(a+b\csc^{-1}(cx)\right)$$

$$= dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)-d^2\left(a+b\csc^{-1}(cx)\right)$$

$$= dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)-d^2\left(a+b\csc^{-1}(cx)\right)$$

$$= \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+dex^2\left(a+b\csc^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\csc^{-1}(cx)\right)-$$

$$= \frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+dex^2\left(a+b\csc^{-1}(cx\right.$$

$$= \frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+$$

$$= \frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+$$

$$= \frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+$$

$$= \frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+$$

**Mathematica [A]**
time = 0.25, size = 157, normalized size = 0.84

$$adex^2+\frac{1}{4}ae^2x^4+\frac{bdex\left(\sqrt{1-\frac{1}{c^2x^2}}+cx\csc^{-1}(cx)\right)}{c}+\frac{be^2x\left(\sqrt{1-\frac{1}{c^2x^2}}\,(2+c^2x^2)+3c^3x^3\csc^{-1}(cx)\right)}{12c^3}+ad^2\log(x)+\frac{1}{2}ibd^2\left(\csc^{-1}(cx)\left(\csc^{-1}(cx)+2i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 + (b*d*e*x*(Sqrt[1 - 1/(c^2*x^2)] + c*x*ArcCsc[c*
x]))/c + (b*e^2*x*(Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2) + 3*c^3*x^3*ArcCsc[c
*x]))/(12*c^3) + a*d^2*Log[x] + (I/2)*b*d^2*(ArcCsc[c*x]*(ArcCsc[c*x] + (2*
I)*Log[1 - E^((2*I)*ArcCsc[c*x])]) + PolyLog[2, E^((2*I)*ArcCsc[c*x])])
```

**Maple [A]**
time = 8.67, size = 301, normalized size = 1.62

| method | result |
|---|---|
| derivativedivides | $ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln(cx)+\frac{ib\,d^2\mathrm{arccsc}(cx)^2}{2}+b\,\mathrm{arccsc}(cx)\,de\,x^2+\frac{b\,\mathrm{arccsc}(cx)e^2x^4}{4}+\frac{b\sqrt{\frac{c^2x^2\cdots}{c^2\cdots}}\cdots}{\cdots}$ |
| default | $ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln(cx)+\frac{ib\,d^2\mathrm{arccsc}(cx)^2}{2}+b\,\mathrm{arccsc}(cx)\,de\,x^2+\frac{b\,\mathrm{arccsc}(cx)e^2x^4}{4}+\frac{b\sqrt{\frac{c^2x^2\cdots}{c^2\cdots}}\cdots}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*d*e*x^2+1/4*a*e^2*x^4+a*d^2*ln(c*x)+1/2*I*b*d^2*arccsc(c*x)^2+b*arccsc(c*
x)*d*e*x^2+1/4*b*arccsc(c*x)*e^2*x^4+b/c*((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e*x+
1/12*b/c*((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*x^3-I*b/c^2*d*e+1/6*b/c^3*((c^2*x^
2-1)/c^2/x^2)^(1/2)*e^2*x-1/6*I*b/c^4*e^2-b*d^2*arccsc(c*x)*ln(1-I/c/x-(1-1
/c^2/x^2)^(1/2))-b*d^2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I*b*d^2*
polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+I*b*d^2*polylog(2,-I/c/x-(1-1/c^2/x^2)
^(1/2))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e^2 + a*d*x^2*e + a*d^2*log(x) + 1/8*(4*I*b*c^4*d^2*log(-c*x + 1)
*log(x) + 4*I*b*c^4*d^2*log(x)^2 + 4*I*b*c^4*d^2*dilog(c*x) + 4*I*b*c^4*d^2
```

```
*dilog(-c*x) - I*(4*b*d*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*e + 32*b*d^2*
integrate(1/4*log(x)/(c^2*x^3 - x), x) + b*(x^2/c^2 + log(c*x + 1)/c^4 + lo
g(c*x - 1)/c^4)*e^2)*c^4 + 2*(b*c^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
*e^2 + I*b*c^4*e^2*log(c))*x^4 + 8*c^4*integrate(1/4*(b*x^4*e^2 + 4*b*d*x^2
*e + 4*b*d^2*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) + (I*b*c
^2*e^2 + 8*(b*c^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e + I*b*c^4*e*log
(c))*d)*x^2 + (-I*b*c^4*x^4*e^2 - 4*I*b*c^4*d*x^2*e - 4*I*b*c^4*d^2*log(x))
*log(c^2*x^2) + (4*I*b*c^4*d^2*log(x) + 4*I*b*c^2*d*e + I*b*e^2)*log(c*x +
1) + (4*I*b*c^2*d*e + I*b*e^2)*log(c*x - 1) - 2*(-I*b*c^4*x^4*e^2 - 4*I*b*c
^4*d*x^2*e - 4*(b*c^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^4*log
(c))*d^2)*log(x))/c^4
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arccsc(c*x))/x, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsc}(cx))\,(d + ex^2)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**2/x, x)
```

**Giac [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]Undef/
Unsigned
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x, x)
```

# 3.97 $\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3} \, dx$

**Optimal**. Leaf size=189

$$-\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{4}bc^2d^2\csc^{-1}(cx)+ibde\csc^{-1}(cx)^2-\frac{d^2(a+b\csc^{-1}(cx))}{2x^2}+\frac{1}{2}e^2x^2(a+$$

```
[Out] 1/4*b*c^2*d^2*arccsc(c*x)+I*b*d*e*arccsc(c*x)^2-1/2*d^2*(a+b*arccsc(c*x))/x
^2+1/2*e^2*x^2*(a+b*arccsc(c*x))-2*b*d*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x
^2)^(1/2))^2)+2*b*d*e*arccsc(c*x)*ln(1/x)-2*d*e*(a+b*arccsc(c*x))*ln(1/x)+I
*b*d*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-1/4*b*c*d^2*(1-1/c^2/x^2)^(
1/2)/x+1/2*b*e^2*x*(1-1/c^2/x^2)^(1/2)/c
```

**Rubi** [A]
time = 0.30, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5349, 272, 45, 4815, 12, 6874, 270, 327, 222, 2363, 4721, 3798, 2221, 2317, 2438}

$$-\frac{d^2(a+b\csc^{-1}(cx))}{2x^2}-2de\log\left(\frac{1}{x}\right)(a+b\csc^{-1}(cx))+\frac{1}{2}e^2x^2(a+b\csc^{-1}(cx))-\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2d^2\csc^{-1}(cx)+\frac{be^2x\sqrt{1-\frac{1}{c^2x^2}}}{2c}+ibde\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)+ibde\csc^{-1}(cx)^2-2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+2bde\log\left(\frac{1}{x}\right)\csc^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] -1/4*(b*c*d^2*Sqrt[1 - 1/(c^2*x^2)])/x + (b*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)/(2
*c) + (b*c^2*d^2*ArcCsc[c*x])/4 + I*b*d*e*ArcCsc[c*x]^2 - (d^2*(a + b*ArcCs
c[c*x]))/(2*x^2) + (e^2*x^2*(a + b*ArcCsc[c*x]))/2 - 2*b*d*e*ArcCsc[c*x]*Lo
g[1 - E^((2*I)*ArcCsc[c*x])] + 2*b*d*e*ArcCsc[c*x]*Log[x^(-1)] - 2*d*e*(a +
 b*ArcCsc[c*x])*Log[x^(-1)] + I*b*d*e*PolyLog[2, E^((2*I)*ArcCsc[c*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2363

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[-e, 2]), x
] - Dist[b*(n/Rt[-e, 2]), Int[ArcSin[Rt[-e, 2]*(x/Sqrt[d])]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4815

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcSin[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*
x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] &
& IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx = -\text{Subst}\left(\int \frac{\left(e+dx^2\right)^2 \left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right) - 2de\left(a+b\csc^{-1}(cx)\right)$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right) - 2de\left(a+b\csc^{-1}(cx)\right)$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right) - 2de\left(a+b\csc^{-1}(cx)\right)$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\csc^{-1}(cx)\right) - 2de\left(a+b\csc^{-1}(cx)\right)$$

$$= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} - \frac{d^2(a+b\csc^{-1}(cx))}{2x^2} + \frac{1}{2}e^2x^2(a$$

$$= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{4}bc^2d^2\csc^{-1}(cx) - \frac{d^2(a+b\,\mathrm{cs}}{2x}$$

$$= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{4}bc^2d^2\csc^{-1}(cx) + ibde\csc^{-1}($$

$$= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{4}bc^2d^2\csc^{-1}(cx) + ibde\csc^{-1}($$

$$= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{4}bc^2d^2\csc^{-1}(cx) + ibde\csc^{-1}($$

$$= -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{4}bc^2d^2\csc^{-1}(cx) + ibde\csc^{-1}($$

**Mathematica [A]**
time = 0.49, size = 194, normalized size = 1.03

$$\frac{1}{4}\left(-\frac{2ad^2}{x^2}+2ae^2x^2-\frac{2bd^2\csc^{-1}(cx)}{x^2}+\frac{2be^2x\left(\sqrt{1-\frac{1}{c^2x^2}}+cx\csc^{-1}(cx)\right)}{c}-\frac{bd^2\left(-1+c^2x^2+c^2x^2\sqrt{-1+c^2x^2}\,\mathrm{ArcTan}\left(\sqrt{-1+c^2x^2}\right)\right)}{c\sqrt{1-\frac{1}{c^2x^2}}\,x^3}-8bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+8ade\log(x)+4ibde\left(\csc^{-1}(cx)^2+\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*d^2*ArcCsc[c*x])/x^2 + (2*b*e^2*x*(Sqr
t[1 - 1/(c^2*x^2)] + c*x*ArcCsc[c*x]))/c - (b*d^2*(-1 + c^2*x^2 + c^2*x^2*S
qrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]]))/(c*Sqrt[1 - 1/(c^2*x^2)]*x^3
) - 8*b*d*e*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + 8*a*d*e*Log[x] + (
4*I)*b*d*e*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/4
```

**Maple [A]**
time = 6.22, size = 309, normalized size = 1.63

| method | result |
|---|---|
| derivativedivides | $c^2\left(\frac{a\,x^2e^2}{2c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{2ade\ln(cx)}{c^2}+\frac{i b \mathrm{arccsc}(cx)^2de}{c^2}-\frac{b\,d^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{b\,d^2\mathrm{arccsc}(cx)}{4}-\frac{b\,d^2\mathrm{arccsc}(cx)}{2c^2x^2}\right.$ |
| default | $c^2\left(\frac{a\,x^2e^2}{2c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{2ade\ln(cx)}{c^2}+\frac{i b \mathrm{arccsc}(cx)^2de}{c^2}-\frac{b\,d^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{b\,d^2\mathrm{arccsc}(cx)}{4}-\frac{b\,d^2\mathrm{arccsc}(cx)}{2c^2x^2}\right.$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(1/2*a/c^2*x^2*e^2-1/2*a*d^2/c^2/x^2+2*a/c^2*d*e*ln(c*x)+I*b/c^2*arccsc
(c*x)^2*d*e-1/4*b*d^2/c/x*((c^2*x^2-1)/c^2/x^2)^(1/2)+1/4*b*d^2*arccsc(c*x)
-1/2*b*d^2/c^2/x^2*arccsc(c*x)+1/2*b/c^2*arccsc(c*x)*e^2*x^2+1/2*b/c^3*((c^
2*x^2-1)/c^2/x^2)^(1/2)*e^2*x-1/2*I*b/c^4*e^2-2*b/c^2*d*e*arccsc(c*x)*ln(1-
I/c/x-(1-1/c^2/x^2)^(1/2))-2*b/c^2*d*e*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)
^(1/2))+2*I*b/c^2*d*e*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+2*I*b/c^2*d*e*p
olylog(2,I/c/x+(1-1/c^2/x^2)^(1/2)))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*d^2*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) + 1/2*a*x^2*
e^2 + 2*a*d*e*log(x) - 1/2*a*d^2/x^2 + 1/4*(4*I*b*c^2*d*e*log(-c*x + 1)*log
(x) + 4*I*b*c^2*d*e*log(x)^2 + 4*I*b*c^2*d*dilog(c*x)*e + 4*I*b*c^2*d*dilog
(-c*x)*e - I*(16*b*d*e*integrate(1/2*log(x)/(c^2*x^3 - x), x) + b*(log(c*x
+ 1)/c^2 + log(c*x - 1)/c^2)*e^2)*c^2 + 2*(b*c^2*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1))*e^2 + I*b*c^2*e^2*log(c))*x^2 + 4*c^2*integrate(1/2*(b*x^2*e^
2 + 4*b*d*e*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) + I*b*e^2
*log(c*x - 1) + (-I*b*c^2*x^2*e^2 - 4*I*b*c^2*d*e*log(x))*log(c^2*x^2) + (4
*I*b*c^2*d*e*log(x) + I*b*e^2)*log(c*x + 1) - 2*(-I*b*c^2*x^2*e^2 - 4*(b*c^
2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e + I*b*c^2*e*log(c))*d)*log(x))/
c^2
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arccsc(c*x))/x^3, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsc}(cx)) (d + ex^2)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**2/x**3, x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsc(c*x) + a)/x^3, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^3, x)
```

# 3.98 $\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$

**Optimal**. Leaf size=565

$$\frac{x(a+b\csc^{-1}(cx))}{e}+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce}-\frac{\sqrt{-d}\,(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{\sqrt{-d}\,(a}{}$$

[Out] x*(a+b*arccsc(c*x))/e+b*arctanh((1-1/c^2/x^2)^(1/2))/c/e-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)

**Rubi** [A]
time = 1.10, antiderivative size = 565, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5349, 4817, 4723, 272, 65, 214, 4757, 4825, 4615, 2221, 2317, 2438}

$$-\frac{\sqrt{-d}\,(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{\sqrt{-d}\,(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\,(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}+\frac{\sqrt{-d}\,(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}+\frac{x(a+b\csc^{-1}(cx))}{e}-\frac{ib\sqrt{-d}\,\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}}+\frac{ib\sqrt{-d}\,\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}}-\frac{ib\sqrt{-d}\,\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}+\frac{ib\sqrt{-d}\,\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]

[Out] (x*(a + b*ArcCsc[c*x]))/e + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e) - (Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) - ((I/2)*b*Sqrt[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(3/2) + ((I/2)*b*Sqrt[-d]*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(3/2) - ((I/2)*b*Sqrt[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(3/2) + ((I/2)*b*Sqrt[-d]*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(3/2)

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{ex^2} - \frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e} + \frac{d\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{e} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce} + \frac{d\text{Subst}\left(\int \left(\frac{a+b}{2\sqrt{e}\left(\sqrt{}\right.}\right.\right.}{}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{e} + \frac{d\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2e^{3/2}} + \frac{d\text{Subst}\left(\int \frac{a+b\sin^{-1}(}{\sqrt{e}+\sqrt{-}}\right.}{2e^{3/2}}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{e} + \frac{d\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\,\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2e^{3/2}} + \frac{d\text{Subst}\left(\right.}{}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{e} + \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} + \frac{d\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-}\right.}{2e^{3}}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{e} + \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} - \frac{\sqrt{-d}\,(a+b\csc^{-1}(cx))\log\left(\right.}{2e^{3/2}}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{e} + \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} - \frac{\sqrt{-d}\,(a+b\csc^{-1}(cx))\log\left(\right.}{2e^{3/2}}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{e} + \frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} - \frac{\sqrt{-d}\,(a+b\csc^{-1}(cx))\log\left(\right.}{2e^{3/2}}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1260 vs. $2(565) = 1130$.

time = 1.25, size = 1260, normalized size = 2.23

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

```
[Out] ((I/4)*((-4*I)*a*c*Sqrt[e]*x - (4*I)*b*c*Sqrt[e]*x*ArcCsc[c*x] + (4*I)*a*c*
Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 - (I*
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi +
 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2
*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + b*c*Sqrt[d]*Pi*Log[1 + (Sqrt[e] - Sqrt
[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*c*Sqrt[d]*ArcCsc[c*x]*Log
[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*c*Sqr
t[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - b*c*Sqrt[d]*Pi*Log[1 + (-S
qrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*b*c*Sqrt[d]*Ar
cCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])
)] - 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1
+ (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - b*c*Sqrt[d]
*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*
b*c*Sqrt[d]*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] + 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
 b*c*Sqrt[d]*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] - 2*b*c*Sqrt[d]*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] + b*c*Sqrt[d]*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - b*c*Sqrt[d]*Pi*L
og[Sqrt[e] + (I*Sqrt[d])/x] - (4*I)*b*Sqrt[e]*Log[Cos[ArcCsc[c*x]/2]] + (4*
I)*b*Sqrt[e]*Log[Sin[ArcCsc[c*x]/2]] + (2*I)*b*c*Sqrt[d]*PolyLog[2, (Sqrt[e
] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)*b*c*Sqrt[d]*Pol
yLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)
*b*c*Sqrt[d]*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x])))] + (2*I)*b*c*Sqrt[d]*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sq
rt[d]*E^(I*ArcCsc[c*x]))]))/(c*e^(3/2))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 3.69, size = 407, normalized size = 0.72

$$\frac{ax}{e}-\frac{ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}}+\frac{bx\,\mathrm{arccsc}(cx)}{e}+\frac{cbd\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+(-2c^2d-4e)\_Z^2+c^2d\right)}\frac{\left(\_R1^2c^2d+4\_R1^2e-c^2d\right)\left(i\right.}{}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x)

[Out] a/e*x-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b*x*arccsc(c*x)/e+1/8*c*b/e^2*d*sum((_R1^2*c^2*d+4*_R1^2*e-c^2*d)/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/c*b/e*ln(I/c/x+(1-1/c^2/x^2)^(1/2)-1)+1/c*b/e*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/8*c*b/e^2*d*sum((_R1^2*c^2*d-c^2*d-4*e)/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")

[Out] -(sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2) - x*e^(-1))*a + b*integrate(x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(x^2*e + d), x)

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")

[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/(x^2*e + d), x)

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\operatorname{acsc}(cx))}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))/(d + e*x**2), x)
```

**Giac** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2),x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2), x)
```

# 3.99 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx$

**Optimal**. Leaf size=507

$$\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{(a+b\csc^{-1}(c}{}$$

```
[Out] -(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e+1/2*(a+b*arccsc(c*
x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
)))/e+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsc(c*x))
*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))
/e+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e-1/2*I*b*polylog(2,-I*
c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e-1/2*I
*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^
(1/2)))/e-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(c^2*d+e)^(1/2)))/e-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))
*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e
```

**Rubi** [A]
time = 0.98, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5349, 4817, 4721, 3798, 2221, 2317, 2438, 4825, 4615}

$$\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}+\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}-\frac{\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{e}-\frac{ib\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e}+\frac{ib\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sq
rt[c^2*d + e])])/(2*e) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*Ar
cCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcCsc[c*x])*Log[
1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) +
((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(2*e) - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])
])/e - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - S
qrt[c^2*d + e])])/e - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/
(Sqrt[e] - Sqrt[c^2*d + e])])/e - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I
*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e - ((I/2)*b*PolyLog[2, (I*c*S
qrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e + ((I/2)*b*PolyL
og[2, E^((2*I)*ArcCsc[c*x])])/e
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x\,(e+dx^2)}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{ex} - \frac{dx\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e\,(e+dx^2)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)}{e} + \frac{d\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\,\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \right.\right.}{e}$$

$$= \frac{i(a+b\csc^{-1}(cx))^2}{2be} + \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)}{e} - \frac{\sqrt{-d}\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{i(a+b\csc^{-1}(cx))^2}{2be} - \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} + \frac{b\text{Subst}\left(\int \log\left(1-\right.\right.}{}$$

$$= -\frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} - \frac{(ib)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i\csc^{-1}(cx)}\right)}{2e}$$

$$= \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}}{\sqrt{e}-}\right.}{2e}$$

$$= \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}}{\sqrt{e}-}\right.}{2e}$$

$$= \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}}{\sqrt{e}-}\right.}{2e}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1123 vs. $2(507) = 1014$.
time = 0.32, size = 1123, normalized size = 2.21

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

```
[Out] (I*b*Pi^2 - (4*I)*b*Pi*ArcCsc[c*x] + (8*I)*b*ArcCsc[c*x]^2 - (16*I)*b*ArcSi
n[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt
[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I)*b*ArcSin[Sqrt[1
 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(P
i + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*b*Pi*Log[1 + (Sqrt[e] - Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e]
 - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 - (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sq
rt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*S
qrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] - 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Arc
Csc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2
]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b
*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*
b*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*
x]))] + 8*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcCsc[c*x]*Log
[1 - E^((2*I)*ArcCsc[c*x])] + 2*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 2*b*Pi*
Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*a*Log[d + e*x^2] + (4*I)*b*PolyLog[2, (Sqr
t[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2,
 (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*Poly
Log[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (4*I
)*b*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
 (4*I)*b*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(8*e)
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.31, size = 420, normalized size = 0.83

| method | result |
|---|---|
| derivativedivides | $\frac{a c^2 \ln\left(c^2 e x^2+c^2 d\right)}{2e}-\frac{b c^2 \operatorname{arccsc}(cx) \ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{i b c^2 \operatorname{dilog}\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}-\frac{i b c^2 \operatorname{dilog}\left(\frac{i}{cx}+\sqrt{\right.}}{e}$ |

| | |
|---|---|
| default | $\frac{a\,c^2 \ln\left(c^2 e\,x^2+c^2 d\right)}{2e}-\frac{b\,c^2 \operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{ib\,c^2 \operatorname{dilog}\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}-\frac{ib\,c^2 \operatorname{dilog}\left(\frac{i}{cx}+\sqrt{1}\right.}{e}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*a*c^2/e*ln(c^2*e*x^2+c^2*d)-b*c^2/e*arccsc(c*x)*ln(1+I/c/x+(1-1/
c^2/x^2)^(1/2))+I*b*c^2/e*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-I*b*c^2*dilog(
I/c/x+(1-1/c^2/x^2)^(1/2))/e-1/4*I*b*c^2*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2
*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+di
log((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4
*e)*_Z^2+c^2*d))/e-1/4*I*b*c^4*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arc
csc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/
x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))*d/e)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*e^(-1)*log(x^2*e + d) + b*integrate(x*arctan2(1, sqrt(c*x + 1)*sqrt(c
*x - 1))/(x^2*e + d), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsc(c*x) + a*x)/(x^2*e + d), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\operatorname{acsc}(cx))}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2), x)
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2), x)
```

# 3.100 $\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx$

**Optimal**. Leaf size=529

$$-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{(a+b\csc^{-1}(c$$

```
[Out] -1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/
e^(1/2)-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccsc(c*x))*ln(1
+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)
^(1/2)/e^(1/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,I*c*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(
1/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

**Rubi** [A]
time = 0.78, antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5339, 4757, 4825, 4615, 2221, 2317, 2438}

$$-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{ib\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{ib\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{ib\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{ib\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2),x]
```

```
[Out] -1/2*((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCsc[c*x])*Log[1 + (I*
c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqr
t[e]) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt
[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcCsc[c*x])*Log[1
+ (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d
]*Sqrt[e]) - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[
e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, (I*c*Sqrt[
-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) - (
(I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*
d + e])])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCs
c[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5339

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\sin(x)}\,dx, x, \text{cs}\right.}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}\,e^{ix}}\,dx, x, \csc^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{e^{ix}(a+}{\frac{\sqrt{e}}{c}+\frac{\sqrt{c^2d+}}{c}}\right.}{}$$

$$= -\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}}{\sqrt{e}-\sqrt{}}\right.}{2\sqrt{-d}\sqrt{e}}$$

$$= -\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}}{\sqrt{e}-\sqrt{}}\right.}{2\sqrt{-d}\sqrt{e}}$$

$$= -\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}}{\sqrt{e}-\sqrt{}}\right.}{2\sqrt{-d}\sqrt{e}}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1068 vs. 2(529) = 1058.
time = 0.31, size = 1068, normalized size = 2.02

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2),x]
```

```
[Out] ((-1/4*I)*((4*I)*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*ArcSin[Sqrt[1 - (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi
```

```
+ 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (8*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c
*x])/4])/Sqrt[c^2*d + e]] + b*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - 2*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] - b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] + 2*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))] - 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*
Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - b*Pi*
Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*b*Ar
cCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))
] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e]
+ Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + b*Pi*Log[1 + (Sqrt[e] +
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*ArcCsc[c*x]*Log[1 +
(Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*ArcSin[Sqr
t[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])
/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - b*Pi*
Log[Sqrt[e] + (I*Sqrt[d])/x] + (2*I)*b*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e
])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)*b*PolyLog[2, (-Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (2*I)*b*PolyLog[2, -((Sqrt[e] + S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (2*I)*b*PolyLog[2, (Sqrt[
e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]))/(Sqrt[d]*Sqrt[e])
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.95, size = 272, normalized size = 0.51

$$\frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} - \frac{cb\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2 d\,\_Z^4+\left(-2c^2 d-4e\right)\_Z^2+c^2 d\right)} \frac{i \operatorname{arccsc}(cx) \ln\left(\frac{\_R1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-}{}\right.}{\_R1\left(\_R1^2 c^2 d-c^2 d-2e\right)}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d),x)
```

```
[Out] a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*c*b*sum(1/_R1/(_R1^2*c^2*d-c^2*d-
2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/
x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*
d))-1/2*c*b*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1
-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Roo
tOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] a*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/sqrt(d) + b*integrate(arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1))/(x^2*e + d), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(x^2*e + d), x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x**2), x)
```

**Giac** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2),x)
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2), x)
```

## 3.101 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)}\,dx$

**Optimal**. Leaf size=479

$$\frac{i(a+b\csc^{-1}(cx))^2}{2bd}-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(c}}{\sqrt{e}-\sqrt{c^2d+}}\right.}{2d}$$

```
[Out] 1/2*I*(a+b*arccsc(c*x))^2/b/d-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsc(c*x))
*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))
/d-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,-I*c*(
I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d+1/2*I*b*
polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/
2)))/d+1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
```

**Rubi** [A]
time = 0.76, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5349, 4817, 4825, 4615, 2221, 2317, 2438}

$$-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}+\frac{i(a+b\csc^{-1}(cx))^2}{2bd}+\frac{ib\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] ((I/2)*(a + b*ArcCsc[c*x])^2)/(b*d) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqr
t[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcC
sc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e
])])/(2*d) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/
(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqr
t[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) + ((I/2)*b*Pol
yLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d
 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/d + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/d + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\,(d+ex^2)}\,dx = -\mathrm{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)$$

$$= -\mathrm{Subst}\left(\int \left(-\frac{\sqrt{-d}\,\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)}+\frac{\sqrt{-d}\,\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx,x,\frac{1}{x}\right)$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx,x,\frac{1}{x}\right)}{2\sqrt{-d}}+\frac{\mathrm{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx,x,\frac{1}{x}\right)}{2\sqrt{-d}}$$

$$= -\frac{\mathrm{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\,\sin(x)}\,dx,x,\csc^{-1}(cx)\right)}{2\sqrt{-d}}+\frac{\mathrm{Subst}\bigg(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\,\sin(x)}\,dx,x,\mathrm{cs}}{2\sqrt{-d}}$$

$$= \frac{i(a+b\csc^{-1}(cx))^2}{2bd}+\frac{\mathrm{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}\,e^{ix}}\,dx,x,\csc^{-1}(cx)\right)}{2\sqrt{-d}}+\mathrm{Subs}$$

$$= \frac{i(a+b\csc^{-1}(cx))^2}{2bd}-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-(a+b\csc^{-1}(c$$

$$= \frac{i(a+b\csc^{-1}(cx))^2}{2bd}-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-(a+b\csc^{-1}(c$$

$$= \frac{i(a+b\csc^{-1}(cx))^2}{2bd}-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-(a+b\csc^{-1}(c$$

**Mathematica [A]**
time = 0.62, size = 407, normalized size = 0.85

$$\frac{4a\log(x)-2a\log\left(d+ex^2\right)+ib\left(2\csc^{-1}(cx)^2+4\mathrm{ArcSin}\left(\sqrt{-\frac{e}{c^2d}}\right)\mathrm{ArcTan}\left(\frac{\sqrt{e\left(c^2d+e\right)}}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)+2i\csc^{-1}(cx)\log\left(1-\frac{\left(c^2d+2e-2\sqrt{e\left(c^2d+e\right)}\right)e^{2i\csc^{-1}(cx)}}{c^2d}\right)-2i\mathrm{ArcSin}\left(\sqrt{-\frac{e}{c^2d}}\right)\log\left(1-\frac{\left(c^2d+2e-2\sqrt{e\left(c^2d+e\right)}\right)e^{2i\csc^{-1}(cx)}}{c^2d}\right)+2i\csc^{-1}(cx)\log\left(1-\frac{\left(c^2d+2\left(e+\sqrt{e\left(c^2d+e\right)}\right)\right)e^{2i\csc^{-1}(cx)}}{c^2d}\right)+2i\mathrm{ArcSin}\left(\sqrt{-\frac{e}{c^2d}}\right)\log\left(1-\frac{\left(c^2d+2\left(e+\sqrt{e\left(c^2d+e\right)}\right)\right)e^{2i\csc^{-1}(cx)}}{c^2d}\right)+\mathrm{PolyLog}\left(2,\frac{\left(c^2d+2e-2\sqrt{e\left(c^2d+e\right)}\right)e^{2i\csc^{-1}(cx)}}{c^2d}\right)+\mathrm{PolyLog}\left(2,\frac{\left(c^2d+2\left(e+\sqrt{e\left(c^2d+e\right)}\right)\right)e^{2i\csc^{-1}(cx)}}{c^2d}\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (4*a*Log[x] - 2*a*Log[d + e*x^2] + I*b*(2*ArcCsc[c*x]^2 + 4*ArcSin[Sqrt[-(e
/(c^2*d))]]*ArcTan[Sqrt[e*(c^2*d + e)]/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x)] + (2*
I)*ArcCsc[c*x]*Log[1 - ((c^2*d + 2*e - 2*Sqrt[e*(c^2*d + e)])*E^((2*I)*ArcC
```

```
sc[c*x]))/(c^2*d)] - (2*I)*ArcSin[Sqrt[-(e/(c^2*d))]]*Log[1 - ((c^2*d + 2*e
 - 2*Sqrt[e*(c^2*d + e)])*E^((2*I)*ArcCsc[c*x]))/(c^2*d)] + (2*I)*ArcCsc[c*
x]*Log[1 - ((c^2*d + 2*(e + Sqrt[e*(c^2*d + e)]))*E^((2*I)*ArcCsc[c*x]))/(c
^2*d)] + (2*I)*ArcSin[Sqrt[-(e/(c^2*d))]]*Log[1 - ((c^2*d + 2*(e + Sqrt[e*(
c^2*d + e)]))*E^((2*I)*ArcCsc[c*x]))/(c^2*d)] + PolyLog[2, ((c^2*d + 2*e -
2*Sqrt[e*(c^2*d + e)])*E^((2*I)*ArcCsc[c*x]))/(c^2*d)] + PolyLog[2, ((c^2*d
 + 2*(e + Sqrt[e*(c^2*d + e)]))*E^((2*I)*ArcCsc[c*x]))/(c^2*d)]))/(4*d)
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.05, size = 2875, normalized size = 6.00

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 2875 |
| default | Expression too large to display | 2875 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/c^2*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1
/2)+2*e))*e*arccsc(c*x)/d^2+2*I*b/c^4*arccsc(c*x)^2*e/d^3*(e*(c^2*d+e))^(1/
2)+1/8*I*b*c^2*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*polylog(2,d*c^2*(I/c/x+(1-1/
c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))-4*I*b/c^2*arccsc(c*x)^
2/d^2/(c^2*d+e)*e^2+4*b/c^2*e^2*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c
^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^2/(c^2*d+e)+2*b/c^4*e^3*ln(1
-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arc
csc(c*x)/d^3/(c^2*d+e)-2*b/c^4*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e*arccsc(c*x)/d^3*(e*(c^2*d+e))^(1/2)+1/4*b
*c^2*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+
2*e))*arccsc(c*x)/e/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+I*b/c^4*polylog(2,d*c^2*(
I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e/d^3*(e*(c
^2*d+e))^(1/2)-2*b/c^4*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e
*(c^2*d+e))^(1/2)+2*e))*e^2*arccsc(c*x)/d^3-b/c^2*ln(1-d*c^2*(I/c/x+(1-1/c^
2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^2*(e*(c^2*
d+e))^(1/2)+I*b/c^2*arccsc(c*x)^2/d^2*(e*(c^2*d+e))^(1/2)+I*b/c^2*polylog(2
,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e/d
^2+I*b/c^4*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d
+e))^(1/2)+2*e))*e^2/d^3+1/4*I*b*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*polylog(2,
d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))-3/4*
I*b*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1
/2)+2*e))/d/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-1/4*b*c^2*(e*(c^2*d+e))^(1/2)/e/(
c^2*d+e)*arccsc(c*x)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(
c^2*d+e))^(1/2)+2*e))-1/8*I*b*c^2*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2
))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/e/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-2*I
*b/c^4*e^3*arccsc(c*x)^2/d^3/(c^2*d+e)-2*I*b/c^2*e^2*polylog(2,d*c^2*(I/c/x
+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^2/(c^2*d+e)-I*
```

```
b/c^4*e^3*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+
e))^(1/2)+2*e))/d^3/(c^2*d+e)+3*b/c^2*e*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/
2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^2/(c^2*d+e)*(e*(c^2*
d+e))^(1/2)+2*b/c^4*e^2*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(
e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^3/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-I*b/
c^4*e^2*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e)
)^(1/2)+2*e))/d^3/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-2*I*b/c^4*e^2*arccsc(c*x)^2
/d^3/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-3*I*b/c^2*arccsc(c*x)^2/d^2/(c^2*d+e)*(e
*(c^2*d+e))^(1/2)*e-3/2*I*b/c^2*e*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2
))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^2/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-1
/2*a/d*ln(c^2*e*x^2+c^2*d)+a/d*ln(c*x)-5/4*I*b*polylog(2,d*c^2*(I/c/x+(1-1/
c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d/(c^2*d+e)*e-5/2*I*b*
arccsc(c*x)^2/d/(c^2*d+e)*e-I*b*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*arccsc(c*x)
^2+2*I*b/c^2*arccsc(c*x)^2/d^2*e+1/2*I*b/c^2*polylog(2,d*c^2*(I/c/x+(1-1/c^
2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^2*(e*(c^2*d+e))^(1/2)+
2*I*b/c^4*arccsc(c*x)^2*e^2/d^3-1/2*b*(e*(c^2*d+e))^(1/2)/d/(c^2*d+e)*arccs
c(c*x)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2
)+2*e))+5/2*b*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e
))^(1/2)+2*e))*arccsc(c*x)/d*e/(c^2*d+e)+3/2*b*ln(1-d*c^2*(I/c/x+(1-1/c^2/x
^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d/(c^2*d+e)*(e*
(c^2*d+e))^(1/2)+1/2*I*b/d*sum((_R1^2*c^2*d-2*c^2*d-4*e)/(_R1^2*c^2*d-c^2*d
-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c
/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2
*d))+I*b*arccsc(c*x)^2/d-1/2*b*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d+1/4*I*b*polylog(2,d*c^2*(I/c/
x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d+1/2*b*c^2*ln(
1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*ar
ccsc(c*x)/(c^2*d+e)-1/4*I*b*c^2*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))
^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/(c^2*d+e)-1/2*I*b*c^2*arccsc(c*x)^2/(
c^2*d+e)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(x^2*e + d)/d - 2*log(x)/d) + b*integrate(arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))/(x^3*e + d*x), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(x^3*e + d*x), x)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{x(d + ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x*(d + e*x**2)), x)
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)), x)
```

# 3.102 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx$

**Optimal**. Leaf size=572

$$-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d}-\frac{a}{dx}-\frac{b\csc^{-1}(cx)}{dx}-\frac{\sqrt{e}\,(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\,(a+b\csc^{-1}(cx))}{2}$$

```
[Out] -a/d/x-b*arccsc(c*x)/d/x-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*(a+b*
arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*
d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/
2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2
*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*I*b*polyl
og(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*
e^(1/2)/(-d)^(3/2)-b*c*(1-1/c^2/x^2)^(1/2)/d
```

**Rubi** [A]

time = 1.02, antiderivative size = 572, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5349, 4817, 4715, 267, 4757, 4825, 4615, 2221, 2317, 2438}

$$-\frac{\sqrt{e}\,(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\,(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\,(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\,(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}-\frac{a}{dx}-\frac{ib\sqrt{e}\,\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}+\frac{ib\sqrt{e}\,\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{ib\sqrt{e}\,\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}+\frac{ib\sqrt{e}\,\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d}-\frac{b\csc^{-1}(cx)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -((b*c*Sqrt[1 - 1/(c^2*x^2)])/d) - a/(d*x) - (b*ArcCsc[c*x])/(d*x) - (Sqrt[
e]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] -
Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcCsc[c*x])*Log[1 + (I
*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2))
 - (Sqrt[e]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(S
qrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcCsc[c*x])*L
og[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-
d)^(3/2)) - ((I/2)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))
/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*PolyLog[2, (I*
c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(3/2) - ((
I/2)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*PolyLog[2, (I*c*Sqrt[-d]*E^(
I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)^(3/2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{d} - \frac{e\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{a}{dx} - \frac{b\text{Subst}\left(\int \sin^{-1}\left(\frac{x}{c}\right)\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+\cdots}{2\sqrt{e}\left(\cdots\right.}\cdots\right.\right.}{d}$$

$$= -\frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} + \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\cdots}\cdots\right.}{2d}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sin(x)}\,dx, x, \csc^{-1}(\cdots\right.}{2d}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{e^{ix}(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{c^2d+e}}{c}-i\sqrt{-d}\,e^{ix}}\,d\cdots\right.}{2d}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} - \frac{\sqrt{e}\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}\cdots}}{\sqrt{e}-\sqrt{c^2d\cdots}}\cdots\right.}{2(-d)^{3/2}}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} - \frac{\sqrt{e}\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}\cdots}}{\sqrt{e}-\sqrt{c^2d\cdots}}\cdots\right.}{2(-d)^{3/2}}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\csc^{-1}(cx)}{dx} - \frac{\sqrt{e}\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}\cdots}}{\sqrt{e}-\sqrt{c^2d\cdots}}\cdots\right.}{2(-d)^{3/2}}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1241 vs. 2(572) = 1144.

time = 1.16, size = 1241, normalized size = 2.17

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -(a/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + b*(-((c*Sqrt
[1 - 1/(c^2*x^2)]*x + ArcCsc[c*x])/(d*x)) + (Sqrt[e]*(Pi^2 - 4*Pi*ArcCsc[c*
x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d
 + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqr
t[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
(4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log
[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*Arc
Csc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d]
)/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*
x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(16*d^(3/2)) - (Sqrt[e]*(Pi^
2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])
/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sq
rt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c
*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sq
rt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(16*d^(3/2)))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 2.22, size = 332, normalized size = 0.58

$$-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}}-\frac{a}{dx}-\frac{cb\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\mathrm{arccsc}(cx)}{dx}+\frac{cbe\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+(-2c^2d-4e)\_Z^2+c^2d\right)}\frac{i\,\mathrm{arccsc}(cx}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccsc(c*x))/x^2/(e*x^2+d),x)

[Out] -a*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-a/d/x-c*b/d*((c^2*x^2-1)/c^2/x^2)^(1/2)-b*arccsc(c*x)/d/x+1/2*c*b*e/d*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2*c*b*e/d*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")

[Out] -a*(arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/d^(3/2) + 1/(d*x)) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(x^4*e + d*x^2), x)

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")

[Out] integral((b*arccsc(c*x) + a)/(x^4*e + d*x^2), x)

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{acsc}\,(cx)}{x^2\,(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**2*(d + e*x**2)), x)
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]Evalua
tion time
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)), x)
```

# 3.103 $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

**Optimal**. Leaf size=628

$$\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2\left(a+b\csc^{-1}(cx)\right)}{2e^2}-\frac{bd\text{ArcTan}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{5/2}\sqrt{c^2d+e}}-\frac{d\left(a+b\csc^{-1}(\right.}{}$$

```
[Out] 1/2*d*(a+b*arccsc(c*x))/e^2/(e+d/x^2)+1/2*x^2*(a+b*arccsc(c*x))/e^2+2*d*(a+
b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e^3-d*(a+b*arccsc(c*x))*
ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/
e^3-d*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-d*(a+b*arccsc(c*x))*ln(
1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3
-I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e^3+I*b*d*polylog(2,-I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+I*b*d*p
olylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
)))/e^3+I*b*d*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))/e^3+I*b*d*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*b*d*arctan((c^2*d+e)^(1/2)/c/x/e
^(1/2)/(1-1/c^2/x^2)^(1/2))/e^(5/2)/(c^2*d+e)^(1/2)+1/2*b*x*(1-1/c^2/x^2)^(
1/2)/c/e^2
```

**Rubi** [A]
time = 1.14, antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5349, 4817, 4723, 270, 4721, 3798, 2221, 2317, 2438, 4813, 385, 211, 4825, 4615}

$$-\frac{d\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}-\frac{d\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}+\frac{2d\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{2e^2\left(\frac{d}{x^2}+e\right)}+\frac{x^2\left(a+b\csc^{-1}(cx)\right)}{2e^2}-\frac{bd\text{ArcTan}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\,x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{5/2}\sqrt{c^2d+e}}+\frac{ibd\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}+\frac{ibd\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}+\frac{ibd\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}+\frac{ibd\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2ce^2}-\frac{ibd\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (b*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c*e^2) + (d*(a + b*ArcCsc[c*x]))/(2*e^2*(e +
 d/x^2)) + (x^2*(a + b*ArcCsc[c*x]))/(2*e^2) - (b*d*ArcTan[Sqrt[c^2*d + e]/
(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d + e]) - (d*(a +
 b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^
2*d + e])])/e^3 - (d*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[
c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcCsc[c*x])*Log[1 - (
I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (d*(a +
```

```
 b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^
2*d + e])])/e^3 + (2*d*(a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/
e^3 + (I*b*d*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt
[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sq
rt[e] - Sqrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*Ar
cCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, (I*c*Sqrt
[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (I*b*d*PolyLog[
2, E^((2*I)*ArcCsc[c*x])])/e^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4813

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x]
 - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x^3\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e^2x^3} - \frac{2d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e^3x} + \frac{d^2x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2} + \right.\right.$$

$$= \frac{(2d)\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3} -$$

$$= \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2} + \frac{(2d)\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \right.}{e^3}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2} - \frac{id(a+b\csc^{-1}(cx}{be^3}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2} - \frac{id(a+b\csc^{-1}(cx}{be^3}$$

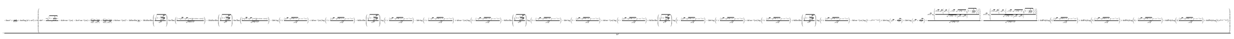
$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{}}{c\sqrt{e}}\right.}{2e^{5/2}\sqrt{c}}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{}}{c\sqrt{e}}\right.}{2e^{5/2}\sqrt{c}}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{}}{c\sqrt{e}}\right.}{2e^{5/2}\sqrt{c}}$$

$$= \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{}}{c\sqrt{e}}\right.}{2e^{5/2}\sqrt{c}}$$

**Mathematica** [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1480 vs. $2(628) = 1256$.
time = 4.09, size = 1480, normalized size = 2.36



Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*(I*d*Pi
^2 - (2*e*Sqrt[1 - 1/(c^2*x^2)]*x)/c - (4*I)*d*Pi*ArcCsc[c*x] - 2*e*x^2*Arc
Csc[c*x] + (d^(3/2)*ArcCsc[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (d^(3/2)*ArcCsc[
c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (8*I)*d*ArcCsc[c*x]^2 - 2*d*ArcSin[1/(c*x)]
 - (16*I)*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)
*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I
)*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d]
+ Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*d*Pi*Log[1 + (
Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]
*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*d*A
rcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*d*Pi*Log[1 + (-Sqrt[e] + Sqrt[
c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 + (-Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*d*ArcSin[Sqrt[1 +
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*d*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Arc
Csc[c*x]))] - 2*d*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))] + 8*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[
2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*
d*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + 2*d*Pi*Log[Sqrt[e] - (I*Sqrt
[d])/x] + 2*d*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + (d*Sqrt[e]*Log[(2*Sqrt[d]*S
qrt[e]*(Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^
2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]])/Sqrt[-(c^2*d) - e]
+ (d*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(
c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqr
t[e]*x))]])/Sqrt[-(c^2*d) - e] + (4*I)*d*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*d*PolyLog[2, (-Sqrt[e] + Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*d*PolyLog[2, -((Sqrt[e] +
Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (4*I)*d*PolyLog[2, (Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*d*PolyLog[2,
E^((2*I)*ArcCsc[c*x])]))/e^3
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.61, size = 764, normalized size = 1.22

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\,c^{6}x^{2}}{2e^{2}}-\frac{a\,c^{6}d\ln\left(c^{2}e\,x^{2}+c^{2}d\right)}{e^{3}}-\frac{a\,c^{8}d^{2}}{2e^{3}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{b\,c^{8}d\,\mathrm{arccsc}(cx)\,x^{2}}{e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{b\,c^{8}\mathrm{arccsc}(cx)\,x^{4}}{2e\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{b\,c^{7}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,dx}{2e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{b\,c^{7}\sqrt{\frac{c^{2}x^{2}-\cdots}{c^{2}x^{2}}}}{2e\left(c^{2}e\,x^{2}+\cdots\right.}\cdots}{\cdots}$ |
| default | $\frac{\frac{a\,c^{6}x^{2}}{2e^{2}}-\frac{a\,c^{6}d\ln\left(c^{2}e\,x^{2}+c^{2}d\right)}{e^{3}}-\frac{a\,c^{8}d^{2}}{2e^{3}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{b\,c^{8}d\,\mathrm{arccsc}(cx)\,x^{2}}{e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{b\,c^{8}\mathrm{arccsc}(cx)\,x^{4}}{2e\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{b\,c^{7}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,dx}{2e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{b\,c^{7}\sqrt{\frac{c^{2}x^{2}-\cdots}{c^{2}x^{2}}}}{2e\left(c^{2}e\,x^{2}+\cdots\right.}\cdots}{\cdots}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(1/2*a*c^6*x^2/e^2-a*c^6*d/e^3*ln(c^2*e*x^2+c^2*d)-1/2*a*c^8*d^2/e^3/
(c^2*e*x^2+c^2*d)+b*c^8/e^2/(c^2*e*x^2+c^2*d)*d*arccsc(c*x)*x^2+1/2*b*c^8/e
/(c^2*e*x^2+c^2*d)*arccsc(c*x)*x^4+1/2*b*c^7/e^2/(c^2*e*x^2+c^2*d)*((c^2*x^
2-1)/c^2/x^2)^(1/2)*d*x+1/2*b*c^7/e/(c^2*e*x^2+c^2*d)*((c^2*x^2-1)/c^2/x^2)
^(1/2)*x^3-1/2*I*b*c^6/e^2/(c^2*e*x^2+c^2*d)*d-1/2*I*b*c^6/e/(c^2*e*x^2+c^2
*d)*x^2+1/2*I*b*c^6*(e*(c^2*d+e))^(1/2)/e^3/(c^2*d+e)*arctanh(1/4*(2*d*c^2*
(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+e^2)^(1/2))*d+1/2*I*b*c
^6*d/e^3*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)
*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/
2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+2*I*b*c^6*d/e^3
*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))+2*b*c^6*d/e^3*arccsc(c*x)*ln(1+I/c/x+(1-1
/c^2/x^2)^(1/2))-2*I*b*c^6*d/e^3*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))+1/2*I*b
*c^8*d^2/e^3*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I
/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),
_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d)))
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x^2*e^(-2) - 2*d*e^(-3)*log(x^2*e + d) - d^2/(x^2*e^4 + d*e^3))*a + b*
integrate(x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(x^4*e^2 + 2*d*x^2*e
+ d^2), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arccsc(c*x) + a*x^5)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a + b\operatorname{acsc}(cx))}{(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**5*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

# 3.104 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$

**Optimal**. Leaf size=593

$$\frac{-a-b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)}+\frac{b\text{ArcTan}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}}+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{(a+b\,\mathrm{c}}{}$$

[Out] 1/2*(-a-b*arccsc(c*x))/e/(e+d/x^2)-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e^2+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e^2-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/e^(3/2)/(c^2*d+e)^(1/2)

**Rubi** [A]

time = 1.06, antiderivative size = 590, normalized size of antiderivative = 0.99, number of steps used = 29, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5349, 4817, 4721, 3798, 2221, 2317, 2438, 4813, 385, 211, 4825, 4615}

$$\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}-\frac{a+b\csc^{-1}(cx)}{2e\left(\frac{d}{x^2}+e\right)}-\frac{\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{e^2}+\frac{b\text{ArcTan}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\,x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{3/2}\sqrt{c^2d+e}}-\frac{ib\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2}+\frac{ib\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)}{2e^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]

[Out] -1/2*(a + b*ArcCsc[c*x])/(e*(e + d/x^2)) + (b*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(3/2)*Sqrt[c^2*d + e]) + ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/e^

```
2 - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt
[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(
Sqrt[e] - Sqrt[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(
I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, (I*
c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^2 + ((I/2)*b*
PolyLog[2, E^((2*I)*ArcCsc[c*x])])/e^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
```

```
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4813

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x]
 - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x\,(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e^2x} - \frac{dx\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e\,(e+dx^2)^2} - \frac{dx\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e^2\,(e+dx^2)}\right) dx$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)}{e^2} + \frac{d\text{Subst}\left(\int \left(-\right.\right.}{}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be^2} + \frac{(2i)\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1-e^{2ix}}\,dx, x, \csc^{-1}(cx)\right)}{e^2}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2be^2} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}} - \frac{(a+}{}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}} - \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\right.}{e^2}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\right.}{2e^2}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\right.}{2e^2}$$

$$= -\frac{a+b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{3/2}\sqrt{c^2d+e}} + \frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\right.}{2e^2}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1442 vs. 2(593) = 1186.
time = 1.53, size = 1442, normalized size = 2.43

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (I*b*Pi^2 + (4*a*d)/(d + e*x^2) - (4*I)*b*Pi*ArcCsc[c*x] + (2*b*Sqrt[d]*Arc
Csc[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (2*b*Sqrt[d]*ArcCsc[c*x])/(Sqrt[d] + I*
Sqrt[e]*x) + (8*I)*b*ArcCsc[c*x]^2 - 4*b*ArcSin[1/(c*x)] - (16*I)*b*ArcSin[
Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e
])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I)*b*ArcSin[Sqrt[1 +
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi
+ 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*b*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] -
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc
[c*x]))] - 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*P
i*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*
ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]
))] + 8*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e
] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcCsc[c*x]*Log[1
 - E^((2*I)*ArcCsc[c*x])] + 2*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 2*b*Pi*Lo
g[Sqrt[e] + (I*Sqrt[d])/x] + (2*b*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] +
 c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(
c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + (2*b*Sqrt[e]*Lo
g[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqr
t[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt
[-(c^2*d) - e] + 4*a*Log[d + e*x^2] + (4*I)*b*PolyLog[2, (Sqrt[e] - Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2, (-Sqrt[e] + S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyLog[2, -((Sqrt
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + (4*I)*b*PolyLog[2,
 (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*b*PolyL
og[2, E^((2*I)*ArcCsc[c*x])])/(8*e^2)
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.03, size = 563, normalized size = 0.95

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a c^6 d}{2e^2(c^2 e x^2+c^2 d)}+\frac{a c^4 \ln(c^2 e x^2+c^2 d)}{2e^2}-\frac{b c^6 x^2 \operatorname{arccsc}(cx)}{2e(c^2 e x^2+c^2 d)}-\frac{i b c^4 \sqrt{e(c^2 d+e)}\, \operatorname{arctanh}\left(\frac{2 d c^2\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2 x^2}}\right)^2-2}{4\sqrt{c^2 d e+e^2}}\right)}{2e^2(c^2 d+e)}}{}$ |
| default | $\frac{\frac{a c^6 d}{2e^2(c^2 e x^2+c^2 d)}+\frac{a c^4 \ln(c^2 e x^2+c^2 d)}{2e^2}-\frac{b c^6 x^2 \operatorname{arccsc}(cx)}{2e(c^2 e x^2+c^2 d)}-\frac{i b c^4 \sqrt{e(c^2 d+e)}\, \operatorname{arctanh}\left(\frac{2 d c^2\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2 x^2}}\right)^2-2}{4\sqrt{c^2 d e+e^2}}\right)}{2e^2(c^2 d+e)}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/2*a*c^6*d/e^2/(c^2*e*x^2+c^2*d)+1/2*a*c^4/e^2*ln(c^2*e*x^2+c^2*d)-
1/2*b*c^6*x^2*arccsc(c*x)/e/(c^2*e*x^2+c^2*d)-1/2*I*b*c^4*(e*(c^2*d+e))^(1/
2)/e^2/(c^2*d+e)*arctanh(1/4*(2*d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d
-4*e)/(c^2*d*e+e^2)^(1/2))-b*c^4/e^2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(
1/2))+I*b*c^4/e^2*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-I*b*c^4/e^2*dilog(I/c/
x+(1-1/c^2/x^2)^(1/2))-1/4*I*b*c^4/e^2*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c
^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilo
g((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e
)*_Z^2+c^2*d))-1/4*I*b*c^6/e^2*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arc
csc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/
x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))*d)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e^(-2)*log(x^2*e + d) + d/(x^2*e^3 + d*e^2))*a + b*integrate(x^3*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*x^3*arccsc(c*x) + a*x^3)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
[Out] Timed out
```

**Giac** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] Timed out
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

## 3.105 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

**Optimal**. Leaf size=134

$$\frac{-a-b\csc^{-1}(cx)}{2e\left(d+ex^2\right)}-\frac{bcx\text{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{2de\sqrt{c^2x^2}}+\frac{bcx\text{ArcTan}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}\sqrt{c^2x^2}}$$

```
[Out] 1/2*(-a-b*arccsc(c*x))/e/(e*x^2+d)-1/2*b*c*x*arctan((c^2*x^2-1)^(1/2))/d/e/
(c^2*x^2)^(1/2)+1/2*b*c*x*arctan(e^(1/2)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2))
/d/e^(1/2)/(c^2*d+e)^(1/2)/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.09, antiderivative size = 131, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$,
Rules used = {5345, 457, 88, 65, 211}

$$-\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)}-\frac{bcx\text{ArcTan}\left(\sqrt{c^2x^2-1}\right)}{2de\sqrt{c^2x^2}}+\frac{bcx\text{ArcTan}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2x^2}\sqrt{c^2d+e}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a + b*ArcCsc[c*x])/(e*(d + e*x^2)) - (b*c*x*ArcTan[Sqrt[-1 + c^2*x^2]
])/(2*d*e*Sqrt[c^2*x^2]) + (b*c*x*ArcTan[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sqrt[
c^2*d + e]])/(2*d*Sqrt[e]*Sqrt[c^2*d + e]*Sqrt[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx &= -\frac{a+b\csc^{-1}(cx)}{2e\,(d+ex^2)} - \frac{(bcx)\int \frac{1}{x\sqrt{-1+c^2x^2}\,(d+ex^2)}\,dx}{2e\sqrt{c^2x^2}} \\
&= -\frac{a+b\csc^{-1}(cx)}{2e\,(d+ex^2)} - \frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\,(d+ex)}\,dx, x, x^2\right)}{4e\sqrt{c^2x^2}} \\
&= -\frac{a+b\csc^{-1}(cx)}{2e\,(d+ex^2)} + \frac{(bcx)\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\,(d+ex)}\,dx, x, x^2\right)}{4d\sqrt{c^2x^2}} - \frac{(bcx)\text{Subst}\left(\right.}{} \\
&= -\frac{a+b\csc^{-1}(cx)}{2e\,(d+ex^2)} + \frac{(bx)\text{Subst}\left(\int \frac{1}{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}\,dx, x, \sqrt{-1+c^2x^2}\right)}{2cd\sqrt{c^2x^2}} - \frac{(bx)\text{Subst}\left(\right.}{} \\
&= -\frac{a+b\csc^{-1}(cx)}{2e\,(d+ex^2)} - \frac{bcx\tan^{-1}\left(\sqrt{-1+c^2x^2}\right)}{2de\sqrt{c^2x^2}} + \frac{bcx\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}\sqrt{c^2x^2}}
\end{aligned}
$$

**Mathematica [C]** Result contains complex when optimal does not.
time = 0.53, size = 286, normalized size = 2.13

$$
-\frac{\frac{2a}{d+ex^2}+\frac{2b\csc^{-1}(cx)}{d+ex^2}-\frac{2b\text{ArcSin}\left(\frac{1}{cx}\right)}{d}+\frac{b\sqrt{e}\log\left(\frac{4ide-4cd\sqrt{e}\left(c\sqrt{d}+i\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x}{b\sqrt{-c^2d-e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{d\sqrt{-c^2d-e}}+\frac{b\sqrt{e}\log\left(\frac{4i\left(-de+cd\sqrt{e}\left(ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b\sqrt{-c^2d-e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d\sqrt{-c^2d-e}}}{4e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*((2*a)/(d + e*x^2) + (2*b*ArcCsc[c*x])/(d + e*x^2) - (2*b*ArcSin[1/(c*
x)])/d + (b*Sqrt[e]*Log[((4*I)*d*e - 4*c*d*Sqrt[e]*(c*Sqrt[d] + I*Sqrt[-(c^
2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)/(b*Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt
[e]*x))])/(d*Sqrt[-(c^2*d) - e]) + (b*Sqrt[e]*Log[((4*I)*(-(d*e) + c*d*Sqrt
[e]*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*Sqrt[-(
c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/(d*Sqrt[-(c^2*d) - e]))/e
```

**Maple [B]** Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(112) = 224$.
time = 7.34, size = 350, normalized size = 2.61

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{a c^4}{2e\left(c^2 e x^2+c^2 d\right)}-\frac{b c^4 \operatorname{arccsc}(cx)}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{bc\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{2e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,xd}-\frac{bc\sqrt{c^2x^2-1}\,\ln\left(-\frac{2\left(-\sqrt{-\frac{c^2d+}{e}}\right.}{}\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{}$ |
| default | $\frac{-\frac{a c^4}{2e\left(c^2 e x^2+c^2 d\right)}-\frac{b c^4 \operatorname{arccsc}(cx)}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{bc\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{2e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,xd}-\frac{bc\sqrt{c^2x^2-1}\,\ln\left(-\frac{2\left(-\sqrt{-\frac{c^2d+}{e}}\right.}{}\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}}}{}$ |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/2*a*c^4/e/(c^2*e*x^2+c^2*d)-1/2*b*c^4/e/(c^2*e*x^2+c^2*d)*arccsc(
c*x)+1/2*b*c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d*arctan(1/(
c^2*x^2-1)^(1/2))-1/4*b*c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
/d/(-(c^2*d+e)/e)^(1/2)*ln(-2*(-(-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-
c^2*d*e)^(1/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1/2)))-1/4*b*c/e*(c^2*x^2-1)^(1/2)
/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d+e)/e)^(1/2)*ln(-2*((-(c^2*d+e)/e)
^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-e*c*x+(-c^2*d*e)^(1/2)
)))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(c^2*x^2*e^2 + c^2*d*e)*integrate(1/2*x*e^(1/2*log(c*x + 1) + 1/2*l
og(c*x - 1))/(c^2*x^4*e^2 + (c^2*d*e - e^2)*x^2 - d*e + (c^2*x^4*e^2 + (c^2
*d*e - e^2)*x^2 - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) + arctan2(1, sq
rt(c*x + 1)*sqrt(c*x - 1)))*b/(x^2*e^2 + d*e) - 1/2*a/(x^2*e^2 + d*e)
```

**Fricas [A]**
time = 0.41, size = 395, normalized size = 2.95

$$\left[-\frac{2\,ac^2d^2+2\,ade+\sqrt{-c^2de-e^2}\,(bx^2e+bd)\log\left(-\frac{c^2d-(c^2x^2-2)e+2\,\sqrt{c^2x^2-1}\,\sqrt{-c^2de-e^2}}{x^2e+d}\right)+2\,(bc^2d^2+bde)\,\mathrm{arccsc}\,(cx)+4\,(bc^2d^2+bx^2e^2+(bc^2dx^2+bd)e)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)}{4\,(c^2d^3e+dx^2e^3+(c^2d^2x^2+d^2)e^2)},\ -\frac{ac^2d^2+ade-\sqrt{c^2de+e^2}\,(bx^2e+bd)\arctan\left(\frac{\sqrt{c^2x^2-1}\,\sqrt{c^2de+e^2}}{c^2d+e}\right)+(bc^2d^2+bde)\,\mathrm{arccsc}\,(cx)+2\,(bc^2d^2+bx^2e^2+(bc^2dx^2+bd)e)\arctan\left(-cx+\sqrt{c^2x^2-1}\right)}{2\,(c^2d^3e+dx^2e^3+(c^2d^2x^2+d^2)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 + 2*a*d*e + sqrt(-c^2*d*e - e^2)*(b*x^2*e + b*d)*log(-(c
^2*d - (c^2*x^2 - 2)*e + 2*sqrt(c^2*x^2 - 1)*sqrt(-c^2*d*e - e^2))/(x^2*e +
 d)) + 2*(b*c^2*d^2 + b*d*e)*arccsc(c*x) + 4*(b*c^2*d^2 + b*x^2*e^2 + (b*c^
2*d*x^2 + b*d)*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d*x^2*e^3
+ (c^2*d^2*x^2 + d^2)*e^2), -1/2*(a*c^2*d^2 + a*d*e - sqrt(c^2*d*e + e^2)*(
b*x^2*e + b*d)*arctan(sqrt(c^2*x^2 - 1)*sqrt(c^2*d*e + e^2)/(c^2*d + e)) +
(b*c^2*d^2 + b*d*e)*arccsc(c*x) + 2*(b*c^2*d^2 + b*x^2*e^2 + (b*c^2*d*x^2 +
 b*d)*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d*x^2*e^3 + (c^2*d^
2*x^2 + d^2)*e^2)]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\mathrm{acsc}\,(cx))}{(d+ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

**Giac [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
```

```
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

# 3.106 $\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$

**Optimal**. Leaf size=566

$$-\frac{e(a+b\csc^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{i(a+b\csc^{-1}(cx))^2}{2bd^2}+\frac{b\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}}-\frac{(a+b\csc^{-1}(cx))\log\left(1-\right.}{2d^2}$$

```
[Out] -1/2*e*(a+b*arccsc(c*x))/d^2/(e+d/x^2)+1/2*I*(a+b*arccsc(c*x))^2/b/d^2-1/2*
(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(
1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsc(c*x))*ln(1-
I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^2-1
/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))
*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,I*c*(I/c/x+(1-
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog
(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
^2+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))/d^2+1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^
(1/2))*e^(1/2)/d^2/(c^2*d+e)^(1/2)
```

**Rubi** [A]
time = 1.03, antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5349, 4817, 4813, 385, 211, 4825, 4615, 2221, 2317, 2438}

$$-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}-\frac{e(a+b\csc^{-1}(cx))}{2d^2\left(\frac{d}{x^2}+e\right)}+\frac{i(a+b\csc^{-1}(cx))^2}{2bd^2}+\frac{b\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\,x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\sqrt{c^2d+e}}+\frac{ib\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] -1/2*(e*(a + b*ArcCsc[c*x]))/(d^2*(e + d/x^2)) + ((I/2)*(a + b*ArcCsc[c*x])
^2)/(b*d^2) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*
x^2)]*x)])/(2*d^2*Sqrt[c^2*d + e]) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt
[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*Arc
Csc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
e])])/(2*d^2) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]
))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcCsc[c*x])*Log[1 + (I*
c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) + ((I/2
)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
```

```
e])])/d^2 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] -
 Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[
c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d
]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^2
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4813

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x]
```

```
- Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{x^3\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{ex\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2} + \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{\sqrt{-d}\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right) dx, x\ldots}{d}$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx\ldots}{2(-d)^{3/2}}$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\cos(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\,\text{si}\ldots}\ldots\right)}{2(-d\ldots}$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}} - \ldots$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}} - \ldots$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}} - \ldots$$

$$= -\frac{e\left(a+b\csc^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d+e}} - \ldots$$

**Mathematica** [F]
time = 40.44, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^2),x]

[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^2), x]

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.96, size = 3036, normalized size = 5.36

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 3036 |
| default | Expression too large to display | 3036 |

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)

[Out] I*b/c^2*arccsc(c*x)^2/d^3*(e*(c^2*d+e))^(1/2)-1/4*b*c^2*(e*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*arccsc(c*x)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))+3*b/c^2*e*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^3/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-1/2*I*b*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arctanh(1/4*(2*d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+e^2)^(1/2))-I*b*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*arccsc(c*x)^2+1/4*I*b*(e*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))-5/2*I*b*arccsc(c*x)^2/d^2/(c^2*d+e)*e-3/4*I*b*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^2/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-5/4*I*b*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^2/(c^2*d+e)*e-1/2*I*b*c^2*arccsc(c*x)^2/d/(c^2*d+e)-1/4*I*b*c^2*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d/(c^2*d+e)+2*I*b/c^2*arccsc(c*x)^2*e/d^3+2*I*b/c^4*arccsc(c*x)^2*e^2/d^4+1/2*I*b/c^2*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^3*(e*(c^2*d+e))^(1/2)+3/2*b*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^2/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-2*I*b/c^4*e^2*arccsc(c*x)^2/d^4/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-1/8*I*b*c^2*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d/e/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-3/2*I*b/c^2*e*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^3/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+2*b/c^4*e^2*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^4/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+1/4*b*c^2*ln(1-d*c^2

```
*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*
x)/d/e/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+1/8*I*b*c^2*(e*(c^2*d+e))^(1/2)/d/e/(c
^2*d+e)*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e)
)^(1/2)+2*e))-I*b/c^4*e^2*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^4/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-3*I*b/c^2
*e*arccsc(c*x)^2/d^3/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-I*b/c^4*e^3*polylog(2,d*
c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/d^4/(c
^2*d+e)-2*I*b/c^2*e^2*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-
2*(e*(c^2*d+e))^(1/2)+2*e))/d^3/(c^2*d+e)+I*b/c^4*polylog(2,d*c^2*(I/c/x+(1
-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e/d^4*(e*(c^2*d+e))
^(1/2)+4*b/c^2*e^2*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^
2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^3/(c^2*d+e)+2*b/c^4*e^3*ln(1-d*c^2*(I/c/x
+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^4/
(c^2*d+e)-2*b/c^4*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2
*d+e))^(1/2)+2*e))*e*arccsc(c*x)/d^4*(e*(c^2*d+e))^(1/2)+2*I*b/c^4*arccsc(c
*x)^2*e/d^4*(e*(c^2*d+e))^(1/2)-4*I*b/c^2*e^2*arccsc(c*x)^2/d^3/(c^2*d+e)-1
/2*a/d^2*ln(c^2*e*x^2+c^2*d)+a/d^2*ln(c*x)-1/2*b*(e*(c^2*d+e))^(1/2)/d^2/(c
^2*d+e)*arccsc(c*x)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c
^2*d+e))^(1/2)+2*e))+5/2*b*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-
2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^2*e/(c^2*d+e)+1/2*b*c^2*ln(1-d*c^
2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c
*x)/d/(c^2*d+e)-b/c^2*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*
(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^3*(e*(c^2*d+e))^(1/2)-2*b/c^2*ln(1-d*c
^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(
c*x)/d^3*e-2*b/c^4*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^
2*d+e))^(1/2)+2*e))*e^2*arccsc(c*x)/d^4+I*b/c^2*polylog(2,d*c^2*(I/c/x+(1-1
/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e/d^3+I*b/c^4*polylog
(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*e
^2/d^4-2*I*b/c^4*e^3*arccsc(c*x)^2/d^4/(c^2*d+e)-1/2*b*c^2*x^2*e*arccsc(c*x
)/d^2/(c^2*e*x^2+c^2*d)+1/2*a*c^2/d/(c^2*e*x^2+c^2*d)+I*b*arccsc(c*x)^2/d^2
-1/2*b*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2
)+2*e))*arccsc(c*x)/d^2+1/2*I*b/d^2*sum((_R1^2*c^2*d-2*c^2*d-4*e)/(_R1^2*c^
2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog
((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)
*_Z^2+c^2*d))+1/4*I*b*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-
2*(e*(c^2*d+e))^(1/2)+2*e))/d^2
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(d*x^2*e + d^2) - log(x^2*e + d)/d^2 + 2*log(x)/d^2) + b*integrate
(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(x^5*e^2 + 2*d*x^3*e + d^2*x), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(x^5*e^2 + 2*d*x^3*e + d^2*x), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

**Giac [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

# 3.107 $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$

**Optimal**. Leaf size=803

$$-\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\,\sqrt{e}\,-\frac{d}{x}\right)}+\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\,\sqrt{e}\,+\frac{d}{x}\right)}+\frac{x(a+b\csc^{-1}(cx))}{e^2}+\frac{b\tanh^{-1}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2}+\frac{b\sqrt{d}\,\tanh^{-1}}{}$$

```
[Out] x*(a+b*arccsc(c*x))/e^2+b*arctanh((1-1/c^2/x^2)^(1/2))/c/e^2-3/4*(a+b*arccs
c(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^
(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+
b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^
2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-
3/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*I*b*po
lylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))*(-d)^(1/2)/e^(5/2)+3/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-1/4*d*(a+b*arccsc(c*
x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*d*(a+b*arccsc(c*x))/e^2/(d/x+(-d)^(1/
2)*e^(1/2))+1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^
(1/2)/(1-1/c^2/x^2)^(1/2))*d^(1/2)/e^2/(c^2*d+e)^(1/2)+1/4*b*arctanh((c^2*d
+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))*d^(1/
2)/e^2/(c^2*d+e)^(1/2)
```

**Rubi** [A]

time = 2.27, antiderivative size = 803, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5349, 4817, 4723, 272, 65, 214, 4757, 4827, 739, 212, 4825, 4615, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(d*(a + b*ArcCsc[c*x]))/(e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*Arc
Csc[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcCsc[c*x]))/e^2 +
 (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e^2) + (b*Sqrt[d]*ArcTanh[(c^2*d - (
```

```
Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4
*e^2*Sqrt[c^2*d + e]) + (b*Sqrt[d]*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(
c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*e^2*Sqrt[c^2*d + e])
- (3*Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/
(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCsc[c*x])
*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*
e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCs
c[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*Arc
Csc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d +
e])])/(4*e^(5/2)) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*
ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4)*b*Sqrt[-d]
*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/
e^(5/2) - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x
]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4)*b*Sqrt[-d]*PolyLog[2
, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4723

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(d*(m + 1))), x] - Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
```

```
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e^2x^2} - \frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right) dx, x$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{a}{}\right.}{}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{e^2} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce^2} + \frac{d\text{Subst}\left(\int \left(\frac{a+b\sin}{2\sqrt{e}\left(\sqrt{e}\right.}\right.\right.}{}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}} + \frac{d\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}}\right.}{2e^{5/2}}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\csc^{-1}(cx))}{e^2} + \frac{d\text{Subst}}{}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\csc^{-1}(cx))}{e^2} + \frac{b\tanh^{-}}{}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\csc^{-1}(cx))}{e^2} + \frac{b\tanh^{-}}{}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\csc^{-1}(cx))}{e^2} + \frac{b\tanh^{-}}{}$$

$$= -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\csc^{-1}(cx))}{e^2} + \frac{b\tanh^{-}}{}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1634 vs. 2(803) = 1606.
time = 6.04, size = 1634, normalized size = 2.03

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a*x)/e^2 + (a*d*x)/(2*e^2*(d + e*x^2)) - (3*a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(2*e^(5/2)) + b*(-1/4*(d*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*
x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*((-
I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d)
 - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^2 - (d*(-(
ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[
(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[
1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(
c^2*d) - e]))/Sqrt[d]))/(4*e^2) + (3*Sqrt[d]*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*A
rcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[
(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] +
 (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))
] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*A
rcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Lo
g[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi
*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)
*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sq
rt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]
*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 8
*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8
*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] +
 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(32*e^(5/2)) - (3*Sqrt[d]*(Pi^2 - 4*
Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/S
qrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e]
- (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
```

```
*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(32*e^(5/2)) + ((Arc
Csc[c*x]*Cot[ArcCsc[c*x]/2])/2 + Log[Cos[ArcCsc[c*x]/2]] - Log[Sin[ArcCsc[c
*x]/2]] + (ArcCsc[c*x]*Tan[ArcCsc[c*x]/2])/2)/(c*e^2))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 12.79, size = 1888, normalized size = 2.35

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] a/e^2*x+1/2*c^2*a/e^2*d*x/(c^2*e*x^2+c^2*d)-3/2*a/e^2*d/(d*e)^(1/2)*arctan(
e*x/(d*e)^(1/2))+c^2*b*x^3*arccsc(c*x)/e/(c^2*e*x^2+c^2*d)+3/2*c^2*b*x*arcc
sc(c*x)/e^2/(c^2*e*x^2+c^2*d)*d+1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e
)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2))/e^2/d-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1
/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2))/e^2/d^2*(e*(c^2*d+e))^(1/2)+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2))/e/d^2+1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+
2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2*d+e)/d*(e*(c^2*d+e))^(1/2)-1/c^2*b*((c^2*d
+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*
d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d+1/c^4*b*((c^2*
d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c
*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^2*(e*(c^2*d+e
))^(1/2)-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x
+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2
*d+e)/d^2+1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/
c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/
e^2/d+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1
-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e^2/d^
2*(e*(c^2*d+e))^(1/2)+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*
arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*
d)^(1/2))/e/d^2-1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arct
an((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(
1/2))/e^2/(c^2*d+e)/d*(e*(c^2*d+e))^(1/2)-1/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c
^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*
(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)-1/c^4*b
*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(
1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^2-1/c*b
```

```
/e^2*ln(I/c/x+(1-1/c^2/x^2)^(1/2)-1)+1/c*b/e^2*ln(1+I/c/x+(1-1/c^2/x^2)^(1/
2))-3/16*c*b/e^3*d*sum((_R1^2*c^2*d-c^2*d-4*e)/_R1/(_R1^2*c^2*d-c^2*d-2*e)*
(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-
1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+3
/16*c*b/e^3*d*sum((_R1^2*c^2*d+4*_R1^2*e-c^2*d)/_R1/(_R1^2*c^2*d-c^2*d-2*e)
*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1
-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2) - 2*x*e^(-2) - d*x/(x^2*
e^3 + d*e^2))*a + b*integrate(x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(
x^4*e^2 + 2*d*x^2*e + d^2), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsc(c*x) + a*x^4)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\,\mathrm{acsc}\,(cx))}{(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**4*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

# 3.108 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

**Optimal**. Leaf size=765

$$\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}-\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d+e}\,\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\,e\sqrt{c^2d+e}}-\frac{b\tanh^{-1}\left(\frac{c^2}{c\sqrt{d}\,\sqrt{c}}\right)}{4\sqrt{d}\,e}$$

```
[Out] -1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arccsc(c*x))*ln(1+I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d
)^(1/2)-1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arccsc(c*x))*ln(1
+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3
/2)/(-d)^(1/2)-1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*I*b*polylog(2,I*c*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(
1/2)-1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(
a+b*arccsc(c*x))/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(-a-b*arccsc(c*x))/e/(d/x+
(-d)^(1/2)*e^(1/2))-1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c
^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e/d^(1/2)/(c^2*d+e)^(1/2)-1/4*b*arctanh(
(c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))
/e/d^(1/2)/(c^2*d+e)^(1/2)
```

**Rubi** [A]
time = 1.13, antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5349, 4757, 4827, 739, 212, 4825, 4615, 2221, 2317, 2438}

$$-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}\,e^{3/2}}-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}-\frac{ib\,\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{ib\,\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}\,e^{3/2}}-\frac{ib\,\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{ib\,\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}\,e^{3/2}}-\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{c^2d+e}}\right)}{4\sqrt{d}\,e\sqrt{c^2d+e}}-\frac{b\tanh^{-1}\left(\frac{c^2d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{1-\frac{1}{c^2x^2}}\,\sqrt{c^2d+e}}\right)}{4\sqrt{d}\,e\sqrt{c^2d+e}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcCsc[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCsc[c*x])/(4
*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c
*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)]])/(4*Sqrt[d]*e*Sqrt[c^2*d +
 e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]
*Sqrt[1 - 1/(c^2*x^2)]])/(4*Sqrt[d]*e*Sqrt[c^2*d + e]) - ((a + b*ArcCsc[c*
```

```
x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/
(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcC
sc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcC
sc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e
])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I
*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((I/4)*
b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e]
)])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]
))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*PolyLog[2, (
(-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*
e^(3/2)) + ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] +
Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
```

```
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\,\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\sin^{-1}\right.}{2e\left(-de-\right.}\right.\right.$$

$$= \frac{d\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e} + \frac{d\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\,\sqrt{e}-dx\right)\sqrt{1-}}\right.}{4ce}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{4e^{3/2}}$$

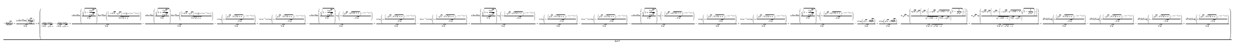
$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d+e}\,\sqrt{1-}}\right.}{4\sqrt{d}\,e\sqrt{c^2d+e}}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d+e}\,\sqrt{1-}}\right.}{4\sqrt{d}\,e\sqrt{c^2d+e}}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d+e}\,\sqrt{1-}}\right.}{4\sqrt{d}\,e\sqrt{c^2d+e}}$$

$$= \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d+e}\,\sqrt{1-}}\right.}{4\sqrt{d}\,e\sqrt{c^2d+e}}$$

**Mathematica [A]**
time = 1.83, size = 1482, normalized size = 1.94



Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 b*((2*ArcCsc[c*x])/(I*Sqrt[d] - Sqrt[e]*x) - (2*ArcCsc[c*x])/(I*Sqrt[d] +
Sqrt[e]*x) + (8*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((
(-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]])/Sq
rt[d] - (8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*S
qrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]])/Sqrt[d] -
(I*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/S
qrt[d] + ((2*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))])/Sqrt[d] + (I*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))])/Sqrt[d] - ((2*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d
 + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + ((4*I)*ArcSin[Sqrt[1 + (I*
Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + (I*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((2*I)*ArcCsc[c*x]*Log[1 - (Sqr
t[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((4*I)*Ar
cSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (I*Pi*Log[1 + (Sqrt[e] +
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + ((2*I)*ArcCsc[c
*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqr
t[d] + ((4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sq
rt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (I*Pi*Lo
g[Sqrt[e] - (I*Sqrt[d])/x])/Sqrt[d] + (I*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x])/S
qrt[d] - ((2*I)*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*((-I)*c*Sqrt[d]
 - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[
d] + I*Sqrt[e]*x))])/(Sqrt[d]*Sqrt[-(c^2*d) - e]) + ((2*I)*Sqrt[e]*Log[(2*S
qrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 -
1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/(Sqrt[d]*S
qrt[-(c^2*d) - e]) + (2*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))])/Sqrt[d] - (2*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (2*PolyLog[2, -((Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))])/Sqrt[d] + (2*PolyLog[2, (Sqrt[e]
+ Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d]))/(8*e^(3/2))
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.82, size = 1722, normalized size = 2.25

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e*x/(c^2*e*x^2+c^2*d)+1/2*a/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)
)-1/2*c^2*b*x*arccsc(c*x)/(c^2*e*x^2+c^2*d)/e-1/2/c^2*b*((c^2*d+2*(e*(c^2*d
+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/d^2+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2
*d+e))^(1/2)+2*e)*d)^(1/2))/e/d^3*(e*(c^2*d+e))^(1/2)-1/c^4*b*((c^2*d+2*(e*
(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^
2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^3-1/2/c^2*b*((c^2*d+2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(
e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)+1/c^2
*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)
^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^2-1/c^
4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2
)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^3*(e*
(c^2*d+e))^(1/2)+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan
h((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/
2))*e/(c^2*d+e)/d^3-1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*
arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*
d)^(1/2))/e/d^2-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan
((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/
2))/e/d^3*(e*(c^2*d+e))^(1/2)-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d
)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/
2)-2*e)*d)^(1/2))/d^3+1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2
)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e
)*d)^(1/2))/e/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)+1/c^2*b*(-(c^2*d-2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2
*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^2+1/c^4*b*(-(c^2*d-2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d
+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)+1/c
^4*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^
2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))*e/(c^2*d+e)/d^3
-1/4*c*b/e*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(
1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Ro
otOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/4*c*b/e*sum(_R1/(_R1^2*c^2*d-
c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R
1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^
2+c^2*d))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/sqrt(d) - x/(x^2*e^2 + d*e))*a + b*
integrate(x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(x^4*e^2 + 2*d*x^2*e
+ d^2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{acsc}(cx))}{(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

**Giac [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2 + d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

# 3.109 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx$

**Optimal**. Leaf size=762

$$\frac{-a-b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}+\frac{b\tanh^{-1}\left(\frac{c^2\cdots}{c\sqrt{d}\sqrt{c^2\cdots}}\right.}{4d^{3/2}\sqrt{c\cdots}}$$

```
[Out] 1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/
c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e
^(1/2)+1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arccsc(c*x))*ln(1+
I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^
(3/2)/e^(1/2)+1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,I*c*(I/c/x
+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1
/2)+1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+
(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/
x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(-
a-b*arccsc(c*x))/d/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(a+b*arccsc(c*x))/d/(d/x+(
-d)^(1/2)*e^(1/2))+1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^
2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(1/2)+1/4*b*arctanh((c^
2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^
(3/2)/(c^2*d+e)^(1/2)
```

**Rubi** [A]
time = 2.09, antiderivative size = 759, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {5339, 4817, 4757, 4827, 739, 212, 4825, 4615, 2221, 2317, 2438}

$$\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4d^{3/2}\sqrt{c^2d+e}}+\frac{b\tanh^{-1}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d+e}}\right)}{4d^{3/2}\sqrt{c^2d+e}}+\frac{ib\text{Li}_2\left(-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\text{Li}_2\left(\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\text{Li}_2\left(-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\text{Li}_2\left(\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(a + b*ArcCsc[c*x])/(d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcCsc[c*x])
/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)
/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d
+ e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e
]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d + e]) + ((a + b*ArcCsc[c*x
```

```
])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(
4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*Arc
Csc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*A
rcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d
+ e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]
*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) +
((I/4)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*Ar
cCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e]) + ((I/4)*b*P
olyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])
/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x])
)/(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
```

```
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5339

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d\,(e+dx^2)^2} + \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{d\,(e+dx^2)}\right)\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \right.}{}$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)\right) - \frac{1}{4}\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}+dx\right)}\right.$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a+b\sin^{-1}\left(\right.}{2d\sqrt{e}\left(\sqrt{e}-\sqrt{\right.}}\right.\right.$$

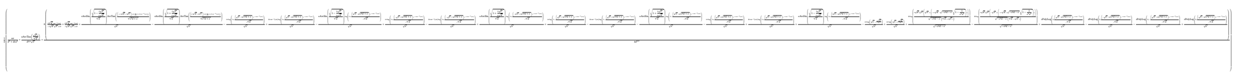
$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{de}{c^2}-x^2}\,dx, x, \frac{-d+\frac{\sqrt{-}}{}}{\sqrt{1-}}\right.}{4cd}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d+e}\,\sqrt{1-}}\right.}{4d^{3/2}\sqrt{c^2d+e}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d+e}\,\sqrt{1-}}\right.}{4d^{3/2}\sqrt{c^2d+e}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d+e}\,\sqrt{1-}}\right.}{4d^{3/2}\sqrt{c^2d+e}}$$

**Mathematica** [**A**]
time = 1.91, size = 1477, normalized size = 1.94



Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*((2*Sqrt[d]*ArcCsc[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) + (2*Sqrt[d]*Arc
Csc[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + (8*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])
/4])/Sqrt[c^2*d + e]])/Sqrt[e] - (8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqr
t[c^2*d + e]])/Sqrt[e] - (I*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + ((2*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] - ((4*I)*ArcSin[Sqrt[
1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + (I*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] - ((2*I)*ArcCsc[c*x]*Log[1
 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + (
(4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] +
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + (I*Pi*Log[1 - (
Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] - ((2*I)
*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))])/Sqrt[e] - ((4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Lo
g[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] -
 (I*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/
Sqrt[e] + ((2*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))])/Sqrt[e] + ((4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[
d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*
x]))])/Sqrt[e] - (I*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x])/Sqrt[e] + (I*Pi*Log[Sq
rt[e] + (I*Sqrt[d])/x])/Sqrt[e] + ((2*I)*Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] +
c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c
^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] - ((2*I)*Log[(2*Sq
rt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1
/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))])/Sqrt[-(c^2*
d) - e] + (2*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))])/Sqrt[e] - (2*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^
(I*ArcCsc[c*x]))])/Sqrt[e] - (2*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x])))])/Sqrt[e] + (2*PolyLog[2, (Sqrt[e] + Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e]))/(4*d^(3/2)))/2
```

**Maple** [**F(-1)**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(3/2) + x/(d*x^2*e + d^2)) + b*
integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(x^4*e^2 + 2*d*x^2*e + d^
2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{(d + ex^2)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

**Giac [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)}{\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^2, x)
```

# 3.110 $\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx$

**Optimal**. Leaf size=806

$$-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2}-\frac{a}{d^2x}-\frac{b\csc^{-1}(cx)}{d^2x}+\frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\,\sqrt{e}\,-\frac{d}{x}\right)}-\frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\,\sqrt{e}\,+\frac{d}{x}\right)}-\frac{be\tanh^{-1}\left(\frac{c^2d-}{c\sqrt{d}\,\sqrt{c^2d}}\right.}{4d^{5/2}\sqrt{c^2}}$$

```
[Out] -a/d^2/x-b*arccsc(c*x)/d^2/x+3/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*(
a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(
c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2
)-3/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*I*b*polylog(2,-I*c*(I/c/x+(1
-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)
-3/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*I*b*p
olylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))*e^(1/2)/(-d)^(5/2)+1/4*e*(a+b*arccsc(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2)
)-1/4*e*(a+b*arccsc(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))-1/4*b*e*arctanh((c^2
*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(
5/2)/(c^2*d+e)^(1/2)-1/4*b*e*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)
/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)-b*c*(1-1/c^2/
x^2)^(1/2)/d^2
```

**Rubi** [A]
time = 2.21, antiderivative size = 806, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5349, 4817, 4715, 267, 4757, 4827, 739, 212, 4825, 4615, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -((b*c*Sqrt[1 - 1/(c^2*x^2)])/d^2) - a/(d^2*x) - (b*ArcCsc[c*x])/(d^2*x) +
(e*(a + b*ArcCsc[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b*ArcCsc
[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*
Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(5/2)*
```

```
Sqrt[c^2*d + e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*S
qrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(5/2)*Sqrt[c^2*d + e]) + (3*Sq
rt[e]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCsc[c*x])*Log[1
 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(
5/2)) + (3*Sqrt[e]*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*
x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCs
c[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e]
)])/(4*(-d)^(5/2)) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*
ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(5/2) - (((3*I)/4)*b*Sqrt[
e]*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]
)/(-d)^(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)^(5/2) - (((3*I)/4)*b*Sqrt[e]*Pol
yLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)
^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4715

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSin[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
```

```
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{x^4\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{d^2} + \frac{e^2\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} - \frac{2e\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right)\,dx, x,$$

$$= -\frac{\text{Subst}\left(\int \left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\text{Sub}}{}$$

$$= -\frac{a}{d^2x} - \frac{b\text{Subst}\left(\int \sin^{-1}\left(\frac{x}{c}\right)\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{}{2\sqrt{}}\right.\right.}{d^2}$$

$$= -\frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd^2} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}}\right.}{d^2}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{e(a+b\csc^{-1}(cx}{4d^2\left(\sqrt{-d}\,\sqrt{e}+\right.}$$

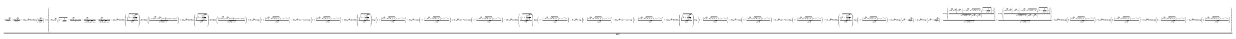
$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{e(a+b\csc^{-1}(cx}{4d^2\left(\sqrt{-d}\,\sqrt{e}+\right.}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{e(a+b\csc^{-1}(cx}{4d^2\left(\sqrt{-d}\,\sqrt{e}+\right.}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{e(a+b\csc^{-1}(cx}{4d^2\left(\sqrt{-d}\,\sqrt{e}+\right.}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\csc^{-1}(cx)}{d^2x} + \frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{e(a+b\csc^{-1}(cx}{4d^2\left(\sqrt{-d}\,\sqrt{e}+\right.}$$

**Mathematica** [A]
time = 1.76, size = 1525, normalized size = 1.89



Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]] + b*(-8*c*Sqrt[d]*Sqrt[1 - 1/(c^2*x^2)] - (8*Sqrt[d]*ArcC
sc[c*x])/x - (2*Sqrt[d]*e*ArcCsc[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) - (2*Sq
rt[d]*e*ArcCsc[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - 24*Sqrt[e]*ArcSin[Sqrt[1 -
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(
Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + 24*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*Ar
cCsc[c*x])/4])/Sqrt[c^2*d + e]] + (3*I)*Sqrt[e]*Pi*Log[1 + (Sqrt[e] - Sqrt[
c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (6*I)*Sqrt[e]*ArcCsc[c*x]*Log[
1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (12*I)*Sqr
t[e]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (3*I)*Sqrt[e]*Pi*Log[1 + (
-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (6*I)*Sqrt[e]*
ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] - (12*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log
[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (3*I)*Sq
rt[e]*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 + (6*I)*Sqrt[e]*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + (12*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] + (3*I)*Sqrt[e]*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] - (6*I)*Sqrt[e]*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (12*I)*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqr
t[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] + (3*I)*Sqrt[e]*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - (3*I)
*Sqrt[e]*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] - ((2*I)*e*Log[(2*Sqrt[d]*Sqrt[e]*
(Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)
)/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + ((2*I
)*e*Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) -
e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]
)/Sqrt[-(c^2*d) - e] - 6*Sqrt[e]*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] + 6*Sqrt[e]*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 6*Sqrt[e]*PolyLog[2, -((Sqrt[e] + Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])))] - 6*Sqrt[e]*PolyLog[2, (Sqrt[
e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]))/(8*d^(5/2))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 17.08, size = 1784, normalized size = 2.21

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x)
```

```
[Out] -1/2*a*e/d^2*x*c^2/(c^2*e*x^2+c^2*d)-3/2*a*e/d^2/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2))-a/d^2/x-b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arct
an((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(
1/2))/d^5*(e*(c^2*d+e))^(1/2)+b/c^2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^
(1/2)*e^2*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(
1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)+b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)
*d)^(1/2)*e^3*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e
))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)-1/2*b/c^2*((c^2*d+2*(e*(c^2*d+e))^(1/
2)+2*e)*d)^(1/2)*e*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4-b/c^4*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d
)^(1/2)*e^2*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))
^(1/2)+2*e)*d)^(1/2))/d^5-1/2*b/c^2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^
(1/2)*e*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/
2)-2*e)*d)^(1/2))/d^4-b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^
2*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e
)*d)^(1/2))/d^5-1/2*b*x*c^2*arccsc(c*x)*e/(c^2*e*x^2+c^2*d)/d^2+b/c^4*((c^2
*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2)
)*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5*(e*(c^2*d+e))^(1/2)+
b/c^2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctanh((I/c/x+(1-1/c
^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+
e)+b/c^4*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^3*arctanh((I/c/x+(1-
1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5/(c^2
*d+e)-1/2*b/c^2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arctanh((I/c/
x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4
/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-b/c^4*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^
(1/2)*e^2*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2))/d^5/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+1/2*b/c^2*(-(c^2*d-2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/
((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)*(e*(c^2*d+e))^(
1/2)+b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctan((I/c/x+(
1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(
c^2*d+e)*(e*(c^2*d+e))^(1/2)+3/4*c*b*e/d^2*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e
)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(
1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
+3/4*c*b*e/d^2*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x
-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=
RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-b*arccsc(c*x)/d^2/x-c*b/d^2*(
(c^2*x^2-1)/c^2/x^2)^(1/2)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((3*x^2*e + 2*d)/(d^2*x^3*e + d^3*x) + 3*arctan(x*e^(1/2)/sqrt(d))*e
^(1/2)/d^(5/2)) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(x^6*
e^2 + 2*d*x^4*e + d^2*x^2), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(x^6*e^2 + 2*d*x^4*e + d^2*x^2), x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acsc}(cx)}{x^2 (d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**2*(d + e*x**2)**2), x)
```

**Giac [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

# 3.111 $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

**Optimal**. Leaf size=727

$$\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x}-\frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{b\text{ArcTan}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{5/2}\sqrt{c^2d+e}}+\frac{b(c^2d+2e)\,\text{Ar}\cdots}{8e\cdots}$$

```
[Out] 1/4*(-a-b*arccsc(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arccsc(c*x))/e^2/(e+d/x^2)+1
/8*b*(c^2*d+2*e)*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/e^
(5/2)/(c^2*d+e)^(3/2)-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
/e^3+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsc(c*
x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))/e^3+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1
/2))^2)/e^3-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(
e^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,-I*c*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*I*b*po
lylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)
))/e^3+1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/e^(5/2
)/(c^2*d+e)^(1/2)+1/8*b*c*d*(1-1/c^2/x^2)^(1/2)/e^2/(c^2*d+e)/(e+d/x^2)/x
```

**Rubi** [A]
time = 1.22, antiderivative size = 727, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5349, 4817, 4721, 3798, 2221, 2317, 2438, 4813, 390, 385, 211, 4825, 4615}

$$\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}-\frac{a+b\csc^{-1}(cx)}{2e^2\left(\frac{d}{x^2}+e\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(\frac{d}{x^2}+e\right)^2}-\frac{\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{e^3}+\frac{b(c^2d+2e)\,\text{ArcTan}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\,x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{8e^{5/2}\left(c^2d+e\right)^{3/2}}+\frac{b\,\text{ArcTan}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\,x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{5/2}\sqrt{c^2d+e}}-\frac{ib\,\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\,\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\,\text{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\,\text{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3}+\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2x\left(c^2d+e\right)\left(\frac{d}{x^2}+e\right)}+\frac{ib\,\text{Li}_2\left(e^{2i\csc^{-1}(cx)}\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*d*Sqrt[1 - 1/(c^2*x^2)])/(8*e^2*(c^2*d + e)*(e + d/x^2)*x) - (a + b*Ar
cCsc[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcCsc[c*x])/(2*e^2*(e + d/x^2)) +
(b*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(5/2)*
Sqrt[c^2*d + e]) + (b*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[
1 - 1/(c^2*x^2)]*x)])/(8*e^(5/2)*(c^2*d + e)^(3/2)) + ((a + b*ArcCsc[c*x])*
Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e
```

```
^3) + ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e
] - Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]
*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcCsc[
c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])]
)/(2*e^3) - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])])/e^3 - ((I/
2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
 e])])/e^3 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
- Sqrt[c^2*d + e])])/e^3 - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-
d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + ((I/2)*b*PolyLog[
2, E^((2*I)*ArcCsc[c*x])])/e^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4721

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 4813

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x]
 - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x\,(e+dx^2)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e^3x} - \frac{dx\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e\,(e+dx^2)^3} - \frac{dx\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e^2\,(e+dx^2)^2} - \frac{d\text{:}\dots}{}\right.\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\text{Subst}\left(\dots\right.}{}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int (a+bx)\cot(x)\,dx, x, \csc^{-1}(cx)\right)}{e^3}$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\,(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\csc^{-1}(cx)}{2be^3}$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\,(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\csc^{-1}(cx)}{2be^3}$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\,(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c}}{c\sqrt{e}\sqrt{}}\right.}{2e^{5/2}\sqrt{c^2}}$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\,(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c}}{c\sqrt{e}\sqrt{}}\right.}{2e^{5/2}\sqrt{c^2}}$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\,(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c}}{c\sqrt{e}\sqrt{}}\right.}{2e^{5/2}\sqrt{c^2}}$$

$$= \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2\,(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c}}{c\sqrt{e}\sqrt{}}\right.}{2e^{5/2}\sqrt{c^2}}$$

**Mathematica** [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2053 vs. $2(727) = 1454$.
time = 7.20, size = 2053, normalized size = 2.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x
^2])/(2*e^3) + b*((((7*I)/16)*Sqrt[d]*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*Sqrt[e]
+ e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c
*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^
2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2)
- (((7*I)/16)*Sqrt[d]*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSi
n[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] +
 Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d]
 - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2) - (d*((I*c*Sqrt[e]
*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))
- ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*S
qrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*
(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I
)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)) - (d*(((-I)*
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e
]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(
d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[
e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d +
e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)) + ((I/16
)*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e
])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*Arc
Csc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e
])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sq
rt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*
Log[Sqrt[e] + (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/e^3
+ ((I/16)*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi +
2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2
```

```
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[
e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1
 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (
4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/
e^3)
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.24, size = 1520, normalized size = 2.09

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1520 |
| default | Expression too large to display | 1520 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(-1/4*a*c^10*d^2/e^3/(c^2*e*x^2+c^2*d)^2+a*c^8*d/e^3/(c^2*e*x^2+c^2*d
)+1/2*a*c^6/e^3*ln(c^2*e*x^2+c^2*d)-1/2*b*c^12/e^2/(c^2*e*x^2+c^2*d)^2/(c^2
*d+e)*d^2*arccsc(c*x)*x^2-3/4*b*c^12/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*d*arcc
sc(c*x)*x^4+1/8*b*c^11/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x
^2)^(1/2)*d^2*x+1/8*b*c^11/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*((c^2*x^2-1)/c^2
/x^2)^(1/2)*d*x^3-1/2*b*c^10/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*d*arccsc(c*x)*
x^2-3/4*b*c^10/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccsc(c*x)*x^4-1/8*I*b*c^10/e
^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*d^2-I*b*c^8/e^3/(c^2*d+e)*d*dilog(I/c/x+(1
-1/c^2/x^2)^(1/2))-3/4*I*b*c^6*(e*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)^2*arctanh(
1/4*(2*d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+e^2)^(1/2)
)-1/8*I*b*c^10/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*x^4-5/8*I*b*c^8*(e*(c^2*d+e))^
(1/2)/e^3/(c^2*d+e)^2*arctanh(1/4*(2*d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*
c^2*d-4*e)/(c^2*d*e+e^2)^(1/2))*d-b*c^6/e^2/(c^2*d+e)*arccsc(c*x)*ln(1+I/c/
x+(1-1/c^2/x^2)^(1/2))+I*b*c^6/e^2/(c^2*d+e)*dilog(1+I/c/x+(1-1/c^2/x^2)^(1
/2))-1/4*I*b*c^8/e^3/(c^2*d+e)*d*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c
^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1
-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2
+c^2*d))-1/4*I*b*c^6/e^2/(c^2*d+e)*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d
-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_
R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z
^2+c^2*d))-I*b*c^6/e^2/(c^2*d+e)*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))+I*b*c^8/e
^3/(c^2*d+e)*d*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-b*c^8/e^3/(c^2*d+e)*d*arc
```

```
csc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I*b*c^8/e^2/(c^2*d+e)*sum((_R1
^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1
/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4
+(-2*c^2*d-4*e)*_Z^2+c^2*d))*d-1/4*I*b*c^10/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)
*d*x^2-1/4*I*b*c^10/e^3/(c^2*d+e)*d^2*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)
*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1
-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d)))
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*e^(-3)*log(x^2*e + d) + (4*d*x^2*e + 3*d^2)/(x^4*e^5 + 2*d*x^2*e^4 +
 d^2*e^3))*a + b*integrate(x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(x^6
*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arccsc(c*x) + a*x^5)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e +
 d^3), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

**Giac** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

**Mupad** [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

# 3.112 $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

**Optimal**. Leaf size=157

$$-\frac{bcx\sqrt{-1+c^2x^2}}{8e\left(c^2d+e\right)\sqrt{c^2x^2}\left(d+ex^2\right)}+\frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{bc\left(c^2d+2e\right)x\text{ArcTan}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\left(c^2d+e\right)^{3/2}\sqrt{c^2x^2}}$$

```
[Out] 1/4*x^4*(a+b*arccsc(c*x))/d/(e*x^2+d)^2+1/8*b*c*(c^2*d+2*e)*x*arctan(e^(1/2
)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2))/d/e^(3/2)/(c^2*d+e)^(3/2)/(c^2*x^2)^(1
/2)-1/8*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {270, 5347, 12, 457, 79, 65, 211}

$$\frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{bcx\left(c^2d+2e\right)\text{ArcTan}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}\sqrt{c^2x^2}\left(c^2d+e\right)^{3/2}}-\frac{bcx\sqrt{c^2x^2-1}}{8e\sqrt{c^2x^2}\left(c^2d+e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*c*x*Sqrt[-1 + c^2*x^2])/(e*(c^2*d + e)*Sqrt[c^2*x^2]*(d + e*x^2)) +
 (x^4*(a + b*ArcCsc[c*x]))/(4*d*(d + e*x^2)^2) + (b*c*(c^2*d + 2*e)*x*ArcTa
n[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*d + e]])/(8*d*e^(3/2)*(c^2*d + e)^(
3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{x^4(a+b\csc^{-1}(cx))}{4d\,(d+ex^2)^2} + \frac{(bcx)\int \frac{x^3}{4d\sqrt{-1+c^2x^2}\,(d+ex^2)^2}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{x^4(a+b\csc^{-1}(cx))}{4d\,(d+ex^2)^2} + \frac{(bcx)\int \frac{x^3}{\sqrt{-1+c^2x^2}\,(d+ex^2)^2}\,dx}{4d\sqrt{c^2x^2}}$$

$$= \frac{x^4(a+b\csc^{-1}(cx))}{4d\,(d+ex^2)^2} + \frac{(bcx)\text{Subst}\left(\int \frac{x}{\sqrt{-1+c^2x}\,(d+ex)^2}\,dx, x, x^2\right)}{8d\sqrt{c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{8e\,(c^2d+e)\,\sqrt{c^2x^2}\,(d+ex^2)} + \frac{x^4(a+b\csc^{-1}(cx))}{4d\,(d+ex^2)^2} + \frac{(bc(c^2d+2e)\,x)\,\text{Subst}}{16de\,(}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{8e\,(c^2d+e)\,\sqrt{c^2x^2}\,(d+ex^2)} + \frac{x^4(a+b\csc^{-1}(cx))}{4d\,(d+ex^2)^2} + \frac{(b(c^2d+2e)\,x)\,\text{Subst}\,(}{8cde}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{8e\,(c^2d+e)\,\sqrt{c^2x^2}\,(d+ex^2)} + \frac{x^4(a+b\csc^{-1}(cx))}{4d\,(d+ex^2)^2} + \frac{bc(c^2d+2e)\,x\tan^{-1}\Big(}{8de^{3/2}\,(c^2d+}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.74, size = 390, normalized size = 2.48

$$\frac{\frac{4ad}{(d+ex^2)^2} - \frac{8a}{d+ex^2} - \frac{2bce\sqrt{1-\frac{1}{c^2x^2}}\,x}{(c^2d+e)(d+ex^2)} - \frac{4b(d+2ex^2)\csc^{-1}(cx)}{(d+ex^2)^2} + \frac{4b\text{ArcSin}\left(\frac{1}{cx}\right)}{d} + \frac{b\sqrt{e}\,(c^2d+2e)\log\left(\frac{16d\sqrt{-c^2d-e}\,e^{3/2}\left(i\sqrt{e}+c\left(c\sqrt{d}-i\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(c^2d+2e)\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right)}{d(-c^2d-e)^{3/2}} + \frac{b\sqrt{e}\,(c^2d+2e)\log\left(-\frac{16d\sqrt{-c^2d-e}\,e^{3/2}\left(-\sqrt{e}+c\left(-ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(c^2d+2e)\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right)}{d(-c^2d-e)^{3/2}}}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((4*a*d)/(d + e*x^2)^2 - (8*a)/(d + e*x^2) - (2*b*c*e*Sqrt[1 - 1/(c^2*x^2)]
*x)/((c^2*d + e)*(d + e*x^2)) - (4*b*(d + 2*e*x^2)*ArcCsc[c*x])/(d + e*x^2)
^2 + (4*b*ArcSin[1/(c*x)])/d + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(16*d*Sqrt[-(c^
2*d) - e]*e^(3/2)*(I*Sqrt[e] + c*(c*Sqrt[d] - I*Sqrt[-(c^2*d) - e]*Sqrt[1 -
 1/(c^2*x^2)])*x))/(b*(c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(-(c^2*d)
 - e)^(3/2)) + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(-16*d*Sqrt[-(c^2*d) - e]*e^(3/
2)*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)]
)*x))/(b*(c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(-(c^2*d) - e)^(3/2)))
/(16*e^2)
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1830 vs. $2(135) = 270$.
time = 1.48, size = 1831, normalized size = 11.66

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1831 |
| default | Expression too large to display | 1831 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a*c^6*(-1/2/e^2/(c^2*e*x^2+c^2*d)+1/4*d*c^2/e^2/(c^2*e*x^2+c^2*d)^2)
-1/2*b*c^6*arccsc(c*x)/e^2/(c^2*e*x^2+c^2*d)+1/4*b*c^8*arccsc(c*x)*d/e^2/(c
^2*e*x^2+c^2*d)^2-1/4*b*c^7*(c^2*x^2-1)^(1/2)/e/((c^2*x^2-1)/c^2/x^2)^(1/2)
/x*d/(c^2*d+e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))*arctan(1/
(c^2*x^2-1)^(1/2))-1/4*b*c^7*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)*
x/(c^2*d+e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))*arctan(1/(c^
2*x^2-1)^(1/2))+1/16*b*c^7*(c^2*x^2-1)^(1/2)/e/((c^2*x^2-1)/c^2/x^2)^(1/2)/
x*d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d
*e)^(1/2))*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*
c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))+1/16*b*c^7*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/
c^2/x^2)^(1/2)*x/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(-e*c*x+(-c^2*d*e)^(1/2))/(
e*c*x+(-c^2*d*e)^(1/2))*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^
2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))+1/16*b*c^7*(c^2*x^2-1)^(1/2)/
e/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(e*c*x+(-c
^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))*ln(-2*((-(c^2*d+e)/e)^(1/2)*(c^2*x
^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-e*c*x+(-c^2*d*e)^(1/2)))+1/16*b*c^7
*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/(-(c^2*d+e)/e)^(1/2)/(c^2*
d+e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))*ln(-2*((-(c^2*d+e)/
e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-e*c*x+(-c^2*d*e)^(1/
2)))+1/8*b*c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/(c^2*d+e)/(e*c*x+(
-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))-1/4*b*c^5*(c^2*x^2-1)^(1/2)/((c^
2*x^2-1)/c^2/x^2)^(1/2)/x/(c^2*d+e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*
d*e)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))-1/4*b*c^5*(c^2*x^2-1)^(1/2)*e/((c^2
*x^2-1)/c^2/x^2)^(1/2)*x/d/(c^2*d+e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2
*d*e)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))+1/8*b*c^5*(c^2*x^2-1)^(1/2)/((c^2*
x^2-1)/c^2/x^2)^(1/2)/x/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(-e*c*x+(-c^2*d*e)^(
1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)
*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))+1/8*b*c^5*(c^2*x^2-1)^
(1/2)*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(-e*
c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln(2*((-(c^2*d+e)/e)^(1/2)*(
c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))+1/8*b*
c^5*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/(-(c^2*d+e)/e)^(1/2)/(c
^2*d+e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))*ln(-2*((-(c^2*d+
e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-e*c*x+(-c^2*d*e)^
(1/2)))+1/8*b*c^5*(c^2*x^2-1)^(1/2)*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d/(-(c^
2*d+e)/e)^(1/2)/(c^2*d+e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2)
```

```
)*ln(-2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(
-e*c*x+(-c^2*d*e)^(1/2))))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2*e + d)*a/(x^4*e^4 + 2*d*x^2*e^3 + d^2*e^2) - 1/4*(2*x^2*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e + d*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1)) + 4*(x^4*e^4 + 2*d*x^2*e^3 + d^2*e^2)*integrate(1/4*(2*c^2*x^3*e + c^2*
d*x)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*x^6*e^4 + (2*c^2*d*e^3 -
e^4)*x^4 + (c^2*d^2*e^2 - 2*d*e^3)*x^2 - d^2*e^2 + (c^2*x^6*e^4 + (2*c^2*d*
e^3 - e^4)*x^4 + (c^2*d^2*e^2 - 2*d*e^3)*x^2 - d^2*e^2)*e^(log(c*x + 1) + l
og(c*x - 1))), x))*b/(x^4*e^4 + 2*d*x^2*e^3 + d^2*e^2)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(138) = 276$.
time = 0.60, size = 1021, normalized size = 6.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 8*a*d*x^2*e^3 + (b*c^2*d^3 + 2*b*x^4*e^3 + (b*c^2*d*x
^4 + 4*b*d*x^2)*e^2 + 2*(b*c^2*d^2*x^2 + b*d^2)*e)*sqrt(-c^2*d*e - e^2)*log
(-(c^2*d - (c^2*x^2 - 2)*e + 2*sqrt(c^2*x^2 - 1)*sqrt(-c^2*d*e - e^2))/(x^2
*e + d)) + 4*(b*c^4*d^4 + 2*b*d*x^2*e^3 + (4*b*c^2*d^2*x^2 + b*d^2)*e^2 + 2
*(b*c^4*d^3*x^2 + b*c^2*d^3)*e)*arccsc(c*x) + 8*(b*c^4*d^4 + b*x^4*e^4 + 2*
(b*c^2*d*x^4 + b*d*x^2)*e^3 + (b*c^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + b*d^2)*e^2
 + 2*(b*c^4*d^3*x^2 + b*c^2*d^3)*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 4*(4
*a*c^2*d^2*x^2 + a*d^2)*e^2 + 8*(a*c^4*d^3*x^2 + a*c^2*d^3)*e + 2*(b*c^2*d^
3*e + b*d*x^2*e^3 + (b*c^2*d^2*x^2 + b*d^2)*e^2)*sqrt(c^2*x^2 - 1))/(c^4*d^
5*e^2 + d*x^4*e^6 + 2*(c^2*d^2*x^4 + d^2*x^2)*e^5 + (c^4*d^3*x^4 + 4*c^2*d^
3*x^2 + d^3)*e^4 + 2*(c^4*d^4*x^2 + c^2*d^4)*e^3), -1/8*(2*a*c^4*d^4 + 4*a*
d*x^2*e^3 - (b*c^2*d^3 + 2*b*x^4*e^3 + (b*c^2*d*x^4 + 4*b*d*x^2)*e^2 + 2*(b
*c^2*d^2*x^2 + b*d^2)*e)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*x^2 - 1)*sqrt(
c^2*d*e + e^2)/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*d*x^2*e^3 + (4*b*c^2*d^2*x
^2 + b*d^2)*e^2 + 2*(b*c^4*d^3*x^2 + b*c^2*d^3)*e)*arccsc(c*x) + 4*(b*c^4*d
^4 + b*x^4*e^4 + 2*(b*c^2*d*x^4 + b*d*x^2)*e^3 + (b*c^4*d^2*x^4 + 4*b*c^2*d
^2*x^2 + b*d^2)*e^2 + 2*(b*c^4*d^3*x^2 + b*c^2*d^3)*e)*arctan(-c*x + sqrt(c
^2*x^2 - 1)) + 2*(4*a*c^2*d^2*x^2 + a*d^2)*e^2 + 4*(a*c^4*d^3*x^2 + a*c^2*d
```

```
^3)*e + (b*c^2*d^3*e + b*d*x^2*e^3 + (b*c^2*d^2*x^2 + b*d^2)*e^2)*sqrt(c^2*
x^2 - 1))/(c^4*d^5*e^2 + d*x^4*e^6 + 2*(c^2*d^2*x^4 + d^2*x^2)*e^5 + (c^4*d
^3*x^4 + 4*c^2*d^3*x^2 + d^3)*e^4 + 2*(c^4*d^4*x^2 + c^2*d^4)*e^3)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

# 3.113 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

**Optimal**. Leaf size=193

$$\frac{bcx\sqrt{-1+c^2x^2}}{8d\left(c^2d+e\right)\sqrt{c^2x^2}\,\left(d+ex^2\right)}-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{bcx\mathrm{ArcTan}\left(\sqrt{-1+c^2x^2}\right)}{4d^2e\sqrt{c^2x^2}}+\frac{bc(3c^2d+2e)\,x\mathrm{ArcTan}\left(\frac{\sqrt{e}}{}\right.}{8d^2\sqrt{e}\,\left(c^2d+e\right)^{3/2}}$$

```
[Out] 1/4*(-a-b*arccsc(c*x))/e/(e*x^2+d)^2-1/4*b*c*x*arctan((c^2*x^2-1)^(1/2))/d^
2/e/(c^2*x^2)^(1/2)+1/8*b*c*(3*c^2*d+2*e)*x*arctan(e^(1/2)*(c^2*x^2-1)^(1/2
)/(c^2*d+e)^(1/2))/d^2/(c^2*d+e)^(3/2)/e^(1/2)/(c^2*x^2)^(1/2)+1/8*b*c*x*(c
^2*x^2-1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.15, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5345, 457, 105, 162, 65, 211}

$$-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{bcx\mathrm{ArcTan}\left(\sqrt{c^2x^2-1}\right)}{4d^2e\sqrt{c^2x^2}}+\frac{bcx(3c^2d+2e)\,\mathrm{ArcTan}\left(\frac{\sqrt{e}\,\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}\,\sqrt{c^2x^2}\,\left(c^2d+e\right)^{3/2}}+\frac{bcx\sqrt{c^2x^2-1}}{8d\sqrt{c^2x^2}\,\left(c^2d+e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*x*Sqrt[-1 + c^2*x^2])/(8*d*(c^2*d + e)*Sqrt[c^2*x^2]*(d + e*x^2)) - (a
 + b*ArcCsc[c*x])/(4*e*(d + e*x^2)^2) - (b*c*x*ArcTan[Sqrt[-1 + c^2*x^2]])/
(4*d^2*e*Sqrt[c^2*x^2]) + (b*c*(3*c^2*d + 2*e)*x*ArcTan[(Sqrt[e]*Sqrt[-1 +
c^2*x^2])/Sqrt[c^2*d + e]])/(8*d^2*Sqrt[e]*(c^2*d + e)^(3/2)*Sqrt[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
```

```
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{a+b\csc^{-1}(cx)}{4e(d+ex^2)^2} - \frac{(bcx)\int \frac{1}{x\sqrt{-1+c^2x^2}(d+ex^2)^2}\,dx}{4e\sqrt{c^2x^2}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{4e(d+ex^2)^2} - \frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}(d+ex)^2}\,dx, x, x^2\right)}{8e\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a+b\csc^{-1}(cx)}{4e(d+ex^2)^2} + \frac{(bcx)\text{Subst}\left(\int \frac{-c^2d-e+\frac{1}{2}c^2}{x\sqrt{-1+c^2x}}\right.}{8de(c^2d+e)\sqrt{c}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a+b\csc^{-1}(cx)}{4e(d+ex^2)^2} - \frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}}\right.}{8d^2e\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a+b\csc^{-1}(cx)}{4e(d+ex^2)^2} - \frac{(bx)\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}+\frac{x^2}{c^2}}\,dx, x, \sqrt{\right.}}{4cd^2e\sqrt{c^2x^2}}$$

$$= \frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a+b\csc^{-1}(cx)}{4e(d+ex^2)^2} - \frac{bcx\tan^{-1}\left(\sqrt{-1+c^2x^2}\right)}{4d^2e\sqrt{c^2x^2}} +$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 0.65, size = 385, normalized size = 1.99

$$\frac{1}{16}\left(-\frac{4a}{e(d+ex^2)^2} + \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\,x}{d(c^2d+e)(d+ex^2)} - \frac{4b\csc^{-1}(cx)}{e(d+ex^2)^2} + \frac{4b\text{ArcSin}\left(\frac{1}{cx}\right)}{d^2e} + \frac{b(3c^2d+2e)\log\left(\frac{16d^2\sqrt{-c^2d-e}\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}-i\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d^2(-c^2d-e)^{3/2}\sqrt{e}} + \frac{b(3c^2d+2e)\log\left(-\frac{16d^2\sqrt{-c^2d-e}\sqrt{e}\left(-\sqrt{e}+c\left(-ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2(-c^2d-e)^{3/2}\sqrt{e}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((-4*a)/(e*(d + e*x^2)^2) + (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d + e)*
(d + e*x^2)) - (4*b*ArcCsc[c*x])/(e*(d + e*x^2)^2) + (4*b*ArcSin[1/(c*x)])/
(d^2*e) + (b*(3*c^2*d + 2*e)*Log[(16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(I*Sqrt
[e] + c*(c*Sqrt[d] - I*Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(3*
c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e])
+ (b*(3*c^2*d + 2*e)*Log[(-16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(-Sqrt[e] + c*
((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(3*c^2*d
 + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e]))/16
```

**Maple** [B] Leaf count of result is larger than twice the leaf count of optimal. 1795 vs. $2(168) = 336$.
time = 1.46, size = 1796, normalized size = 9.31

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1796 |
| default | Expression too large to display | 1796 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/4*a*c^6/e/(c^2*e*x^2+c^2*d)^2-1/4*b*c^6/e/(c^2*e*x^2+c^2*d)^2*arc
csc(c*x)-1/4*b*c^5*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/(c^2*d+e
)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))*arctan(1/(c^2*x^2-1)^(
1/2))-1/4*b*c^5*(c^2*x^2-1)^(1/2)*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d/(c^2*d+
e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))*arctan(1/(c^2*x^2-1)^
(1/2))+3/16*b*c^5*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/(-(c^2*d+
e)/e)^(1/2)/(c^2*d+e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln
(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x
+(-c^2*d*e)^(1/2)))+3/16*b*c^5*(c^2*x^2-1)^(1/2)*e/((c^2*x^2-1)/c^2/x^2)^(1
/2)*x/d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c
^2*d*e)^(1/2))*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1
/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))+3/16*b*c^5*(c^2*x^2-1)^(1/2)/((c^2*x^2
-1)/c^2/x^2)^(1/2)/x/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(e*c*x+(-c^2*d*e)^(1/2)
)/(-e*c*x+(-c^2*d*e)^(1/2))*ln(-2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e
+(-c^2*d*e)^(1/2)*c*x-e)/(-e*c*x+(-c^2*d*e)^(1/2)))+3/16*b*c^5*(c^2*x^2-1)^
(1/2)*e/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(e*c
*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))*ln(-2*((-(c^2*d+e)/e)^(1/2)*
(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-e*c*x+(-c^2*d*e)^(1/2)))-1/8*
b*c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(c^2*d+e)/(-e*c*x+(-c^2*d
*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*e-1/4*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2
-1)/c^2/x^2)^(1/2)/x/d/(c^2*d+e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e
)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))*e-1/4*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^
2-1)/c^2/x^2)^(1/2)*x/d^2/(c^2*d+e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*
d*e)^(1/2))*arctan(1/(c^2*x^2-1)^(1/2))*e^2+1/8*b*c^3*(c^2*x^2-1)^(1/2)/((c
^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(-e*c*x+(-c^2*d
*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln(2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^
(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))*e+1/8*b*c^3*(c^2*
x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x/d^2/(-(c^2*d+e)/e)^(1/2)/(c^2*d+
e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln(2*((-(c^2*d+e)/e)^
(1/2)*(c^2*x^2-1)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))
*e^2+1/8*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d+e
)/e)^(1/2)/(c^2*d+e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln(
-2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-e*c*
x+(-c^2*d*e)^(1/2)))*e+1/8*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1
/2)*x/d^2/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(
-c^2*d*e)^(1/2))*ln(-2*((-(c^2*d+e)/e)^(1/2)*(c^2*x^2-1)^(1/2)*e+(-c^2*d*e)
```

```
^(1/2)*c*x-e)/(-e*c*x+(-c^2*d*e)^(1/2)))*e^2)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(c^2*x^4*e^3 + 2*c^2*d*x^2*e^2 + c^2*d^2*e)*integrate(1/4*x*e^(1/2*
log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*x^6*e^3 + (2*c^2*d*e^2 - e^3)*x^4 + (
c^2*d^2*e - 2*d*e^2)*x^2 - d^2*e + (c^2*x^6*e^3 + (2*c^2*d*e^2 - e^3)*x^4 +
 (c^2*d^2*e - 2*d*e^2)*x^2 - d^2*e)*e^(log(c*x + 1) + log(c*x - 1))), x) +
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(x^4*e^3 + 2*d*x^2*e^2 + d^2*e)
- 1/4*a/(x^4*e^3 + 2*d*x^2*e^2 + d^2*e)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(167) = 334.
time = 0.63, size = 888, normalized size = 4.60

$$\left[-\frac{4\,ac^4d^4 + 8\,ac^2d^3e + 4\,ad^2e^2 + (3\,bc^2d^3 + 2\,bx^4e^3 + (3\,bc^2dx^4 + 4\,bdx^2)e^2 + 2\,(3\,bc^2d^2x^2 + bd^2)e)\sqrt{-c^2de - e^2}\,\log\left(-\frac{c^2d - (c^2x^2 - 2)e + 2\,\sqrt{c^2x^2 - 1}\,\sqrt{-c^2de - e^2}}{x^2e + d}\right) + 4\,(bc^4d^4 + 2\,bc^2d^3e + bd^2e^2)\,\mathrm{arccsc}\,(cx) + 8\,(bc^4d^4 + bx^4e^4 + 2\,(bc^2dx^4 + bdx^2)e^3 + (bc^4d^2x^4 + 4\,bc^2d^2x^2 + bd^2)e^2 + 2\,(bc^4d^3x^2 + bc^2d^3)e)\arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) - 2\,(bc^2d^3e + bdx^2e^3 + (bc^2d^2x^2 + bd^2)e^2)\sqrt{c^2x^2 - 1}}{16\,(c^4d^6e + d^2x^4e^5 + 2\,(c^2d^3x^4 + d^3x^2)e^4 + (c^4d^4x^4 + 4\,c^2d^4x^2 + d^4)e^3 + 2\,(c^4d^5x^2 + c^2d^5)e^2)}, -\frac{2\,ac^4d^4 + 4\,ac^2d^3e + 2\,ad^2e^2 - (3\,bc^2d^3 + 2\,bx^4e^3 + (3\,bc^2dx^4 + 4\,bdx^2)e^2 + 2\,(3\,bc^2d^2x^2 + bd^2)e)\sqrt{c^2de + e^2}\arctan\left(\frac{\sqrt{c^2x^2 - 1}\,\sqrt{c^2de + e^2}}{c^2d + e}\right) + 2\,(bc^4d^4 + 2\,bc^2d^3e + bd^2e^2)\,\mathrm{arccsc}\,(cx) + 4\,(bc^4d^4 + bx^4e^4 + 2\,(bc^2dx^4 + bdx^2)e^3 + (bc^4d^2x^4 + 4\,bc^2d^2x^2 + bd^2)e^2 + 2\,(bc^4d^3x^2 + bc^2d^3)e)\arctan\left(-cx + \sqrt{c^2x^2 - 1}\right) - (bc^2d^3e + bdx^2e^3 + (bc^2d^2x^2 + bd^2)e^2)\sqrt{c^2x^2 - 1}}{8\,(c^4d^6e + d^2x^4e^5 + 2\,(c^2d^3x^4 + d^3x^2)e^4 + (c^4d^4x^4 + 4\,c^2d^4x^2 + d^4)e^3 + 2\,(c^4d^5x^2 + c^2d^5)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (3*b*c^2*d^3 + 2*b*x^4*
e^3 + (3*b*c^2*d*x^4 + 4*b*d*x^2)*e^2 + 2*(3*b*c^2*d^2*x^2 + b*d^2)*e)*sqrt
(-c^2*d*e - e^2)*log(-(c^2*d - (c^2*x^2 - 2)*e + 2*sqrt(c^2*x^2 - 1)*sqrt(-
c^2*d*e - e^2))/(x^2*e + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*ar
ccsc(c*x) + 8*(b*c^4*d^4 + b*x^4*e^4 + 2*(b*c^2*d*x^4 + b*d*x^2)*e^3 + (b*c
^4*d^2*x^4 + 4*b*c^2*d^2*x^2 + b*d^2)*e^2 + 2*(b*c^4*d^3*x^2 + b*c^2*d^3)*e
)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^2*d^3*e + b*d*x^2*e^3 + (b*c^2*
d^2*x^2 + b*d^2)*e^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + d^2*x^4*e^5 + 2*(c^2*
d^3*x^4 + d^3*x^2)*e^4 + (c^4*d^4*x^4 + 4*c^2*d^4*x^2 + d^4)*e^3 + 2*(c^4*d
^5*x^2 + c^2*d^5)*e^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e + 2*a*d^2*e^2 - (
3*b*c^2*d^3 + 2*b*x^4*e^3 + (3*b*c^2*d*x^4 + 4*b*d*x^2)*e^2 + 2*(3*b*c^2*d^
2*x^2 + b*d^2)*e)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*x^2 - 1)*sqrt(c^2*d*e
 + e^2)/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*arccsc(c*x
) + 4*(b*c^4*d^4 + b*x^4*e^4 + 2*(b*c^2*d*x^4 + b*d*x^2)*e^3 + (b*c^4*d^2*x
^4 + 4*b*c^2*d^2*x^2 + b*d^2)*e^2 + 2*(b*c^4*d^3*x^2 + b*c^2*d^3)*e)*arctan
(-c*x + sqrt(c^2*x^2 - 1)) - (b*c^2*d^3*e + b*d*x^2*e^3 + (b*c^2*d^2*x^2 +
b*d^2)*e^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + d^2*x^4*e^5 + 2*(c^2*d^3*x^4 +
d^3*x^2)*e^4 + (c^4*d^4*x^4 + 4*c^2*d^4*x^2 + d^4)*e^3 + 2*(c^4*d^5*x^2 + c
^2*d^5)*e^2)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

# 3.114 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^3}\,dx$

**Optimal**. Leaf size=704

$$-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x}+\frac{e^2\left(a+b\csc^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)}+\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^3}+\frac{b\sqrt{e}\,\mathrm{ArcTan}\left(\frac{\ \ }{c\sqrt{\ \ }}\right.}{d^3\sqrt{c}}$$

```
[Out] 1/4*e^2*(a+b*arccsc(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arccsc(c*x))/d^3/(e+d/x^2)
+1/2*I*(a+b*arccsc(c*x))^2/b/d^3-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsc(
c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1
/2)))/d^3-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polyl
og(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))
/d^3+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/8*b*(c^2*d+2*e)*
arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))*e^(1/2)/d^3/(c^2*d+
e)^(3/2)+b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))*e^(1/2)/
d^3/(c^2*d+e)^(1/2)-1/8*b*c*e*(1-1/c^2/x^2)^(1/2)/d^2/(c^2*d+e)/(e+d/x^2)/x
```

**Rubi** [A]
time = 1.14, antiderivative size = 704, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {5349, 4817, 4813, 390, 385, 211, 4825, 4615, 2221, 2317, 2438}

$$-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}-\frac{\left(a+b\csc^{-1}(cx)\right)\log\left(1+\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}+\frac{e^2\left(a+b\csc^{-1}(cx)\right)}{4d^3\left(\frac{d}{x^2}+e\right)^2}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{d^3\left(\frac{d}{x^2}+e\right)}+\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^3}-\frac{b\sqrt{e}\left(c^2d+2e\right)\mathrm{ArcTan}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\,x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{8d^3\left(c^2d+e\right)^{3/2}}+\frac{b\sqrt{e}\,\mathrm{ArcTan}\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\,x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d^3\sqrt{c^2d+e}}+\frac{ib\,\mathrm{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\mathrm{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\mathrm{Li}_2\left(-\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\mathrm{Li}_2\left(\frac{ic\sqrt{-d}\,e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3}-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2x\left(c^2d+e\right)\left(\frac{d}{x^2}+e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] -1/8*(b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(d^2*(c^2*d + e)*(e + d/x^2)*x) + (e^2*(
a + b*ArcCsc[c*x]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcCsc[c*x]))/(d^3*(e
 + d/x^2)) + ((I/2)*(a + b*ArcCsc[c*x])^2)/(b*d^3) + (b*Sqrt[e]*ArcTan[Sqrt
[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(d^3*Sqrt[c^2*d + e]) - (
b*Sqrt[e]*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x
^2)]*x)])/(8*d^3*(c^2*d + e)^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqr
t[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*Ar
cCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
```

```
e])])/(2*d^3) - ((a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x
]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcCsc[c*x])*Log[1 + (I
*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) + ((I/
2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d +
 e])])/d^3 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e]
- Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-
d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4813

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSin[c*x])/(2*e*(p + 1))), x]
 - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 - c^2*x^2], x],
 x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{x^5\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{e^2x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2ex\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} + \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right) d$$

$$= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \frac{x\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\text{Subst}}{}$$

$$= \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\,\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{\sqrt{-d}}{2d\left(\sqrt{}\right.}\right.\right.}{d^2}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \frac{a}{\sqrt{}}\right.}{}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{e}\,\tan^{-1}}{d}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\csc^{-}}{2bd^3}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\csc^{-}}{2bd^3}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\csc^{-}}{2bd^3}$$

$$= -\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2\left(c^2d+e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b\csc^{-}}{2bd^3}$$

**Mathematica [F]**
time = 60.54, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^3), x]
```

**Maple [C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 17.46, size = 5294, normalized size = 7.52

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 5294 |
| default | Expression too large to display | 5294 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*x^2*e + 3*d)/(d^2*x^4*e^2 + 2*d^3*x^2*e + d^4) - 2*log(x^2*e + d)
/d^3 + 4*log(x)/d^3) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/
(x^7*e^3 + 3*d*x^5*e^2 + 3*d^2*x^3*e + d^3*x), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(x^7*e^3 + 3*d*x^5*e^2 + 3*d^2*x^3*e + d^3*x),
 x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

# 3.115 $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

**Optimal**. Leaf size=1144

$$-\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{\sqrt{-d}\,(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}+\frac{3(a\ \cdots}{16e^2\ (\cdots}$$

```
[Out] -3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arccsc(c*x))*ln(1+I*c*
(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(
-d)^(1/2)-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arccsc(c*x))*
ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/
e^(5/2)/(-d)^(1/2)-3/16*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*I*b*polylog(2,I*c
*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/
(-d)^(1/2)-3/16*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(
e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*I*b*polylog(2,I*c*(I/c/x+
(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/
2)-1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1
-1/c^2/x^2)^(1/2))/e/(c^2*d+e)^(3/2)/d^(1/2)-1/16*b*arctanh((c^2*d+(-d)^(1/
2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e/(c^2*d+e)^(3
/2)/d^(1/2)+1/16*(a+b*arccsc(c*x))*(-d)^(1/2)/e^(3/2)/(-d/x+(-d)^(1/2)*e^(1
/2))^2+3/16*(a+b*arccsc(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arccs
c(c*x))*(-d)^(1/2)/e^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2-3/16*(a+b*arccsc(c*x)
)/e^2/(d/x+(-d)^(1/2)*e^(1/2))-3/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/
c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2*d+e)^(1/2)-
3/16*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/
c^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2*d+e)^(1/2)-1/16*b*c*(-d)^(1/2)*(1-1/c^2/x^
2)^(1/2)/e^(3/2)/(c^2*d+e)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(-d)^(1/2)*(1
-1/c^2/x^2)^(1/2)/e^(3/2)/(c^2*d+e)/(d/x+(-d)^(1/2)*e^(1/2))
```

**Rubi** [A]
time = 1.47, antiderivative size = 1144, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {5349, 4757, 4827, 745, 739, 212, 4825, 4615, 2221, 2317, 2438}

Antiderivative was successfully verified.

[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]

[Out] -1/16*(b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] - d/x)) - (b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcCsc[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcCsc[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcCsc[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (3*(a + b*ArcCsc[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) - (3*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) - (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2)) + (((3*I)/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(5/2))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
```

```
&& NeQ[m, -1]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = -\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{(e+dx^2)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{d^3\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-dx\right)^3} - \frac{3d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2} -\right.\right.$$

$$= \frac{(3d)\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2} + \frac{(3d)\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}+dx\right)^2}\,dx\right.}{16e^2}$$

$$= \frac{\sqrt{-d}\,(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\,(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)^2}$$

$$= -\frac{bc\sqrt{-d}\,\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\,\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{}}{16e}$$

$$= -\frac{bc\sqrt{-d}\,\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\,\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{}}{16e}$$

$$= -\frac{bc\sqrt{-d}\,\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\,\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{}}{16e}$$

$$= -\frac{bc\sqrt{-d}\,\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\,\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{}}{16e}$$

$$= -\frac{bc\sqrt{-d}\,\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\,\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{}}{16e}$$

**Mathematica** [A]
time = 6.05, size = 2067, normalized size = 1.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*((5*(-(ArcCsc[c*x]/((-I)*Sqrt[
d]*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(
Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))
/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]
))/(16*e^2) + (5*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(
c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt
[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*
Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*e^2) + ((I/16)*Sqrt[d]*((I*
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqr
t[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c
*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqr
t[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d
+ e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/e^2 - ((I/16)*Sq
rt[d]*(((-I)*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqr
t[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcS
in[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e
]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))
/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/e^2 - (3
*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e]
)/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcC
sc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c
*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Arc
Csc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*L
og[Sqrt[e] + (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(128*
Sqrt[d]*e^(5/2)) + (3*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSi
n[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e]
)*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e
] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log
```

```
[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*A
rcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c
^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (S
qrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sq
rt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e]
)/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCs
c[c*x])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] -
Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcC
sc[c*x])]))/(128*Sqrt[d]*e^(5/2)))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 11.32, size = 3188, normalized size = 2.79

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 3188 |
| default | Expression too large to display | 3188 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(-3/4*b*c^3*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/
x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e^
2/(c^2*d+e)/d^2*(e*(c^2*d+e))^(1/2)+3/8*b*c^5*(-(c^2*d-2*(e*(c^2*d+e))^(1/2
)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d
+e))^(1/2)-2*e)*d)^(1/2))/e^2/(c^2*d+e)^2/d*(e*(c^2*d+e))^(1/2)-3/8*b*c^11*
x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/e^2*d^2*arccsc(c*x)-5/8*b*c^11*x^3/(c^2*e*x
^2+c^2*d)^2/(c^2*d+e)/e*d*arccsc(c*x)-1/8*b*c^10*x^2/(c^2*e*x^2+c^2*d)^2/(c
^2*d+e)/e*((c^2*x^2-1)/c^2/x^2)^(1/2)*d-3/8*b*c^9*x/(c^2*e*x^2+c^2*d)^2/(c^
2*d+e)/e*d*arccsc(c*x)-3/8*a*c^9/(c^2*e*x^2+c^2*d)^2*d/e^2*x-5/4*b*c^3*((c^
2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))
*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^2+b*c*((c^2
*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*
c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3-5/4*b*c^
3*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^
(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d^2+b*
c*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^
(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3-
3/8*b*c^5*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c
^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2*d+
e)/d+3/4*b*c^5*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(
1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2
*d+e)^2/d-3/8*b*c^5*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/
c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/
```

```
e^2/(c^2*d+e)/d+3/4*b*c^5*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arct
an((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(
1/2))/e/(c^2*d+e)^2/d+b*c*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arct
an((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(
1/2))/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^(1/2)-b*c*((c^2*d+2*(e*(c^2*d+e))^(1/2)
+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^(1/2)-1/8*b*c^10*x^4
/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*((c^2*x^2-1)/c^2/x^2)^(1/2)-5/8*b*c^9*x^3/(c
^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccsc(c*x)-b*c*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^3+7/4*b*c^3*((c^2*d+2*(e*(c^2*d+e))^(1/2
)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d
+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^2-b*c*(-(c^2*d-2*(e*(c^2*d+e))^(1/2
)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d
+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/d^3+7/4*b*c^3*(-(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c
^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/d^2-3/16*b*c^8/e^2/(c^2*d+e)*d*su
m(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1
/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4
+(-2*c^2*d-4*e)*_Z^2+c^2*d))-3/16*b*c^8/e^2/(c^2*d+e)*d*sum(1/_R1/(_R1^2*c^
2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog
((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)
*_Z^2+c^2*d))-5/8*a*c^9/(c^2*e*x^2+c^2*d)^2/e*x^3+5/4*b*c^3*(-(c^2*d-2*(e*(
c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2
*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^(1/
2)+3/4*b*c^3*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-
1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2
*d+e)/d^2*(e*(c^2*d+e))^(1/2)-3/8*b*c^5*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2))/e^2/(c^2*d+e)^2/d*(e*(c^2*d+e))^(1/2)+b*c*((c^2*d+2*(e*(
c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2
*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)
-5/4*b*c^3*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/
c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^
2/e/d^2*(e*(c^2*d+e))^(1/2)-b*c*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2
)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e
)*d)^(1/2))/e/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)+3/8*a*c^5/e^2/(d*e)^(1/2)*a
rctan(e*x/(d*e)^(1/2))-3/16*b*c^6/e/(c^2*d+e)*sum(_R1/(_R1^2*c^2*d-c^2*d-2*
e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-
(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d)
)-3/16*b*c^6/e/(c^2*d+e)*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*l
n((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2)
)/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d)))
```

**Maxima [F]**

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) - (5*x^3*e + 3*d*x)/(x^4*
e^4 + 2*d*x^2*e^3 + d^2*e^2))*a + b*integrate(x^4*arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1))/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsc(c*x) + a*x^4)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e +
 d^3), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

# 3.116 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

**Optimal**. Leaf size=1144

$$-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\,\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\sqrt{e}\,\left(c^2d+e\right)\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{a+b\csc^{-1}(c}{16\sqrt{-d}\,\sqrt{e}\,\left(\sqrt{-d}\right.}$$

```
[Out] 1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/
c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(3/2)+1/16*b*arctanh((c^2*d+(-d)^(1/2)*e^
(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(
3/2)+1/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arccsc(c*x))*ln(1+
I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^
(3/2)/e^(3/2)+1/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arccsc(c*
x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))/(-d)^(3/2)/e^(3/2)-1/16*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*I*b*polylog(2,
I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^
(3/2)/e^(3/2)+1/16*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*I*b*polylog(2,-I*c*(I/
c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e
^(3/2)+1/16*(a+b*arccsc(c*x))/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))^
2+1/16*(a+b*arccsc(c*x))/d/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*(-a-b*arccsc(c*
x))/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+1/16*(-a-b*arccsc(c*x))/d
/e/(d/x+(-d)^(1/2)*e^(1/2))-1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d
^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/e/(c^2*d+e)^(1/2)-1/16*
b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x
^2)^(1/2))/d^(3/2)/e/(c^2*d+e)^(1/2)-1/16*b*c*(1-1/c^2/x^2)^(1/2)/(c^2*d+e)
/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(1-1/c^2/x^2)^(1/2)/
(c^2*d+e)/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))
```

**Rubi** [A]

time = 2.86, antiderivative size = 1144, normalized size of antiderivative = 1.00, number of steps used = 63, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5349, 4817, 4757, 4827, 745, 739, 212, 4825, 4615, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[1 - 1/(c^2*x^2)])/(Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*S
qrt[e] - d/x)) - (b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d +
e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcCsc[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sq
rt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcCsc[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] -
d/x)) - (a + b*ArcCsc[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] + d/x)^2
) - (a + b*ArcCsc[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2
*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)]
)])/(16*d^(3/2)*(c^2*d + e)^(3/2)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])
/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*e*Sqrt[
c^2*d + e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2
*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d + e)^(3/2)) - (b*ArcTan
h[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2
*x^2)])])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) + ((a + b*ArcCsc[c*x])*Log[1 - (I*
c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*
e^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(S
qrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCsc[c*x])*
Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*
(-d)^(3/2)*e^(3/2)) - ((a + b*ArcCsc[c*x])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCs
c[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((I/16)*b*
PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])]
)/((-d)^(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x
]))/(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(3/2)*e^(3/2)) + ((I/16)*b*PolyLog[
2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^
(3/2)*e^(3/2)) - ((I/16)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/((-d)^(3/2)*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
```

```
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5349

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{(e+dx^2)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d\,(e+dx^2)^3} + \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{d\,(e+dx^2)^2}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{(e+dx^2)^3}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{2e(-de-d^2x^2)}\right)dx,\right.}{d}$$

$$= -\frac{3\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e} - \frac{3\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right.}{16e}$$

$$= \frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-\right.}}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}\right.}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}\right.}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}\right.}$$

$$= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}\right.}$$

**Mathematica [A]**
time = 6.04, size = 2075, normalized size = 1.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(-1/16*(-(ArcCsc[c*x]/((-I)*Sqrt[d]
*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sq
rt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(
Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])/
(d*e) - (-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/Sqr
t[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d)
 - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x
))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(16*d*e) - ((I/16)*((I*c*Sqrt[e]*Sqrt[1 -
 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsc[
c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqrt[e]) +
 (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c*Sqrt[d
] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*Sqrt[d]
 + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + ((I/16)*(((-I)*c*Sqr
t[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))
 - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqr
t[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e] +
c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*(I
*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) - (Pi^2 - 4*Pi
*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/
Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*
E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])
/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[
c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
 (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e] +
(I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(128*d^(3/2)*e^(3/2
)) + (Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 + (I*Sqr
t[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*Arc
Csc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e] +
```

```
Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*Sqrt[e]
)/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*
Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(128*d
^(3/2)*e^(3/2)))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.88, size = 2342, normalized size = 2.05

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 2342 |
| default | Expression too large to display | 2342 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/8*a*c^3/d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/8*b*c^9*x/(c^2*e
*x^2+c^2*d)^2/(c^2*d+e)/e*d*arccsc(c*x)-1/8*b*c^3*((c^2*d+2*(e*(c^2*d+e))^(
1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^2+1/4*b*c*((c^2*d+2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*
(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3-1/8*b*c^3*(-(c^2*d-2*(e*(
c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2
*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e/(c^2*d+e)/d^2+1/4*b*c*(-(c^2*d-2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((
-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3+1/4*b*c*(-(c^
2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*
c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/d^3*(e*(c^2*d
+e))^(1/2)-1/4*b*c*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c
/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c
^2*d+e)^2/d^3*(e*(c^2*d+e))^(1/2)+1/8*b*c^9*x^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+
e)*arccsc(c*x)+1/8*b*c^8*x^4/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/d*e*((c^2*x^2-1)
/c^2/x^2)^(1/2)+1/8*b*c^7*x^3/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/d*e*arccsc(c*x)
-1/4*b*c*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^
2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/d^
3+1/4*b*c^3*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1
/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)
^2/d^2-1/4*b*c*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(
1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*
```

```
d+e)/d^3+1/4*b*c^3*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c
/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(
c^2*d+e)^2/d^2+1/8*b*c^3*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arcta
n((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1
/2))/(c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^(1/2)+1/4*b*c*((c^2*d+2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c
^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)-1/8*b*c^3
*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(
1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/e/d^2*(e
*(c^2*d+e))^(1/2)-1/4*b*c*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arct
an((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(
1/2))/e/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)-1/16*b*c^6/e/(c^2*d+e)*sum(_R1/(_
R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1
)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2
*d-4*e)*_Z^2+c^2*d))-1/16*b*c^6/e/(c^2*d+e)*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*
e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-
(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d)
)-1/16*b*c^4/d/(c^2*d+e)*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*l
n((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2)
)/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/16*b*c^4/d/(c^2
*d+e)*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2
/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^
2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/8*a*c^7/(c^2*e*x^2+c^2*d)^2/e*x+1/8*
a*c^7/(c^2*e*x^2+c^2*d)^2/d*x^3+1/8*b*c^8*x^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)
*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/8*b*c^7*x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arcc
sc(c*x))
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*(arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(3/2) + (x^3*e - d*x)/(d*x^4*e^
3 + 2*d^2*x^2*e^2 + d^3*e)) + b*integrate(x^2*arctan2(1, sqrt(c*x + 1)*sqrt
(c*x - 1))/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

# 3.117 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx$

**Optimal**. Leaf size=1134

$$-\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{\sqrt{e}\,(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

```
[Out] -1/16*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1
-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(3/2)-1/16*b*e*arctanh((c^2*d+(-d)^(1/
2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d
+e)^(3/2)-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arccsc(c*x))*
ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/
(-d)^(5/2)/e^(1/2)-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arcc
sc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)
^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*I*b*polyl
og(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))
/(-d)^(5/2)/e^(1/2)-3/16*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*I*b*polylog(2,I*
c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5
/2)/e^(1/2)+1/16*(a+b*arccsc(c*x))*e^(1/2)/(-d)^(3/2)/(-d/x+(-d)^(1/2)*e^(1
/2))^2-5/16*(a+b*arccsc(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arccs
c(c*x))*e^(1/2)/(-d)^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+5/16*(a+b*arccsc(c*x)
)/d^2/(d/x+(-d)^(1/2)*e^(1/2))+5/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/
c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)+5/16
*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/
x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)-1/16*b*c*e^(1/2)*(1-1/c^2/x^2)^(1/2)/(-
d)^(3/2)/(c^2*d+e)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*e^(1/2)*(1-1/c^2/x^2)
^(1/2)/(-d)^(3/2)/(c^2*d+e)/(d/x+(-d)^(1/2)*e^(1/2))
```

**Rubi** [A]

time = 3.54, antiderivative size = 1134, normalized size of antiderivative = 1.00, number of steps used = 81, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5339, 4817, 4757, 4827, 745, 739, 212, 4825, 4615, 2221, 2317, 2438}

$$-\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{e}(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{\sqrt{e}(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}+\frac{5b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{dc^2+e}}-\frac{be\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(dc^2+e)^{3/2}}+\frac{5b\tanh^{-1}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{dc^2+e}}-\frac{be\tanh^{-1}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(dc^2+e)^{3/2}}-\frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3(a+b\csc^{-1}(cx))\log\left(\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3(a+b\csc^{-1}(cx))\log\left(\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3ib\text{Li}_2\left(-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3ib\text{Li}_2\left(\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3ib\text{Li}_2\left(-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3ib\text{Li}_2\left(\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/((-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]
*Sqrt[e] - d/x)) - (b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*
d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcCsc[c*x]))/(16*(-d)^(
3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcCsc[c*x]))/(16*d^2*(Sqrt[-d
]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcCsc[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*
Sqrt[e] + d/x)^2) + (5*(a + b*ArcCsc[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] + d/x
)) - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]
*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) + (5*b*ArcTanh[(c^
2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)
])])/(16*d^(5/2)*Sqrt[c^2*d + e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e]
)/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d
 + e)^(3/2)) + (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[
c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) - (3*(a +
b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCsc[c*x])*Log[1 + (I*c*Sq
rt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt
[e]) - (3*(a + b*ArcCsc[c*x])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCsc[c*x])*
Log[1 + (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*
(-d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*
PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(
(-d)^(5/2)*Sqrt[e]) - (((3*I)/16)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcCsc
[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^(5/2)*Sqrt[e]) + (((3*I)/16)*b*
PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcCsc[c*x]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(
(-d)^(5/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4615

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

Rule 4757

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

Rule 4817

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 4825

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
```

```
FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 4827

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^n/(e*(m + 1))), x] -
Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSin[c*x])^(n - 1
)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 5339

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{x^4\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{e^2\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2e\left(a+b\sin^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} + \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{d^2\left(e+dx^2\right)}\right)\,dx, x, \frac{1}{x}\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\text{Subst}\left(\int \frac{a\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{3\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\text{Subst}\left(\int \frac{a+b\sin^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\cdots}{\cdots}$$

$$= \frac{\sqrt{e}\left(a+b\csc^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5\left(a+b\csc^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}\left(a+b\csc^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right.\cdots}$$

$$= -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \cdots$$

$$= -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \cdots$$

$$= -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \cdots$$

$$= -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \cdots$$

**Mathematica** [A]
time = 6.03, size = 2060, normalized size = 1.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*((-3*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*
Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqr
t[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(S
qrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/
(16*d^2) - (3*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x
)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(
c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqr
t[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*d^2) + ((I/16)*((I*c*Sqrt[e]*S
qrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) -
ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqr
t[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] + c*(c
*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*((-I)*
Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/d^(3/2) - ((I/16)*(((-I)*c*
Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*
x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d*
Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e]
 + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)
*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/d^(3/2) - (3*(Pi^2 - 4*
Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4]
)/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e
])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*S
qrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e]
+ (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(
I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^
(I*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(128*d^(5/2)*Sqr
t[e]) + (3*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 +
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi +
 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (-Sqrt
```

```
[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[
1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (4*I)*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e
])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sq
rt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 + (I*S
qrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] -
(4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 8*PolyLog[2, (Sqrt[e] - Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*PolyLog[2, (Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))
/(128*d^(5/2)*Sqrt[e]))
```

**Maple** [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.70, size = 3181, normalized size = 2.81

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 3181 |
| default | Expression too large to display | 3181 |

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(3/8*a*c^3/d^2*x/(c^2*e*x^2+c^2*d)+b/c^3*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)
+2*e)*d)^(1/2)*e*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*
d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-7/4*b/c*(-(c^2
*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))
*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)^2*(e*(c^2*
d+e))^(1/2)-b/c^3*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctan((
I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2)
)/d^5/(c^2*d+e)^2*(e*(c^2*d+e))^(1/2)-b/c^3*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*e*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+
e))^(1/2)+2*e)*d)^(1/2))/d^5/(c^2*d+e)*(e*(c^2*d+e))^(1/2)-1/8*b*c^6*x^4/(c
^2*e*x^2+c^2*d)^2/(c^2*d+e)/d^2*((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2+5/8*b*c^5*x
/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)/d*e*arccsc(c*x)+3/8*b*c^5*x^3/(c^2*e*x^2+c^2
*d)^2/(c^2*d+e)/d^2*e^2*arccsc(c*x)-1/8*b*c^6*x^2/(c^2*e*x^2+c^2*d)^2/(c^2*
d+e)/d*((c^2*x^2-1)/c^2/x^2)^(1/2)*e-5/4*b*c*((c^2*d+2*(e*(c^2*d+e))^(1/2)+
2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3-5/4*b*c*(-(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c
^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2*e/d^3-5/8*b*c*(-(c^2*d-2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2
*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^(1/2)+5/8
*b*c*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^
2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)^2/d^3*
(e*(c^2*d+e))^(1/2)+b/c^3*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*a
```

```
rctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d
)^(1/2))/d^5/(c^2*d+e)-9/4*b/c*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*
e^2*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2))/d^4/(c^2*d+e)^2-b/c^3*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(
1/2)*e^3*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1
/2)+2*e)*d)^(1/2))/d^5/(c^2*d+e)^2+5/4*b/c*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2
*e)*d)^(1/2)*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e)
)^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)*(e*(c^2*d+e))^(1/2)+3/8*b*c^7*x^3/(c^2
*e*x^2+c^2*d)^2/(c^2*d+e)/d*e*arccsc(c*x)+7/4*b/c*(-(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)*d)^(1/2)*e*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*
(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)+b/c^3*(-(c^2*d-2*(e*(c^2*d+e)
)^(1/2)+2*e)*d)^(1/2)*e^2*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2
*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)-9/4*b/c*(-(c^2*d-2*(e*(c^
2*d+e))^(1/2)+2*e)*d)^(1/2)*e^2*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c
^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)^2-b/c^3*(-(c^2*d-2*
(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e^3*arctan((I/c/x+(1-1/c^2/x^2)^(1/2))*c*
d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/(c^2*d+e)^2+7/4*b/c*((c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arctanh((I/c/x+(1-1/c^2/x^2)^(1/
2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)-5/4*b/c*
((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I/c/x+(1-1/c^2/x^2)^(1
/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)*(e*(c^2
*d+e))^(1/2)+5/8*b*c*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctanh((I
/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/
(c^2*d+e)/d^3+5/8*b*c*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan((
I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2)
)/(c^2*d+e)/d^3+7/4*b/c*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*e*arcta
nh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1
/2))/d^4/(c^2*d+e)^2*(e*(c^2*d+e))^(1/2)+b/c^3*((c^2*d+2*(e*(c^2*d+e))^(1/2
)+2*e)*d)^(1/2)*e^2*arctanh((I/c/x+(1-1/c^2/x^2)^(1/2))*c*d/((c^2*d+2*(e*(c
^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^5/(c^2*d+e)^2*(e*(c^2*d+e))^(1/2)+3/8*a*c/d
^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-3/16*b*c^4/d/(c^2*d+e)*sum(1/_R1/(_R
1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)
+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*
d-4*e)*_Z^2+c^2*d))-3/16*b*c^4/d/(c^2*d+e)*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*
(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-
1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+5
/8*b*c^7*x/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccsc(c*x)-3/16*b*c^2/d^2/(c^2*d+
e)*e*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^
2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c
^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-3/16*b*c^2/d^2/(c^2*d+e)*e*sum(_R1/(_
R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1
)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2
*d-4*e)*_Z^2+c^2*d))+1/4*a*c^5*x/d/(c^2*e*x^2+c^2*d)^2)
```

**Maxima** [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((3*x^3*e + 5*d*x)/(d^2*x^4*e^2 + 2*d^3*x^2*e + d^4) + 3*arctan(x*e^(
1/2)/sqrt(d))*e^(-1/2)/d^(5/2)) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt
(c*x - 1))/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

**Fricas** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x
)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(sageVARx)]sym2po
ly/r2sym(
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{\left(e\,x^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^3,x)
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^3, x)
```

# 3.118 $\int x^5 \sqrt{d+ex^2} \left(a+b\csc^{-1}(cx)\right) dx$

**Optimal**. Leaf size=403

$$-\frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}}-\frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+}}{4}$$

```
[Out] 1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*ar
ccsc(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arccsc(c*x))/e^3-8/105*b*c*d^(7/2)*
x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)+1/1
680*b*(105*c^6*d^3-35*c^4*d^2*e+63*c^2*d*e^2+75*e^3)*x*arctanh(e^(1/2)*(c^2
*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(5/2)/(c^2*x^2)^(1/2)-1/840*b*(29*c^
2*d-25*e)*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c^3/e^2/(c^2*x^2)^(1/2)+1/42*
b*x*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/c/e^2/(c^2*x^2)^(1/2)-1/1680*b*(23*c^
4*d^2+12*c^2*d*e-75*e^2)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^5/e^2/(c^2*x
^2)^(1/2)
```

**Rubi** [A]
time = 0.84, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 5347, 12, 1629, 159, 163, 65, 223, 212, 95, 210}

$$\frac{d^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}-\frac{2d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}+\frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{8bcd^{7/2}x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{105e^3\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}\,(29c^2d-25e)\,(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}}+\frac{bx(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}\,(23c^4d^2+12c^2de-75e^2)\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] -1/1680*(b*(23*c^4*d^2 + 12*c^2*d*e - 75*e^2)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d +
 e*x^2])/(c^5*e^2*Sqrt[c^2*x^2]) - (b*(29*c^2*d - 25*e)*x*Sqrt[-1 + c^2*x^2
]*(d + e*x^2)^(3/2))/(840*c^3*e^2*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*
(d + e*x^2)^(5/2))/(42*c*e^2*Sqrt[c^2*x^2]) + (d^2*(d + e*x^2)^(3/2)*(a + b
*ArcCsc[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^3
) + ((d + e*x^2)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^3) - (8*b*c*d^(7/2)*x*ArcT
an[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(105*e^3*Sqrt[c^2*x^2]) +
 (b*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*x*ArcTanh[(Sqrt[e]
*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(1680*c^6*e^(5/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{d^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$= \frac{d^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$= \frac{d^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3} - 2d$$

$$= -\frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\,(d+}{42ce^2\sqrt{c^2x^2}}$$

$$= -\frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - b(29c^2d-$$

$$= -\frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - b(29c^2d-$$

$$= -\frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - b(29c^2d-$$

$$= -\frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - b(29c^2d-$$

$$= -\frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} - b(29c^2d-$$

**Mathematica** [A]
time = 0.33, size = 321, normalized size = 0.80

$$\frac{\sqrt{d+ex^2}\left(16ac^5(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6)+be\sqrt{1-\frac{1}{c^2x^2}}\,x(75e^2+2c^2e(19d+25ex^2)+c^4(-41d^2+22dex^2+40e^2x^4))+16bc^5(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6)\csc^{-1}(cx)\right)}{1680c^5e^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(128c^7d^{7/2}\operatorname{ArcTan}\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{1680c^6e^3\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(16*a*c^5*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6)
 + b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(75*e^2 + 2*c^2*e*(19*d + 25*e*x^2) + c^4*(-
41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)) + 16*b*c^5*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^
2*x^4 + 15*e^3*x^6)*ArcCsc[c*x]))/(1680*c^5*e^3) + (b*Sqrt[1 - 1/(c^2*x^2)]
*x*(128*c^7*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]] +
Sqrt[e]*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*ArcTanh[(Sqrt[
e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(1680*c^6*e^3*Sqrt[-1 + c^2*x
^2])
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int x^{5}(a+b\,\mathrm{arccsc}(cx))\,\sqrt{e\,x^{2}+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(105*e^3*integrate(1/105*(15*c^2*x^7*e^3 + 3*c^2*d*x^5*e^2 - 4*c^2*d^
2*x^3*e + 8*c^2*d^3*x)*e^(1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c
*x - 1))/(c^2*x^2*e^3 + (c^2*x^2*e^3 - e^3)*e^(log(c*x + 1) + log(c*x - 1))
 - e^3), x) + (15*x^6*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^3 + 3*d*x^4
*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 - 4*d^2*x^2*arctan2(1, sqrt(c*
x + 1)*sqrt(c*x - 1))*e + 8*d^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sq
rt(x^2*e + d))*b*e^(-3) + 1/105*(15*(x^2*e + d)^(3/2)*x^4*e^(-1) - 12*(x^2*
e + d)^(3/2)*d*x^2*e^(-2) + 8*(x^2*e + d)^(3/2)*d^2*e^(-3))*a
```

**Fricas [A]**
time = 2.43, size = 868, normalized size = 2.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6720*(128*b*c^7*sqrt(-d)*d^3*log((c^4*d^2*x^4 - 8*c^2*d^2*x^2 + x^4*e^2
+ 4*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(-d) +
8*d^2 - 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e
 + 63*b*c^2*d*e^2 + 75*b*e^3)*e^(1/2)*log(c^4*d^2 + 4*(c^3*d + (2*c^3*x^2 -
 c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^4 - 8*c^2*x^2 +
 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) + 4*(240*a*c^7*x^6*e^3 + 48*a*c^7*d*
x^4*e^2 - 64*a*c^7*d^2*x^2*e + 128*a*c^7*d^3 + 16*(15*b*c^7*x^6*e^3 + 3*b*c
^7*d*x^4*e^2 - 4*b*c^7*d^2*x^2*e + 8*b*c^7*d^3)*arccsc(c*x) - (41*b*c^5*d^2
*e - 5*(8*b*c^5*x^4 + 10*b*c^3*x^2 + 15*b*c)*e^3 - 2*(11*b*c^5*d*x^2 + 19*b
*c^3*d)*e^2)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d))*e^(-3)/c^7, -1/6720*(256*b
*c^7*d^(7/2)*arctan(-1/2*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x
^2*e + d)*sqrt(d)/(c^2*d^2*x^2 - d^2 + (c^2*d*x^4 - d*x^2)*e)) - (105*b*c^6
*d^3 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*e^(1/2)*log(c^4*d^2 + 4*
(c^3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*
c^4*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) - 4*(240*a*c^7*
x^6*e^3 + 48*a*c^7*d*x^4*e^2 - 64*a*c^7*d^2*x^2*e + 128*a*c^7*d^3 + 16*(15*
b*c^7*x^6*e^3 + 3*b*c^7*d*x^4*e^2 - 4*b*c^7*d^2*x^2*e + 8*b*c^7*d^3)*arccsc
(c*x) - (41*b*c^5*d^2*e - 5*(8*b*c^5*x^4 + 10*b*c^3*x^2 + 15*b*c)*e^3 - 2*(
11*b*c^5*d*x^2 + 19*b*c^3*d)*e^2)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d))*e^(-3
)/c^7]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^5, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^5*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

# 3.119 $\int x^3\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=294

$$\frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}-\frac{d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}+\frac{(d+\cdots}{}$$

```
[Out] -1/3*d*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arccs
c(c*x))/e^2+2/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(
1/2))/e^2/(c^2*x^2)^(1/2)-1/120*b*(15*c^4*d^2-10*c^2*d*e-9*e^2)*x*arctanh(e
^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(3/2)/(c^2*x^2)^(1/2)+1/2
0*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/e/(c^2*x^2)^(1/2)+1/120*b*(c^2*d+
9*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/e/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.28, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 5347, 12, 587, 159, 163, 65, 223, 212, 95, 210}

$$-\frac{d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2}+\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^2}+\frac{2bcd^{5/2}x\,\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{15e^2\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}-\frac{bx(15c^4d^2-10c^2de-9e^2)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,(c^2d+9e)\,\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(c^2*d + 9*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e*Sqrt[c^2*
x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e*Sqrt[c^2*x^2]) -
 (d*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) + ((d + e*x^2)^(5/2)*(a
+ b*ArcCsc[c*x]))/(5*e^2) + (2*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d
]*Sqrt[-1 + c^2*x^2])])/(15*e^2*Sqrt[c^2*x^2]) - (b*(15*c^4*d^2 - 10*c^2*d*
e - 9*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(12
0*c^4*e^(3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx = -\frac{d(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+$$

$$= -\frac{d(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+$$

$$= -\frac{d(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}-\frac{d(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{(d+}{}$$

$$= \frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}$$

$$= \frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}$$

$$= \frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}$$

$$= \frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}$$

$$= \frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}}$$

**Mathematica** [A]
time = 0.25, size = 257, normalized size = 0.87

$$\frac{\sqrt{d+ex^2}\left(8ac^3(-2d^2+dex^2+3e^2x^4)+be\sqrt{1-\frac{1}{c^2x^2}}\,x(9e+c^2(7d+6ex^2))+8bc^3(-2d^2+dex^2+3e^2x^4)\csc^{-1}(cx)\right)}{120c^3e^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(-16c^5d^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(-15c^4d^2+10c^2de+9e^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{120c^4e^2\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(-2*d^2 + d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 - 1/(
c^2*x^2)]*x*(9*e + c^2*(7*d + 6*e*x^2)) + 8*b*c^3*(-2*d^2 + d*e*x^2 + 3*e^2
```

```
*x^4)*ArcCsc[c*x]))/(120*c^3*e^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(-16*c^5*d^(
5/2)*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(-15*c^
4*d^2 + 10*c^2*d*e + 9*e^2)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d
+ e*x^2])]))/(120*c^4*e^2*Sqrt[-1 + c^2*x^2])
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int x^3(a+b\,\mathrm{arccsc}(cx))\,\sqrt{e\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*e^2*integrate(1/15*(3*c^2*x^5*e^2 + c^2*d*x^3*e - 2*c^2*d^2*x)*e^(
1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*x^2*e^2 + (c
^2*x^2*e^2 - e^2)*e^(log(c*x + 1) + log(c*x - 1)) - e^2), x) + (3*x^4*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 + d*x^2*arctan2(1, sqrt(c*x + 1)*sqr
t(c*x - 1))*e - 2*d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(x^2*e +
 d))*b*e^(-2) + 1/15*(3*(x^2*e + d)^(3/2)*x^2*e^(-1) - 2*(x^2*e + d)^(3/2)*
d*e^(-2))*a
```

**Fricas [A]**
time = 1.21, size = 721, normalized size = 2.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(16*b*c^5*sqrt(-d)*d^2*log((c^4*d^2*x^4 - 8*c^2*d^2*x^2 + x^4*e^2 -
4*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(-d) + 8*
d^2 - 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) - (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*
b*e^2)*e^(1/2)*log(c^4*d^2 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1
)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^
2 - 3*c^2*d)*e) + 4*(24*a*c^5*x^4*e^2 + 8*a*c^5*d*x^2*e - 16*a*c^5*d^2 + 8*
```

```
(3*b*c^5*x^4*e^2 + b*c^5*d*x^2*e - 2*b*c^5*d^2)*arccsc(c*x) + (7*b*c^3*d*e
+ 3*(2*b*c^3*x^2 + 3*b*c)*e^2)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d))*e^(-2)/c
^5, 1/480*(32*b*c^5*d^(5/2)*arctan(-1/2*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*
x^2 - 1)*sqrt(x^2*e + d)*sqrt(d)/(c^2*d^2*x^2 - d^2 + (c^2*d*x^4 - d*x^2)*e
)) - (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*e^(1/2)*log(c^4*d^2 + 4*(c^3*d
 + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^
4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) + 4*(24*a*c^5*x^4*e^2
 + 8*a*c^5*d*x^2*e - 16*a*c^5*d^2 + 8*(3*b*c^5*x^4*e^2 + b*c^5*d*x^2*e - 2*
b*c^5*d^2)*arccsc(c*x) + (7*b*c^3*d*e + 3*(2*b*c^3*x^2 + 3*b*c)*e^2)*sqrt(c
^2*x^2 - 1))*sqrt(x^2*e + d))*e^(-2)/c^5]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a + b\operatorname{acsc}(cx))\sqrt{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x**3*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^3, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

# 3.120 $\int x\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=195

$$\frac{bx\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e}-\frac{bcd^{3/2}x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}}+\frac{b(3c^2d+$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e-1/3*b*c*d^(3/2)*x*arctan((e*x^2+d)^
(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/6*b*(3*c^2*d+e)*x*arct
anh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(1/2)/(c^2*x^2)^(1/2
)+1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.13, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5345, 457, 104, 163, 65, 223, 212, 95, 210}

$$\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e}-\frac{bcd^{3/2}x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{3e\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{bx(3c^2d+e)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\,\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(6*c*Sqrt[c^2*x^2]) + ((d + e*x^2)
^(3/2)*(a + b*ArcCsc[c*x]))/(3*e) - (b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(
Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*e*Sqrt[c^2*x^2]) + (b*(3*c^2*d + e)*x*ArcT
anh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*Sqrt[e]*Sqrt[
c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
```

```
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{(bcx)\int\frac{\left(d+ex^2\right)^{3/2}}{x\sqrt{-1+c^2x^2}}\,dx}{3e\sqrt{c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{(bcx)\text{Subst}\left(\int\frac{(d+ex)^{3/2}}{x\sqrt{-1+c^2x}}\,dx,x,x\right.}{6e\sqrt{c^2x^2}}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{(bx)\text{Su}}{}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{(bcd^2x}{}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{(bcd^2x}{}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{bcd^{3/2}x}{}$$

$$=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{bcd^{3/2}x}{}$$

**Mathematica** [A]
time = 0.16, size = 196, normalized size = 1.01

$$\frac{\sqrt{d+ex^2}\left(be\sqrt{1-\frac{1}{c^2x^2}}\,x+2ac(d+ex^2)+2bc(d+ex^2)\csc^{-1}(cx)\right)}{6ce}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(2c^3d^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(3c^2d+e)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{6c^2e\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(b*e*Sqrt[1 - 1/(c^2*x^2)]*x + 2*a*c*(d + e*x^2) + 2*b*c*(
d + e*x^2)*ArcCsc[c*x]))/(6*c*e) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(2*c^3*d^(3/2
)*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(3*c^2*d +
```

```
 e)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(6*c^2*e*Sq
rt[-1 + c^2*x^2])
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int x(a + b \operatorname{arccsc}(cx)) \sqrt{e x^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2*e + d)^(3/2)*a*e^(-1) + 1/3*(3*e*integrate(1/3*(c^2*x^3*e + c^2*d*
x)*e^(1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*x^2*e
+ (c^2*x^2*e - e)*e^(log(c*x + 1) + log(c*x - 1)) - e), x) + (x^2*arctan2(1
, sqrt(c*x + 1)*sqrt(c*x - 1))*e + d*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)
))*sqrt(x^2*e + d))*b*e^(-1)
```

**Fricas [A]**
time = 0.66, size = 588, normalized size = 3.02

$$\left[\frac{\left(2\,bc^3\sqrt{-d}\,d\log\left(\frac{c^4d^2x^4-8\,c^2d^2x^2+x^4e^2+4\,(c^2dx^2-x^2e-2\,d)\sqrt{c^2x^2-1}\,\sqrt{x^2e+d}\,\sqrt{-d}+8\,d^2-2\,(3\,c^2dx^4-4\,dx^2)e}{x^4}\right)+(3\,bc^2d+be)e^{\frac{1}{2}}\log\left(c^4d^2+4\,(c^3d+(2\,c^3x^2-c)e)\sqrt{c^2x^2-1}\,\sqrt{x^2e+d}\,e^{\frac{1}{2}}+(8\,c^4x^4-8\,c^2x^2+1)e^2+2\,(4\,c^4dx^2-3\,c^2d)e\right)+4\left(2\,ac^3x^2e+2\,ac^3d+\sqrt{c^2x^2-1}\,bce+2\,(bc^3x^2e+bc^3d)\operatorname{arccsc}(cx)\right)\sqrt{x^2e+d}\right)e^{(-1)}}{24\,c^3}, -\frac{\left(4\,bc^3d^{\frac{3}{2}}\arctan\left(-\frac{(c^2dx^2-x^2e-2\,d)\sqrt{c^2x^2-1}\,\sqrt{x^2e+d}\,\sqrt{d}}{2\,(c^2d^2x^2-d^2+(c^2dx^4-dx^2)e)}\right)-(3\,bc^2d+be)e^{\frac{1}{2}}\log\left(c^4d^2+4\,(c^3d+(2\,c^3x^2-c)e)\sqrt{c^2x^2-1}\,\sqrt{x^2e+d}\,e^{\frac{1}{2}}+(8\,c^4x^4-8\,c^2x^2+1)e^2+2\,(4\,c^4dx^2-3\,c^2d)e\right)-4\left(2\,ac^3x^2e+2\,ac^3d+\sqrt{c^2x^2-1}\,bce+2\,(bc^3x^2e+bc^3d)\operatorname{arccsc}(cx)\right)\sqrt{x^2e+d}\right)e^{(-1)}}{24\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(2*b*c^3*sqrt(-d)*d*log((c^4*d^2*x^4 - 8*c^2*d^2*x^2 + x^4*e^2 + 4*(c
^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(-d) + 8*d^2
- 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) + (3*b*c^2*d + b*e)*e^(1/2)*log(c^4*d^2
 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2)
+ (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) + 4*(2*a*c
^3*x^2*e + 2*a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*x^2*e + b*c^3*d)*
arccsc(c*x))*sqrt(x^2*e + d))*e^(-1)/c^3, -1/24*(4*b*c^3*d^(3/2)*arctan(-1/
2*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(d)/(c^2*
d^2*x^2 - d^2 + (c^2*d*x^4 - d*x^2)*e)) - (3*b*c^2*d + b*e)*e^(1/2)*log(c^4
*d^2 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1
```

```
/2) + (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) - 4*(2
*a*c^3*x^2*e + 2*a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*x^2*e + b*c^3
*d)*arccsc(c*x))*sqrt(x^2*e + d))*e^(-1)/c^3]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\,\mathrm{acsc}\,(cx))\,\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

# 3.121 $\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x)
```

**Rubi** [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x}\,dx$$

**Mathematica** [A]
time = 3.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x, x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccsc}(cx))\,\sqrt{e\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*arcsinh(sqrt(d)*e^(-1/2)/abs(x)) - sqrt(x^2*e + d))*a + b*integra
te(sqrt(x^2*e + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsc(c*x) + a)/x, x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsc}(cx)) \sqrt{d + ex^2}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x, x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x, x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x, x)
```

# 3.122 $\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx$$

**Mathematica** [A]
time = 2.91, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3, x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccsc}(cx))\,\sqrt{e\,x^2+d}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e/sqrt(d) - sqrt(x^2*e + d)*e/d + (x
^2*e + d)^(3/2)/(d*x^2))*a + b*integrate(sqrt(x^2*e + d)*arctan2(1, sqrt(c*
x + 1)*sqrt(c*x - 1))/x^3, x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsc(c*x) + a)/x^3, x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsc}(cx))\sqrt{d + ex^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**3, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^3, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^3, x)
```

# 3.123 $\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

**Mathematica** [A]
time = 8.06, size = 0, normalized size = 0.00

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^2(a+b\,\mathrm{arccsc}(cx))\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(d^2*arcsinh(x*e^(1/2)/sqrt(d))*e^(-3/2) - 2*(x^2*e + d)^(3/2)*x*e^(-1
) + sqrt(x^2*e + d)*d*x*e^(-1))*a + b*integrate(sqrt(x^2*e + d)*x^2*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(x^2*e + d), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\operatorname{acsc}(cx))\sqrt{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^2, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

# 3.124 $\int \sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))\,,x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))\,dx = \int \sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))\,dx$$

**Mathematica** [A]
time = 34.70, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (a+b\,\text{arccsc}(cx))\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(d*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + sqrt(x^2*e + d)*x)*a + b*integ
rate(sqrt(x^2*e + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsc(c*x) + a), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{acsc}(cx))\sqrt{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
[Out] int((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

## 3.125 $\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

**Rubi** [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx$$

**Mathematica** [A]
time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2, x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccsc}(cx))\,\sqrt{e\,x^2+d}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] (arcsinh(x*e^(1/2)/sqrt(d))*e^(1/2) - sqrt(x^2*e + d)/x)*a + b*integrate(sq
rt(x^2*e + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsc(c*x) + a)/x^2, x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsc}\,(cx))\,\sqrt{d + ex^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**2, x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^2, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^2, x)
```

## 3.126 $\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx$

**Optimal**. Leaf size=328

$$-\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}+\frac{2bc^2(c$$

```
[Out] -1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/d/x^3-2/9*b*c*(c^2*d+2*e)*(c^2*x^2-1
)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)-1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+
d)^(1/2)/x^2/(c^2*x^2)^(1/2)+2/9*b*c^2*(c^2*d+2*e)*x*EllipticE(c*x,(-e/c^2/
d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^
(1/2)/(1+e*x^2/d)^(1/2)-1/9*b*(c^2*d+e)*(2*c^2*d+3*e)*x*EllipticF(c*x,(-e/c
^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^
2-1)^(1/2)/(e*x^2+d)^(1/2)
```

**Rubi** [A]
time = 0.29, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {270, 5347, 12, 485, 597, 538, 438, 437, 435, 432, 430}

$$-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}-\frac{bx\sqrt{1-c^2x^2}\left(c^2d+e\right)\left(2c^2d+3e\right)\sqrt{\frac{ex^2}{d}+1}\,F\left(\text{ArcSin}(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d\sqrt{c^2x^2}\,\sqrt{c^2x^2-1}\,\sqrt{d+ex^2}}+\frac{2bc^2x\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{d+ex^2}\,E\left(\text{ArcSin}(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d\sqrt{c^2x^2}\,\sqrt{c^2x^2-1}\,\sqrt{\frac{ex^2}{d}+1}}-\frac{2bc\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}\,\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] (-2*b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d*Sqrt[c^2*x^2
]) - (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*x^2*Sqrt[c^2*x^2]) - ((d +
 e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*d*x^3) + (2*b*c^2*(c^2*d + 2*e)*x*Sqr
t[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d*S
qrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(2*c^
2*d + 3*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -
(e/(c^2*d))])/(9*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3dx^3} + \frac{(bcx)\int -\frac{(d+ex^2)^{3/2}}{3dx^4\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3dx^3} - \frac{(bcx)\int \frac{(d+ex^2)^{3/2}}{x^4\sqrt{-1+c^2x^2}}\,dx}{3d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} - \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3dx^3} + \frac{(bcx)\int}{}$$

$$= -\frac{2bc(c^2d+2e)\,\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} -$$

$$= -\frac{2bc(c^2d+2e)\,\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} -$$

$$= -\frac{2bc(c^2d+2e)\,\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} -$$

$$= -\frac{2bc(c^2d+2e)\,\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} -$$

$$= -\frac{2bc(c^2d+2e)\,\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}} -$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 7.25, size = 247, normalized size = 0.75

$$-\frac{\sqrt{d+ex^2}\left(3a(d+ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}\,x(d+2c^2dx^2+4ex^2)+3b(d+ex^2)\csc^{-1}(cx)\right)}{9dx^3}+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d(c^2d+2e)\,E\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)|-\frac{e}{c^2d}\right)-(2c^4d^2+5c^2de+3e^2)\,F\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)|-\frac{e}{c^2d}\right)\right)}{9\sqrt{-c^2}\,d\sqrt{1-c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^4,x]
```

```
[Out] -1/9*(Sqrt[d + e*x^2]*(3*a*(d + e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2
*c^2*d*x^2 + 4*e*x^2) + 3*b*(d + e*x^2)*ArcCsc[c*x]))/(d*x^3) + ((I/9)*b*c*
```

```
Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d + 2*e)*Elliptic
E[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (2*c^4*d^2 + 5*c^2*d*e + 3*e^2)*
EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d*Sqrt[1 - c
^2*x^2]*Sqrt[d + e*x^2])
```

**Maple** [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccsc}(cx))\,\sqrt{e\,x^2+d}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(x^2*e + d)^(3/2)*a/(d*x^3) - 1/3*(3*d*x^3*integrate(1/3*(c^2*x^2*e +
c^2*d)*e^(1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*d*
x^4 - d*x^2 + (c^2*d*x^4 - d*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x) + (x
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e + d*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)))*sqrt(x^2*e + d))*b/(d*x^3)
```

**Fricas** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acsc}\,(cx))\,\sqrt{d+ex^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**4, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^4, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^4, x)
```

# 3.127 $\int \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{x^6}\,dx$

**Optimal**. Leaf size=453

$$-\frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}}-\frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2}}{25dx}$$

```
[Out] -1/5*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*ar
ccsc(c*x))/d^2/x^3+2/15*b*c*e^2*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*
x^2)^(1/2)-1/45*b*c*e*(2*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^
2*x^2)^(1/2)-1/75*b*c*(8*c^4*d^2+3*c^2*d*e-2*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+
d)^(1/2)/d^2/(c^2*x^2)^(1/2)-1/25*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^4
/(c^2*x^2)^(1/2)-1/45*b*c*e*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^
2)^(1/2)-1/75*b*c*(4*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*
x^2)^(1/2)-2/15*b*c^2*e^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)
+1/45*b*c^2*e*(2*c^2*d+e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)
+1/75*b*c^2*(8*c^4*d^2+3*c^2*d*e-2*e^2)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(
-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+
e*x^2/d)^(1/2)-1/75*b*c^2*(8*c^2*d-e)*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^
(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(
1/2)/(e*x^2+d)^(1/2)-2/45*b*c^2*e*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2
))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)
/(e*x^2+d)^(1/2)+2/15*b*e^2*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c
^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*
x^2+d)^(1/2)
```

**Rubi** [A]

time = 0.41, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {277, 270, 5347, 12, 594, 597, 538, 438, 437, 435, 432, 430}

$$\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}-\frac{bx\sqrt{1-c^2x^2}\,(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,F\left(\text{ArcSin}(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\,\sqrt{c^2x^2-1}\,\sqrt{d+ex^2}}+\frac{bc^2x\sqrt{1-c^2x^2}\,(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}\,E\left(\text{ArcSin}(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\,\sqrt{c^2x^2-1}\,\sqrt{\frac{ex^2}{d}+1}}-\frac{bc\sqrt{c^2x^2-1}\,(12c^2d-e)\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}\,(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}\,(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(b*c*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d +
e*x^2])/(d^2*Sqrt[c^2*x^2]) - (b*c*(12*c^2*d - e)*Sqrt[-1 + c^2*x^2]*Sqrt[d
 + e*x^2])/(225*d*x^2*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^
(3/2))/(25*d*x^4*Sqrt[c^2*x^2]) - ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(
```

```
5*d*x^5) + (2*e*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(15*d^2*x^3) + (b*c^
2*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*El
lipticE[ArcSin[c*x], -(e/(c^2*d))])/(225*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^
2]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(24*c^4*d^2 + 7*c^2*d*e - 30*e^2)*
x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))
])/(225*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
```

```
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{15d^2x^3} + \cdots$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{15d^2x^3} + \cdots$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+\cdots}{\cdots}$$

$$= -\frac{bc\left(12c^2d-e\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{25dx^4\sqrt{c^2x^2}} - \cdots$$

$$= -\frac{bc\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc\left(12c^2d-e\right)\cdots}{225\cdots}$$

$$= -\frac{bc\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc\left(12c^2d-e\right)\cdots}{225\cdots}$$

$$= -\frac{bc\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc\left(12c^2d-e\right)\cdots}{225\cdots}$$

$$= -\frac{bc\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc\left(12c^2d-e\right)\cdots}{225\cdots}$$

$$= -\frac{bc\left(24c^4d^2+19c^2de-31e^2\right)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}} - \frac{bc\left(12c^2d-e\right)\cdots}{225\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 9.78, size = 325, normalized size = 0.72

$$-\frac{\sqrt{d+ex^2}\left(15a\left(3d^2+dex^2-2e^2x^4\right)+bc\sqrt{1-\frac{1}{c^2x^2}}\,x\left(-31e^2x^4+dex^2\left(8+19c^2x^2\right)+3d^2\left(3+4c^2x^2+8c^4x^4\right)\right)+15b\left(3d^2+dex^2-2e^2x^4\right)\csc^{-1}(cx)\right)}{225d^2x^5}+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2d\left(24c^4d^2+19c^2de-31e^2\right)E\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)|-\frac{e}{c^2d}\right)+\left(-24c^6d^3-31c^4d^2e+23c^2de^2+30e^3\right)F\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)|-\frac{e}{c^2d}\right)\right)}{225\sqrt{-c^2}\,d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/225*(Sqrt[d + e*x^2]*(15*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*Sqrt[1 -
1/(c^2*x^2)]*x*(-31*e^2*x^4 + d*e*x^2*(8 + 19*c^2*x^2) + 3*d^2*(3 + 4*c^2*x
^2 + 8*c^4*x^4)) + 15*b*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*ArcCsc[c*x]))/(d^2*x^
5) + ((I/225)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(24*c^
4*d^2 + 19*c^2*d*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d)
)] + (-24*c^6*d^3 - 31*c^4*d^2*e + 23*c^2*d*e^2 + 30*e^3)*EllipticF[I*ArcSi
nh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d
+ e*x^2])
```

**Maple** [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsc}(cx)) \sqrt{e\,x^2 + d}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

```
[Out] int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/15*a*(2*(x^2*e + d)^(3/2)*e/(d^2*x^3) - 3*(x^2*e + d)^(3/2)/(d*x^5)) + 1/
15*(15*d^2*x^5*integrate(1/15*(2*c^2*x^4*e^2 - c^2*d*x^2*e - 3*c^2*d^2)*e^(
1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*d^2*x^6 - d^
2*x^4 + (c^2*d^2*x^6 - d^2*x^4)*e^(log(c*x + 1) + log(c*x - 1))), x) + (2*x
^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 - d*x^2*arctan2(1, sqrt(c*x
+ 1)*sqrt(c*x - 1))*e - 3*d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt
(x^2*e + d))*b/(d^2*x^5)
```

**Fricas** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsc}(cx))\sqrt{d + ex^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**6,x)
```

```
[Out] Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**6, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^6, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2 + d}\,\left(a + b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^6, x)
```

# 3.128 $\int x^3(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=374

$$-\frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}}+\frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+}}{4}$$

```
[Out] -1/5*d*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arccs
c(c*x))/e^2+2/35*b*c*d^(7/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(
1/2))/e^2/(c^2*x^2)^(1/2)-1/560*b*(35*c^6*d^3-35*c^4*d^2*e-63*c^2*d*e^2-25*
e^3)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(3/2)/(c^
2*x^2)^(1/2)+1/840*b*(13*c^2*d+25*e)*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c^
3/e/(c^2*x^2)^(1/2)+1/42*b*x*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/c/e/(c^2*x^2
)^(1/2)-1/560*b*(3*c^4*d^2-38*c^2*d*e-25*e^2)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)
^(1/2)/c^5/e/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.36, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 5347, 12, 587, 159, 163, 65, 223, 212, 95, 210}

$$-\frac{d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2}+\frac{2bcd^{7/2}x\,\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{35e^2\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,(13c^2d+25e)\,(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}}-\frac{bx(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}\,(3c^4d^2-38c^2de-25e^2)\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] -1/560*(b*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e
*x^2])/(c^5*e*Sqrt[c^2*x^2]) + (b*(13*c^2*d + 25*e)*x*Sqrt[-1 + c^2*x^2]*(d
 + e*x^2)^(3/2))/(840*c^3*e*Sqrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e
*x^2)^(5/2))/(42*c*e*Sqrt[c^2*x^2]) - (d*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*
x]))/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^2) + (2*b*c*d^(
7/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(35*e^2*Sqrt[c
^2*x^2]) - (b*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2 - 25*e^3)*x*ArcTanh
[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^6*e^(3/2)*Sqrt[c
^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\, dx = -\frac{d\left(d+ex^2\right)^{5/2} \left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2} \left(a+b\csc^{-1}(cx)\right)}{7e^2} +$$

$$= -\frac{d\left(d+ex^2\right)^{5/2} \left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2} \left(a+b\csc^{-1}(cx)\right)}{7e^2} +$$

$$= -\frac{d\left(d+ex^2\right)^{5/2} \left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{\left(d+ex^2\right)^{7/2} \left(a+b\csc^{-1}(cx)\right)}{7e^2} +$$

$$= \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d\left(d+ex^2\right)^{5/2} \left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{(d}{}$$

$$= \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\,(d+e}{42ce\sqrt{c^2x^2}}$$

$$= -\frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d+2}{}$$

$$= -\frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d+2}{}$$

$$= -\frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d+2}{}$$

$$= -\frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d+2}{}$$

$$= -\frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} + \frac{b(13c^2d+2}{}$$

**<u>Mathematica</u> [A]**
time = 0.33, size = 295, normalized size = 0.79

$$\frac{\sqrt{d+ex^2}\left(-48ac^5(2d-5ex^2)\left(d+ex^2\right)^2+be\sqrt{1-\frac{1}{c^2x^2}}\,x\left(75e^2+2c^2e(82d+25ex^2)+c^4(57d^2+106dex^2+40e^2x^4)\right)-48bc^5(2d-5ex^2)\left(d+ex^2\right)^2\csc^{-1}(cx)\right)}{1680c^5e^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(-32c^7d^{7/2}\mathrm{ArcTan}\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\left(-35c^6d^3+35c^4d^2e+63c^2de^2+25e^3\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{560c^6e^2\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-48*a*c^5*(2*d - 5*e*x^2)*(d + e*x^2)^2 + b*e*Sqrt[1 - 1/
(c^2*x^2)]*x*(75*e^2 + 2*c^2*e*(82*d + 25*e*x^2) + c^4*(57*d^2 + 106*d*e*x^
2 + 40*e^2*x^4)) - 48*b*c^5*(2*d - 5*e*x^2)*(d + e*x^2)^2*ArcCsc[c*x]))/(16
80*c^5*e^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(-32*c^7*d^(7/2)*ArcTan[(Sqrt[d]*S
qrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(-35*c^6*d^3 + 35*c^4*d^2*e +
 63*c^2*d*e^2 + 25*e^3)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*
x^2])]))/(560*c^6*e^2*Sqrt[-1 + c^2*x^2])
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int x^3 \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsc}(cx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/35*(35*e^2*integrate(1/35*(5*c^2*x^7*e^3 + 8*c^2*d*x^5*e^2 + c^2*d^2*x^3*
e - 2*c^2*d^3*x)*e^(1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1
))/(c^2*x^2*e^2 + (c^2*x^2*e^2 - e^2)*e^(log(c*x + 1) + log(c*x - 1)) - e^2
), x) + (5*x^6*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^3 + 8*d*x^4*arctan
2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 + d^2*x^2*arctan2(1, sqrt(c*x + 1)*sq
rt(c*x - 1))*e - 2*d^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(x^2*e
+ d))*b*e^(-2) + 1/35*(5*(x^2*e + d)^(5/2)*x^2*e^(-1) - 2*(x^2*e + d)^(5/2)
*d*e^(-2))*a
```

**Fricas [A]**
time = 2.37, size = 865, normalized size = 2.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] [1/6720*(96*b*c^7*sqrt(-d)*d^3*log((c^4*d^2*x^4 - 8*c^2*d^2*x^2 + x^4*e^2 -
 4*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(-d) + 8
```

```
*d^2 - 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) - 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e
 - 63*b*c^2*d*e^2 - 25*b*e^3)*e^(1/2)*log(c^4*d^2 + 4*(c^3*d + (2*c^3*x^2 -
 c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^4 - 8*c^2*x^2 +
 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) + 4*(240*a*c^7*x^6*e^3 + 384*a*c^7*d
*x^4*e^2 + 48*a*c^7*d^2*x^2*e - 96*a*c^7*d^3 + 48*(5*b*c^7*x^6*e^3 + 8*b*c^
7*d*x^4*e^2 + b*c^7*d^2*x^2*e - 2*b*c^7*d^3)*arccsc(c*x) + (57*b*c^5*d^2*e
+ 5*(8*b*c^5*x^4 + 10*b*c^3*x^2 + 15*b*c)*e^3 + 2*(53*b*c^5*d*x^2 + 82*b*c^
3*d)*e^2)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d))*e^(-2)/c^7, 1/6720*(192*b*c^7
*d^(7/2)*arctan(-1/2*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e
 + d)*sqrt(d)/(c^2*d^2*x^2 - d^2 + (c^2*d*x^4 - d*x^2)*e)) - 3*(35*b*c^6*d^
3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*e^(1/2)*log(c^4*d^2 + 4*(c^
3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4
*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) + 4*(240*a*c^7*x^6
*e^3 + 384*a*c^7*d*x^4*e^2 + 48*a*c^7*d^2*x^2*e - 96*a*c^7*d^3 + 48*(5*b*c^
7*x^6*e^3 + 8*b*c^7*d*x^4*e^2 + b*c^7*d^2*x^2*e - 2*b*c^7*d^3)*arccsc(c*x)
+ (57*b*c^5*d^2*e + 5*(8*b*c^5*x^4 + 10*b*c^3*x^2 + 15*b*c)*e^3 + 2*(53*b*c
^5*d*x^2 + 82*b*c^3*d)*e^2)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d))*e^(-2)/c^7]
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x^3, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

# 3.129 $\int x(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=262

$$\frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{bcd^{5/2}}{}$$

```
[Out] 1/5*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/e-1/5*b*c*d^(5/2)*x*arctan((e*x^2+d)^
(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/40*b*(15*c^4*d^2+10*c^
2*d*e+3*e^2)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(
1/2)/(c^2*x^2)^(1/2)+1/20*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)
^(1/2)+1/40*b*(7*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/(c^2*x^
2)^(1/2)
```

**Rubi** [**A**]
time = 0.19, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5345, 457, 104, 159, 163, 65, 223, 212, 95, 210}

$$\frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{bcd^{5/2}x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{5e\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}}+\frac{bx(15c^4d^2+10c^2de+3e^2)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\,\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,(7c^2d+3e)\,\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*(7*c^2*d + 3*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(40*c^3*Sqrt[c^2*x
^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*Sqrt[c^2*x^2]) + ((
d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e) - (b*c*d^(5/2)*x*ArcTan[Sqrt[d
+ e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(5*e*Sqrt[c^2*x^2]) + (b*(15*c^4*d^
2 + 10*c^2*d*e + 3*e^2)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d +
e*x^2])])/(40*c^4*Sqrt[e]*Sqrt[c^2*x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} + \frac{(bcx)\int\frac{\left(d+ex^2\right)^{5/2}}{x\sqrt{-1+c^2x^2}}\,dx}{5e\sqrt{c^2x^2}}$$

$$= \frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} + \frac{(bcx)\text{Subst}\left(\int\frac{(d+ex)^{5/2}}{x\sqrt{-1+c^2x}}\,dx,x,\right.}{10e\sqrt{c^2x^2}}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{c^2x^2}} + \frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} + \frac{(bx)}{}$$

$$= \frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/}}{20c\sqrt{c^2x^2}}$$

$$= \frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/}}{20c\sqrt{c^2x^2}}$$

$$= \frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/}}{20c\sqrt{c^2x^2}}$$

$$= \frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/}}{20c\sqrt{c^2x^2}}$$

$$= \frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\left(d+ex^2\right)^{3/}}{20c\sqrt{c^2x^2}}$$

**<u>Mathematica</u> [A]**
time = 2.78, size = 235, normalized size = 0.90

$$\frac{\sqrt{d+ex^2}\left(8ac^3\left(d+ex^2\right)^2+be\sqrt{1-\frac{1}{c^2x^2}}\,x\left(3e+c^2\left(9d+2ex^2\right)\right)+8bc^3\left(d+ex^2\right)^2\csc^{-1}(cx)\right)}{40c^3e}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(8c^5d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\left(15c^4d^2+10c^2de+3e^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{40c^4e\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(d + e*x^2)^2 + b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(3*e
+ c^2*(9*d + 2*e*x^2)) + 8*b*c^3*(d + e*x^2)^2*ArcCsc[c*x]))/(40*c^3*e) + (
b*Sqrt[1 - 1/(c^2*x^2)]*x*(8*c^5*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2]
)/Sqrt[d + e*x^2]] + Sqrt[e]*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*ArcTanh[(Sqr
```

```
t[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(40*c^4*e*Sqrt[-1 + c^2*x^2
])
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int x\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccsc}(cx)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] 1/5*(x^2*e + d)^(5/2)*a*e^(-1) + 1/5*(5*e*integrate(1/5*(c^2*x^5*e^2 + 2*c^
2*d*x^3*e + c^2*d^2*x)*e^(1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c
*x - 1))/(c^2*x^2*e + (c^2*x^2*e - e)*e^(log(c*x + 1) + log(c*x - 1)) - e),
 x) + (x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 + 2*d*x^2*arctan2(1,
 sqrt(c*x + 1)*sqrt(c*x - 1))*e + d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
)))*sqrt(x^2*e + d))*b*e^(-1)
```

**Fricas [A]**
time = 1.16, size = 716, normalized size = 2.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] [1/160*(8*b*c^5*sqrt(-d)*d^2*log((c^4*d^2*x^4 - 8*c^2*d^2*x^2 + x^4*e^2 + 4
*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(-d) + 8*d
^2 - 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b
*e^2)*e^(1/2)*log(c^4*d^2 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1)
*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^2
 - 3*c^2*d)*e) + 4*(8*a*c^5*x^4*e^2 + 16*a*c^5*d*x^2*e + 8*a*c^5*d^2 + 8*(b
*c^5*x^4*e^2 + 2*b*c^5*d*x^2*e + b*c^5*d^2)*arccsc(c*x) + (9*b*c^3*d*e + (2
*b*c^3*x^2 + 3*b*c)*e^2)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d))*e^(-1)/c^5, -1
/160*(16*b*c^5*d^(5/2)*arctan(-1/2*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 -
```

```
1)*sqrt(x^2*e + d)*sqrt(d)/(c^2*d^2*x^2 - d^2 + (c^2*d*x^4 - d*x^2)*e)) -
(15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*e^(1/2)*log(c^4*d^2 + 4*(c^3*d + (2
*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^4 - 8
*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) - 4*(8*a*c^5*x^4*e^2 + 16*
a*c^5*d*x^2*e + 8*a*c^5*d^2 + 8*(b*c^5*x^4*e^2 + 2*b*c^5*d*x^2*e + b*c^5*d^
2)*arccsc(c*x) + (9*b*c^3*d*e + (2*b*c^3*x^2 + 3*b*c)*e^2)*sqrt(c^2*x^2 - 1
))*sqrt(x^2*e + d))*e^(-1)/c^5]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\operatorname{acsc}(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
[Out] Integral(x*(a + b*acsc(c*x))*(d + e*x**2)**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

# 3.130 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x}\,dx$

**Optimal**. Leaf size=26

$$\operatorname{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)
```

**Rubi** [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x}\,dx$$

**Mathematica** [A]
time = 3.75, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x, x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arccsc}(cx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/3*(3*d^(3/2)*arcsinh(sqrt(d)*e^(-1/2)/abs(x)) - (x^2*e + d)^(3/2) - 3*sq
rt(x^2*e + d)*d)*a + (e*integrate(sqrt(x^2*e + d)*x*arctan2(1, sqrt(c*x + 1
)*sqrt(c*x - 1)), x) + d*integrate(sqrt(x^2*e + d)*arctan2(1, sqrt(c*x + 1)
*sqrt(c*x - 1))/x, x))*b
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsc(c*x))*sqrt(x^2*e + d)/x, x
)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsc}(cx))\left(d + ex^2\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x, x)
```

# 3.131 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)
```

**Rubi** [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx$$

**Mathematica** [A]
time = 3.04, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3, x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arccsc}(cx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*sqrt(d)*arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e - 3*sqrt(x^2*e + d)*e -
(x^2*e + d)^(3/2)*e/d + (x^2*e + d)^(5/2)/(d*x^2))*a + (e*integrate(sqrt(x^
2*e + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) + d*integrate(sqrt(x
^2*e + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x))*b
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsc(c*x))*sqrt(x^2*e + d)/x^3,
 x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsc}(cx))(d + ex^2)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^3, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^3, x)
```

# 3.132 $\int x^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(x^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

**Rubi** [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

**Mathematica** [A]
time = 8.38, size = 0, normalized size = 0.00

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int x^2\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccsc}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

```
[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] -1/48*(3*d^3*arcsinh(x*e^(1/2)/sqrt(d))*e^(-3/2) - 8*(x^2*e + d)^(5/2)*x*e^
(-1) + 2*(x^2*e + d)^(3/2)*d*x*e^(-1) + 3*sqrt(x^2*e + d)*d^2*x*e^(-1))*a +
 b*integrate((x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e + d*x^2*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(x^2*e + d), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e + a*d*x^2 + (b*x^4*e + b*d*x^2)*arccsc(c*x))*sqrt(x^2*e +
 d), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x^2, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2\,{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

# 3.133 $\int \left(d+e x^2\right)^{3/2} \left(a+b \csc^{-1}(c x)\right) \, dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\left(d+e x^2\right)^{3/2} \left(a+b \csc^{-1}(c x)\right),x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

**Rubi** [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(d+e x^2\right)^{3/2} \left(a+b \csc^{-1}(c x)\right) \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int \left(d+e x^2\right)^{3/2} \left(a+b \csc^{-1}(c x)\right) \, dx=\int \left(d+e x^2\right)^{3/2} \left(a+b \csc^{-1}(c x)\right) \, dx$$

**Mathematica** [A]
time = 35.29, size = 0, normalized size = 0.00

$$\int \left(d+e x^2\right)^{3/2} \left(a+b \csc^{-1}(c x)\right) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \,\mathrm{arccsc}\left(c x \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*d^2*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + 2*(x^2*e + d)^(3/2)*x + 3*
sqrt(x^2*e + d)*d*x)*a + b*integrate((x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
 - 1))*e + d*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(x^2*e + d), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsc(c*x))*sqrt(x^2*e + d), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{acsc}(cx)) \left(d + ex^2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
[Out] int((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

# 3.134 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx$$

**Mathematica** [A]
time = 68.85, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2, x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}(a+b\,\text{arccsc}(cx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*d*arcsinh(x*e^(1/2)/sqrt(d))*e^(1/2) + 3*sqrt(x^2*e + d)*x*e - 2*(x^
2*e + d)^(3/2)/x)*a + (e*integrate(sqrt(x^2*e + d)*arctan2(1, sqrt(c*x + 1)
*sqrt(c*x - 1)), x) + d*integrate(sqrt(x^2*e + d)*arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1))/x^2, x))*b
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsc(c*x))*sqrt(x^2*e + d)/x^2,
 x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsc}(cx))\,(d + ex^2)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**2,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^2, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^2, x)
```

# 3.135 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4} \, dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^4},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^4} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^4} \, dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^4} \, dx$$

**Mathematica** [A]
time = 4.58, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^4} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4, x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\left(a+b\,\mathrm{arccsc}(cx)\right)}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*arcsinh(x*e^(1/2)/sqrt(d))*e^(3/2) + 3*sqrt(x^2*e + d)*x*e^2/d - 2*(
x^2*e + d)^(3/2)*e/(d*x) - (x^2*e + d)^(5/2)/(d*x^3))*a + (e*integrate(sqrt
(x^2*e + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x) + d*integrate(s
qrt(x^2*e + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^4, x))*b
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsc(c*x))*sqrt(x^2*e + d)/x^4,
 x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsc}(cx))(d + ex^2)^{\frac{3}{2}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^4, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^4, x)
```

# 3.136 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx$

**Optimal**. Leaf size=416

$$-\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}}-\frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x}}{25x^4}$$

```
[Out] -1/5*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/d/x^5-1/25*b*c*(e*x^2+d)^(3/2)*(c^2*
x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)-1/75*b*c*(8*c^4*d^2+23*c^2*d*e+23*e^2)*(c^
2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)-4/75*b*c*(c^2*d+2*e)*(c^2*
x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^2/(c^2*x^2)^(1/2)+1/75*b*c^2*(8*c^4*d^2+23*c
^2*d*e+23*e^2)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+
d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-1/75*b*(c^2*
d+e)*(8*c^4*d^2+19*c^2*d*e+15*e^2)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*
x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d
)^(1/2)
```

**Rubi** [A]
time = 0.38, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {270, 5347, 12, 485, 594, 597, 538, 438, 437, 435, 432, 430}

$$-\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5}-\frac{bx\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,F\left(\text{ArcSin}(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{bc^2x\sqrt{1-c^2x^2}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}\,E\left(\text{ArcSin}(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{4bc\sqrt{c^2x^2-1}(c^2d+2e)\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/75*(b*c*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*
x^2])/(d*Sqrt[c^2*x^2]) - (4*b*c*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d +
e*x^2])/(75*x^2*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))
/(25*x^4*Sqrt[c^2*x^2]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*d*x^5)
 + (b*c^2*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*
x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(75*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^
2*x^2]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(8*c^4*d^2 + 19*c^2*d*e + 15*e
^2)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2
*d))])/(75*d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5dx^5} + \frac{(bcx)\int -\frac{(d+ex^2)^{5/2}}{5dx^6\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5dx^5} - \frac{(bcx)\int \frac{(d+ex^2)^{5/2}}{x^6\sqrt{-1+c^2x^2}}\,dx}{5d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5dx^5} + \frac{(bcx)\cdots}{\cdots}$$

$$= -\frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}}\cdots$$

$$= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\cdots}{75\cdots}$$

$$= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\cdots}{75\cdots}$$

$$= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\cdots}{75\cdots}$$

$$= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\cdots}{75\cdots}$$

$$= -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\cdots}{75\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 10.20, size = 303, normalized size = 0.73

$$-\frac{\sqrt{d+ex^2}\left(15a(d+ex^2)^2+bc\sqrt{1-\frac{1}{c^2x^2}}\,x(23e^2x^4+dex^2(11+23c^2x^2)+d^2(3+4c^2x^2+8c^4x^4))+15b(d+ex^2)^2\csc^{-1}(cx)\right)}{75dx^5}+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(8c^4d^2+23c^2de+23e^2)\,E\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)|-\frac{e}{c^2d}\right)-(8c^6d^3+27c^4d^2e+34c^2de^2+15e^3)\,F\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)|-\frac{e}{c^2d}\right)\right)}{75\sqrt{-c^2}\,d\sqrt{1-c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

```
[Out] -1/75*(Sqrt[d + e*x^2]*(15*a*(d + e*x^2)^2 + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2
3*e^2*x^4 + d*e*x^2*(11 + 23*c^2*x^2) + d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4)) +
15*b*(d + e*x^2)^2*ArcCsc[c*x]))/(d*x^5) + ((I/75)*b*c*Sqrt[1 - 1/(c^2*x^2)
]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*EllipticE[
I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (8*c^6*d^3 + 27*c^4*d^2*e + 34*c^2
*d*e^2 + 15*e^3)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-
c^2]*d*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

**Maple** [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsc}(cx)\right)}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*(x^2*e + d)^(5/2)*a/(d*x^5) - 1/5*(5*d*x^5*integrate(1/5*(c^2*x^4*e^2
+ 2*c^2*d*x^2*e + c^2*d^2)*e^(1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*l
og(c*x - 1))/(c^2*d*x^6 - d*x^4 + (c^2*d*x^6 - d*x^4)*e^(log(c*x + 1) + log
(c*x - 1))), x) + (x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 + 2*d*x^
2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e + d^2*arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1)))*sqrt(x^2*e + d))*b/(d*x^5)
```

**Fricas** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**6,x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^6, x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^6,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^6, x)
```

# 3.137 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}\,dx$

**Optimal**. Leaf size=554

$$-\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}-\frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1}}{3675dx^2\sqrt{c^2x^2}}$$

```
[Out] -1/7*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*ar
ccsc(c*x))/d^2/x^5-1/1225*b*c*(30*c^2*d+11*e)*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(
1/2)/d/x^4/(c^2*x^2)^(1/2)-1/49*b*c*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/d/x^6
/(c^2*x^2)^(1/2)-1/3675*b*c*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^
3)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)-1/3675*b*c*(120*c^
4*d^2+159*c^2*d*e-37*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^2)
^(1/2)+1/3675*b*c^2*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3)*x*Ell
ipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^
2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-2/3675*b*(c^2*d+e)*(120*c^6*d^
3+204*c^4*d^2*e+17*c^2*d*e^2-105*e^3)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c
^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*
x^2+d)^(1/2)
```

**Rubi** [A]
time = 0.54, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {277, 270, 5347, 12, 594, 597, 538, 438, 437, 435, 432, 430}

$$\frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7}-\frac{2bx\sqrt{1-c^2x^2}(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\,F\left(\text{ArcSin}(cx)\left|-\frac{e}{c^2d}\right.\right)}{3675d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{bc^2x\sqrt{1-c^2x^2}(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{d+ex^2}\,E\left(\text{ArcSin}(cx)\left|-\frac{e}{c^2d}\right.\right)}{3675d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{bc\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}(30c^2d+11e)(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}(120c^4d^2+159c^2de-37e^2)\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] -1/3675*(b*c*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[-
1 + c^2*x^2]*Sqrt[d + e*x^2])/(d^2*Sqrt[c^2*x^2]) - (b*c*(120*c^4*d^2 + 159
*c^2*d*e - 37*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^2*Sqrt[c^2
*x^2]) - (b*c*(30*c^2*d + 11*e)*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(1225
*d*x^4*Sqrt[c^2*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(5/2))/(49*d*x^
6*Sqrt[c^2*x^2]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(7*d*x^7) + (2*e
*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(35*d^2*x^5) + (b*c^2*(240*c^6*d^3
+ 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x
^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[c^2*x^2]*Sqrt[-1 +
 c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^
2*e + 17*c^2*d*e^2 - 105*e^3)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*Ellip
ticF[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]
*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}\,dx = -\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5} + \cdots$$

$$= -\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5} + \cdots$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\,(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7} + \frac{2e(d \cdots}{\cdots}$$

$$= -\frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,(d+ex \cdots}{49dx^6\sqrt{c^2x^2}}$$

$$= -\frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} - \frac{bc(30c^2d+\cdots}{\cdots}$$

$$= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}\cdots}{3675d^2\sqrt{c^2x^2}\cdots}$$

$$= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}\cdots}{3675d^2\sqrt{c^2x^2}\cdots}$$

$$= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}\cdots}{3675d^2\sqrt{c^2x^2}\cdots}$$

$$= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}\cdots}{3675d^2\sqrt{c^2x^2}\cdots}$$

$$= -\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}\cdots}{3675d^2\sqrt{c^2x^2}\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 10.57, size = 383, normalized size = 0.69

$$-\frac{\sqrt{d+ex^2}\left(105a(5d-2ex^2)(d+ex^2)^2+bc\sqrt{1-\frac{1}{c^2x^2}}\,x(-247e^3x^6+de^2x^4(71+193c^2x^2)+3d^2ex^2(61+83c^2x^2+176c^4x^4)+15d^3(5+6c^2x^2+8c^4x^4+16c^6x^6))+105b(5d-2ex^2)(d+ex^2)^2\csc^{-1}(cx)\right)}{3675d^2x^7}+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)E\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)\middle|-\frac{e}{c^2d}\right)-2(120c^8d^4+324c^6d^3e+221c^4d^2e^2-88c^2de^3-105e^4)F\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)\middle|-\frac{e}{c^2d}\right)\right)}{3675\sqrt{-c^2}\,d^2\sqrt{1-c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

```
[Out] -1/3675*(Sqrt[d + e*x^2]*(105*a*(5*d - 2*e*x^2)*(d + e*x^2)^2 + b*c*Sqrt[1
- 1/(c^2*x^2)]*x*(-247*e^3*x^6 + d*e^2*x^4*(71 + 193*c^2*x^2) + 3*d^2*e*x^2
*(61 + 83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c
^6*x^6)) + 105*b*(5*d - 2*e*x^2)*(d + e*x^2)^2*ArcCsc[c*x]))/(d^2*x^7) + ((
I/3675)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(240*c^6*d^3
 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*EllipticE[I*ArcSinh[Sqrt[-c^2]*
x], -(e/(c^2*d))] - 2*(120*c^8*d^4 + 324*c^6*d^3*e + 221*c^4*d^2*e^2 - 88*c
^2*d*e^3 - 105*e^4)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqr
t[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

**Maple** [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsc}(cx)\right)}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/35*a*(2*(x^2*e + d)^(5/2)*e/(d^2*x^5) - 5*(x^2*e + d)^(5/2)/(d*x^7)) + 1/
35*(35*d^2*x^7*integrate(1/35*(2*c^2*x^6*e^3 - c^2*d*x^4*e^2 - 8*c^2*d^2*x^
2*e - 5*c^2*d^3)*e^(1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1
))/(c^2*d^2*x^8 - d^2*x^6 + (c^2*d^2*x^8 - d^2*x^6)*e^(log(c*x + 1) + log(c
*x - 1))), x) + (2*x^6*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^3 - d*x^4*
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 - 8*d^2*x^2*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1))*e - 5*d^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqr
t(x^2*e + d))*b/(d^2*x^7)
```

**Fricas** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

**Sympy [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^8, x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^8,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^8, x)
```

# 3.138 $\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

**Optimal**. Leaf size=321

$$-\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3}-\frac{2a}{}$$

```
[Out] -2/3*d*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arccs
c(c*x))/e^3-8/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(
1/2))/e^3/(c^2*x^2)^(1/2)+1/120*b*(45*c^4*d^2-10*c^2*d*e+9*e^2)*x*arctanh(e
^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(5/2)/(c^2*x^2)^(1/2)+1/2
0*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/e^2/(c^2*x^2)^(1/2)+d^2*(a+b*arcc
sc(c*x))*(e*x^2+d)^(1/2)/e^3-1/120*b*(19*c^2*d-9*e)*x*(c^2*x^2-1)^(1/2)*(e*
x^2+d)^(1/2)/c^3/e^2/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.72, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 5347, 12, 1629, 159, 163, 65, 223, 212, 95, 210}

$$\frac{d^2\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3}-\frac{2d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3}+\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^3}-\frac{8bcd^{5/2}x\,\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{15e^3\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}}+\frac{bx(45c^4d^2-10c^2de+9e^2)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{c^2x^2}}-\frac{bx\sqrt{c^2x^2-1}\,(19c^2d-9e)\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -1/120*(b*(19*c^2*d - 9*e)*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(c^3*e^2*S
qrt[c^2*x^2]) + (b*x*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e^2*Sqrt[c
^2*x^2]) + (d^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^3 - (2*d*(d + e*x^2)
^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]
))/(5*e^3) - (8*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2
*x^2])])/(15*e^3*Sqrt[c^2*x^2]) + (b*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*x*Ar
cTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(5/2)*S
qrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{d^2\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}}{}$$

$$= \frac{d^2\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}}{}$$

$$= \frac{d^2\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2}}{}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3} - \frac{2d(d+ex^2)^{3/2}\,(}{3e}$$

$$= -\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{}}{}$$

$$= -\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{}}{}$$

$$= -\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{}}{}$$

$$= -\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{}}{}$$

$$= -\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}} + \frac{d^2\sqrt{}}{}$$

**<u>Mathematica</u> [A]**
time = 0.27, size = 259, normalized size = 0.81

$$\frac{\sqrt{d+ex^2}\left(8ac^3(8d^2-4dex^2+3e^2x^4)+be\sqrt{1-\frac{1}{c^2x^2}}\,x(9e+c^2(-13d+6ex^2))+8bc^3(8d^2-4dex^2+3e^2x^4)\csc^{-1}(cx)\right)}{120c^3e^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(64c^5d^{5/2}\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(45c^4d^2-10c^2de+9e^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{120c^4e^3\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 - 1/
(c^2*x^2)]*x*(9*e + c^2*(-13*d + 6*e*x^2)) + 8*b*c^3*(8*d^2 - 4*d*e*x^2 + 3
*e^2*x^4)*ArcCsc[c*x]))/(120*c^3*e^3) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(64*c^5*
d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(45*
c^4*d^2 - 10*c^2*d*e + 9*e^2)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[
d + e*x^2])]))/(120*c^4*e^3*Sqrt[-1 + c^2*x^2])
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+b\,\mathrm{arccsc}(cx))}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*e^3*integrate(1/15*(3*c^2*x^7*e^3 - c^2*d*x^5*e^2 + 4*c^2*d^2*x^3*
e + 8*c^2*d^3*x)*e^(-1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c*x -
1))/(c^2*x^2*e^3 + (c^2*x^2*e^3 - e^3)*e^(log(c*x + 1) + log(c*x - 1)) - e^
3), x) + (3*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 - 4*d*x^2*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e + 8*d^2*arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1)))*sqrt(x^2*e + d))*b*e^(-3) + 1/15*(3*sqrt(x^2*e + d)*x^4*e^(-1) -
 4*sqrt(x^2*e + d)*d*x^2*e^(-2) + 8*sqrt(x^2*e + d)*d^2*e^(-3))*a
```

**Fricas** [A]
time = 1.34, size = 724, normalized size = 2.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(64*b*c^5*sqrt(-d)*d^2*log((c^4*d^2*x^4 - 8*c^2*d^2*x^2 + x^4*e^2 +
4*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(-d) + 8*
d^2 - 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) + (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*
b*e^2)*e^(1/2)*log(c^4*d^2 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1
```

```
)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^
2 - 3*c^2*d)*e) + 4*(24*a*c^5*x^4*e^2 - 32*a*c^5*d*x^2*e + 64*a*c^5*d^2 + 8
*(3*b*c^5*x^4*e^2 - 4*b*c^5*d*x^2*e + 8*b*c^5*d^2)*arccsc(c*x) - (13*b*c^3*
d*e - 3*(2*b*c^3*x^2 + 3*b*c)*e^2)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d))*e^(-
3)/c^5, -1/480*(128*b*c^5*d^(5/2)*arctan(-1/2*(c^2*d*x^2 - x^2*e - 2*d)*sqr
t(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(d)/(c^2*d^2*x^2 - d^2 + (c^2*d*x^4 - d*
x^2)*e)) - (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*e^(1/2)*log(c^4*d^2 + 4*
(c^3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*
c^4*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) - 4*(24*a*c^5*x
^4*e^2 - 32*a*c^5*d*x^2*e + 64*a*c^5*d^2 + 8*(3*b*c^5*x^4*e^2 - 4*b*c^5*d*x
^2*e + 8*b*c^5*d^2)*arccsc(c*x) - (13*b*c^3*d*e - 3*(2*b*c^3*x^2 + 3*b*c)*e
^2)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d))*e^(-3)/c^5]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a + b \operatorname{acsc}(cx))}{\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

# 3.139 $\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

**Optimal**. Leaf size=225

$$\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}}-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2bcd^{3/2}x\text{ArcTan}}{3e^2}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^2+2/3*b*c*d^(3/2)*x*arctan((e*x^2+d
)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/(c^2*x^2)^(1/2)-1/6*b*(3*c^2*d-e)*x*
arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(3/2)/(c^2*x^2)^
(1/2)-d*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/e^2+1/6*b*x*(c^2*x^2-1)^(1/2)*(e*
x^2+d)^(1/2)/c/e/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.21, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.522$, Rules used = {272, 45, 5347, 12, 587, 159, 163, 65, 223, 212, 95, 210}

$$-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}+\frac{2bcd^{3/2}x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^2\sqrt{c^2x^2}}-\frac{bx\left(3c^2d-e\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e*Sqrt[c^2*x^2]) - (d*Sqrt[d
+ e*x^2]*(a + b*ArcCsc[c*x]))/e^2 + ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))
/(3*e^2) + (2*b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x
^2])])/(3*e^2*Sqrt[c^2*x^2]) - (b*(3*c^2*d - e)*x*ArcTanh[(Sqrt[e]*Sqrt[-1
+ c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*e^(3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = -\frac{d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2} + \frac{(bcx)\int \frac{(-2d+e}{3e^2x\sqrt{}}}{\sqrt{}}$$

$$= -\frac{d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2} + \frac{(bcx)\int \frac{(-2d+e}{x\sqrt{}}}{3e^2}$$

$$= -\frac{d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2} + \frac{(bcx)\text{Subst}\Big(}{}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}\,(a+b}{3e^2}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}\,(a+b}{3e^2}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}\,(a+b}{3e^2}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}\,(a+b}{3e^2}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}\,(a+b}{3e^2}$$

**<u>Mathematica</u>** [A]
time = 0.19, size = 200, normalized size = 0.89

$$\frac{\sqrt{d+ex^2}\left(-4acd+be\sqrt{1-\frac{1}{c^2x^2}}\,x+2acex^2+2bc(-2d+ex^2)\csc^{-1}(cx)\right)}{6ce^2} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(-4c^3d^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(-3c^2d+e)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{6c^2e^2\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(-4*a*c*d + b*e*Sqrt[1 - 1/(c^2*x^2)]*x + 2*a*c*e*x^2 + 2*
b*c*(-2*d + e*x^2)*ArcCsc[c*x]))/(6*c*e^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(-4
*c^3*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]
```

```
*(-3*c^2*d + e)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))
/(6*c^2*e^2*Sqrt[-1 + c^2*x^2])
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(a+b \,\mathrm{arccsc}\left(c x \right)\right)}{\sqrt{e \,x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 - d*x^2*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1))*e - 2*d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
 + 3*sqrt(x^2*e + d)*e^2*integrate(1/3*(c^2*x^5*e^2 - c^2*d*x^3*e - 2*c^2*d
^2*x)*e^(-1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*x^
2*e^2 + (c^2*x^2*e^2 - e^2)*e^(log(c*x + 1) + log(c*x - 1)) - e^2), x))*b*e
^(-2)/sqrt(x^2*e + d) + 1/3*(sqrt(x^2*e + d)*x^2*e^(-1) - 2*sqrt(x^2*e + d)
*d*e^(-2))*a
```

**Fricas [A]**
time = 0.62, size = 593, normalized size = 2.64

$$\left[\frac{\left(4\, b c^{3} \sqrt{-d}\, d \log\left(\frac{c^{4} d^{2} x^{4} - 8\, c^{2} d^{2} x^{2} + x^{4} e^{2} - 4\, {\left(c^{2} d x^{2} - x^{2} e - 2\, d\right)} \sqrt{c^{2} x^{2} - 1} \sqrt{x^{2} e + d} \sqrt{-d} + 8\, d^{2} - 2\, {\left(3\, c^{2} d x^{4} - 4\, d x^{2}\right)} e}{x^{4}}\right) - {\left(3\, b c^{2} d - b e\right)} e^{\frac{1}{2}} \log\left(c^{4} d^{2} + 4\, {\left(c^{3} d + {\left(2\, c^{3} x^{2} - c\right)} e\right)} \sqrt{c^{2} x^{2} - 1} \sqrt{x^{2} e + d}\, e^{\frac{1}{2}} + {\left(8\, c^{4} x^{4} - 8\, c^{2} x^{2} + 1\right)} e^{2} + 2\, {\left(4\, c^{4} d x^{2} - 3\, c^{2} d\right)} e\right) + 4\, {\left(2\, a c^{3} x^{2} e - 4\, a c^{3} d + \sqrt{c^{2} x^{2} - 1}\, b c e + 2\, {\left(b c^{3} x^{2} e - 2\, b c^{3} d\right)} \mathrm{arccsc}\left(c x\right)\right)} \sqrt{x^{2} e + d}\right) e^{\left(-2\right)}}{24\, c^{3}}, \frac{\left(8\, b c^{3} d^{\frac{3}{2}} \arctan\left(-\frac{{\left(c^{2} d x^{2} - x^{2} e - 2\, d\right)} \sqrt{c^{2} x^{2} - 1} \sqrt{x^{2} e + d} \sqrt{d}}{2\, {\left(c^{2} d^{2} x^{2} - d^{2} + {\left(c^{2} d x^{4} - d x^{2}\right)} e\right)}}\right) - {\left(3\, b c^{2} d - b e\right)} e^{\frac{1}{2}} \log\left(c^{4} d^{2} + 4\, {\left(c^{3} d + {\left(2\, c^{3} x^{2} - c\right)} e\right)} \sqrt{c^{2} x^{2} - 1} \sqrt{x^{2} e + d}\, e^{\frac{1}{2}} + {\left(8\, c^{4} x^{4} - 8\, c^{2} x^{2} + 1\right)} e^{2} + 2\, {\left(4\, c^{4} d x^{2} - 3\, c^{2} d\right)} e\right) + 4\, {\left(2\, a c^{3} x^{2} e - 4\, a c^{3} d + \sqrt{c^{2} x^{2} - 1}\, b c e + 2\, {\left(b c^{3} x^{2} e - 2\, b c^{3} d\right)} \mathrm{arccsc}\left(c x\right)\right)} \sqrt{x^{2} e + d}\right) e^{\left(-2\right)}}{24\, c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*b*c^3*sqrt(-d)*d*log((c^4*d^2*x^4 - 8*c^2*d^2*x^2 + x^4*e^2 - 4*(c
^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(-d) + 8*d^2
- 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) - (3*b*c^2*d - b*e)*e^(1/2)*log(c^4*d^2
 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2)
+ (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) + 4*(2*a*c
^3*x^2*e - 4*a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*x^2*e - 2*b*c^3*d
)*arccsc(c*x))*sqrt(x^2*e + d))*e^(-2)/c^3, 1/24*(8*b*c^3*d^(3/2)*arctan(-1
/2*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(d)/(c^2
```

```
*d^2*x^2 - d^2 + (c^2*d*x^4 - d*x^2)*e)) - (3*b*c^2*d - b*e)*e^(1/2)*log(c^
4*d^2 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(
1/2) + (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) + 4*(
2*a*c^3*x^2*e - 4*a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*x^2*e - 2*b*
c^3*d)*arccsc(c*x))*sqrt(x^2*e + d))*e^(-2)/c^3]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\operatorname{acsc}(cx))}{\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

# 3.140 $\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

**Optimal**. Leaf size=132

$$\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e}-\frac{bc\sqrt{d}\,x\mathrm{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}}+\frac{bx\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\,\sqrt{c^2x^2}}$$

```
[Out] -b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e/(c^2*x^2
)^(1/2)+b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(1/2)/(c
^2*x^2)^(1/2)+(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/e
```

**Rubi** [A]
time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5345, 457, 132, 65, 223, 212, 12, 95, 210}

$$\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e}-\frac{bc\sqrt{d}\,x\mathrm{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{e\sqrt{c^2x^2}}+\frac{bx\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\,\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e - (b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*
x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(e*Sqrt[c^2*x^2]) + (b*x*ArcTanh[(Sqrt[
e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x\sqrt{-1+c^2x^2}}\,dx}{e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e} + \frac{(bcx)\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x\sqrt{-1+c^2x}}\,dx, x, x^2\right)}{2e\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e} + \frac{(bcx)\text{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\,\sqrt{d+ex}}\,dx, x, x^2\right)}{2\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e} + \frac{(bx)\text{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx, x, \sqrt{-1+c^2x}\right.}{c\sqrt{c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e} - \frac{bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} + \frac{(bx)\text{Subst}}{}$$

$$= \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e} - \frac{bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} + \frac{bx\tanh^{-1}}{}$$

**Mathematica** [A]
time = 0.08, size = 136, normalized size = 1.03

$$\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(c\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{e\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(c*Sqr
t[d]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*ArcTanh
[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(e*Sqrt[-1 + c^2*x^2])
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\text{arccsc}(cx))}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] (e*integrate((c^2*x^3*e + c^2*d*x)*e^(-1/2*log(x^2*e + d) + 1/2*log(c*x + 1
) + 1/2*log(c*x - 1))/(c^2*x^2*e + (c^2*x^2*e - e)*e^(log(c*x + 1) + log(c*
x - 1)) - e), x) + sqrt(x^2*e + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))
*b*e^(-1) + sqrt(x^2*e + d)*a*e^(-1)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(109) = 218.
time = 0.44, size = 466, normalized size = 3.53

$$\left[\frac{\left(bc\sqrt{-d}\log\left(\frac{c^4d^2x^4-8c^2d^2x^2+x^4e^2+4(c^2dx^2-x^2e-2d)\sqrt{c^2x^2-1}\sqrt{x^2e+d}\sqrt{-d}+8d^2-2(3c^2dx^4-4dx^2)e}{x^4}\right)+be^{\frac{1}{2}}\log\left(c^4d^2+4(c^3d+(2c^3x^2-c)e)\sqrt{c^2x^2-1}\sqrt{x^2e+d}e^{\frac{1}{2}}+(8c^4x^4-8c^2x^2+1)e^2+2(4c^4dx^2-3c^2d)e\right)+4(bc\operatorname{arccsc}(cx)+ac)\sqrt{x^2e+d}\right)e^{(-1)}}{4c}, -\frac{\left(2bc\sqrt{d}\arctan\left(-\frac{(c^2dx^2-x^2e-2d)\sqrt{c^2x^2-1}\sqrt{x^2e+d}\sqrt{d}}{2(c^2d^2x^2-d^2+(c^2dx^4-dx^2)e)}\right)-be^{\frac{1}{2}}\log\left(c^4d^2+4(c^3d+(2c^3x^2-c)e)\sqrt{c^2x^2-1}\sqrt{x^2e+d}e^{\frac{1}{2}}+(8c^4x^4-8c^2x^2+1)e^2+2(4c^4dx^2-3c^2d)e\right)-4(bc\operatorname{arccsc}(cx)+ac)\sqrt{x^2e+d}\right)e^{(-1)}}{4c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(-d)*log((c^4*d^2*x^4 - 8*c^2*d^2*x^2 + x^4*e^2 + 4*(c^2*d*x^
2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(-d) + 8*d^2 - 2*(3*
c^2*d*x^4 - 4*d*x^2)*e)/x^4) + b*e^(1/2)*log(c^4*d^2 + 4*(c^3*d + (2*c^3*x^
2 - c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^4 - 8*c^2*x^
2 + 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) + 4*(b*c*arccsc(c*x) + a*c)*sqrt(
x^2*e + d))*e^(-1)/c, -1/4*(2*b*c*sqrt(d)*arctan(-1/2*(c^2*d*x^2 - x^2*e -
2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(d)/(c^2*d^2*x^2 - d^2 + (c^2*d*
x^4 - d*x^2)*e)) - b*e^(1/2)*log(c^4*d^2 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*sq
rt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2 +
 2*(4*c^4*d*x^2 - 3*c^2*d)*e) - 4*(b*c*arccsc(c*x) + a*c)*sqrt(x^2*e + d))*
e^(-1)/c]
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\operatorname{acsc}(cx))}{\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

# 3.141 $\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x)
```

**Rubi** [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

**Mathematica** [A]
time = 1.14, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(x^2*e + d)*x), x)
 - a*arcsinh(sqrt(d)*e^(-1/2)/abs(x))/sqrt(d)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsc(c*x) + a)/(x^3*e + d*x), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{x\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x*sqrt(d + e*x**2)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

# 3.142 $\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

**Optimal**. Leaf size=26

$$\operatorname{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

**Mathematica** [A]
time = 9.87, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x^3\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e/d^(3/2) - sqrt(x^2*e + d)/(d*x^2)
) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(x^2*e + d)*x^
3), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsc(c*x) + a)/(x^5*e + d*x^3), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{x^3 \sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^3\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

# 3.143 $\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

**Rubi** [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

**Mathematica** [A]
time = 69.47, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\,\mathrm{arccsc}(cx))}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(d*arcsinh(x*e^(1/2)/sqrt(d))*e^(-3/2) - sqrt(x^2*e + d)*x*e^(-1))*a +
 b*integrate(x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(x^2*e + d), x
)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)/sqrt(x^2*e + d), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\,\mathrm{acsc}\,(cx))}{\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

# 3.144 $\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

**Optimal**. Leaf size=23

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

**Rubi** [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

**Mathematica** [A]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2], x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + b*integrate(arctan2(1, sqrt(c*x + 1
)*sqrt(c*x - 1))/sqrt(x^2*e + d), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)/sqrt(x^2*e + d), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}{(cx)}}{\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/sqrt(e*x^2 + d), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)}{\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(1/2),x)
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

## 3.145 $\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

**Optimal**. Leaf size=247

$$-\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{dx}+\frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\text{ArcSin}(cx)\,\middle|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}-$$

```
[Out] -(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/d/x-b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2
)/d/(c^2*x^2)^(1/2)+b*c^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1
/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-b
*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)
^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

**Rubi** [A]
time = 0.17, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {270, 5347, 12, 486, 21, 434, 438, 437, 435, 432, 430}

$$-\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{dx}-\frac{bx\sqrt{1-c^2x^2}\,(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,F\left(\text{ArcSin}(cx)\,\middle|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\text{ArcSin}(cx)\,\middle|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{bc\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] -((b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d*Sqrt[c^2*x^2])) - (Sqrt[d + e
*x^2]*(a + b*ArcCsc[c*x]))/(d*x) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^
2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2
]*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2
)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x
^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{dx} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{dx^2\sqrt{-1+c^2x^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{dx} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x^2\sqrt{-1+c^2x^2}}\,dx}{d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{dx} + \frac{(bcx)\int \frac{-e+c^2}{\sqrt{-1+c^2x^2}}}{d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{dx} + \frac{(bcex)\int \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}}{d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{dx} + \frac{(bc^3x)\int \frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}}{d\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{dx} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\right)\int}{d\sqrt{c^2x^2}\,\sqrt{-1}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{dx} + \frac{\left(bc^3x\sqrt{1-c^2x^2}\,\sqrt{d}\right.}{d\sqrt{c^2x^2}\,\sqrt{-1}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{dx} + \frac{bc^2x\sqrt{1-c^2x^2}\,\sqrt{d+}}{d\sqrt{c^2x^2}\,\sqrt{-1-}}$$

**Mathematica** [A]
time = 0.13, size = 140, normalized size = 0.57

$$-\frac{\sqrt{d+ex^2}\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}\,x+b\csc^{-1}(cx)\right)}{dx} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\,E\left(\text{ArcSin}\left(\sqrt{-\frac{e}{d}}\,x\right)\middle|-\frac{c^2d}{e}\right)}{d\sqrt{-\frac{e}{d}}\,\sqrt{1-c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] -((Sqrt[d + e*x^2]*(a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x + b*ArcCsc[c*x]))/(d*x)
) + (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqr
t[-(e/d)]*x], -((c^2*d)/e)])/(d*Sqrt[-(e/d)]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x
^2])
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x^2\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -(d*x*integrate((c^2*x^2*e + c^2*d)*e^(-1/2*log(x^2*e + d) + 1/2*log(c*x +
1) + 1/2*log(c*x - 1))/(c^2*d*x^2 + (c^2*d*x^2 - d)*e^(log(c*x + 1) + log(c
*x - 1)) - d), x) + sqrt(x^2*e + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
)*b/(d*x) - sqrt(x^2*e + d)*a/(d*x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{acsc}\,(cx)}{x^2\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsc(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

# 3.146 $\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

**Optimal**. Leaf size=362

$$-\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{3dx^3}+\frac{2e\sqrt{d+ \cdots}}{}$$

```
[Out] -1/3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arccsc(c*x))*(e*x^2
+d)^(1/2)/d^2/x-1/9*b*c*(2*c^2*d-5*e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2
/(c^2*x^2)^(1/2)-1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^2)^
(1/2)+1/9*b*c^2*(2*c^2*d-5*e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1
)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(
1/2)-2/9*b*(c^2*d-3*e)*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^
2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d
)^(1/2)
```

**Rubi** [A]
time = 0.32, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {277, 270, 5347, 12, 594, 597, 538, 438, 437, 435, 432, 430}

$$\frac{2e\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{3dx^3}-\frac{2bx\sqrt{1-c^2x^2}\,(c^2d-3e)\,(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,F\left(\text{ArcSin}(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d^2\sqrt{c^2x^2}\,\sqrt{c^2x^2-1}\,\sqrt{d+ex^2}}+\frac{bc^2x\sqrt{1-c^2x^2}\,(2c^2d-5e)\sqrt{d+ex^2}\,E\left(\text{ArcSin}(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d^2\sqrt{c^2x^2}\,\sqrt{c^2x^2-1}\,\sqrt{\frac{ex^2}{d}+1}}-\frac{bc\sqrt{c^2x^2-1}\,(2c^2d-5e)\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-\frac{bc\sqrt{c^2x^2-1}\,\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/9*(b*c*(2*c^2*d - 5*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d^2*Sqrt[c^2
*x^2]) - (b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(9*d*x^2*Sqrt[c^2*x^2]) -
 (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/(3*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a
+ b*ArcCsc[c*x]))/(3*d^2*x) + (b*c^2*(2*c^2*d - 5*e)*x*Sqrt[1 - c^2*x^2]*Sq
rt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[c^2*x^2]*Sq
rt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d - 3*e)*(c^2*d + e)*x*Sq
rt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(
9*d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{3d^2x} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{3d^2x^4\sqrt{-\cdots}}\cdots}{\sqrt{c^2x^2}\cdots}$$

$$= -\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{3d^2x} + \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x^4\sqrt{-1\cdots}}\cdots}{3d^2\sqrt{c^2\cdots}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d+ex^2}\,(a+b\,\mathrm{cs}\cdots}{3d^2x}$$

$$= -\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\cdots}{\cdots}$$

$$= -\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\cdots}{\cdots}$$

$$= -\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\cdots}{\cdots}$$

$$= -\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\cdots}{\cdots}$$

$$= -\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\cdots}{\cdots}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 5.53, size = 249, normalized size = 0.69

$$-\frac{\sqrt{d+ex^2}\left(bc\sqrt{1-\frac{1}{c^2x^2}}\,x(d+2c^2dx^2-5ex^2)+3a(d-2ex^2)+3b(d-2ex^2)\csc^{-1}(cx)\right)}{9d^2x^3}+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(2c^2d-5e)\,E\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)|-\frac{e}{c^2d}\right)+2(-c^4d^2+2c^2de+3e^2)\,F\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)|-\frac{e}{c^2d}\right)\right)}{9\sqrt{-c^2}\,d^2\sqrt{1-c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/9*(Sqrt[d + e*x^2]*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 - 5*e*x
^2) + 3*a*(d - 2*e*x^2) + 3*b*(d - 2*e*x^2)*ArcCsc[c*x]))/(d^2*x^3) + ((I/9
```

```
)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2*c^2*d - 5*e)*El
lipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(-(c^4*d^2) + 2*c^2*d*e
+ 3*e^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2
*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x^4\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*sqrt(x^2*e + d)*e/(d^2*x) - sqrt(x^2*e + d)/(d*x^3)) + 1/3*(3*sqrt
(x^2*e + d)*d^2*x^3*integrate(1/3*(2*c^2*x^4*e^2 + c^2*d*x^2*e - c^2*d^2)*e
^(-1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*d^2*x^4 -
 d^2*x^2 + (c^2*d^2*x^4 - d^2*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x) + 2
*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 + d*x^2*arctan2(1, sqrt(c*
x + 1)*sqrt(c*x - 1))*e - d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(s
qrt(x^2*e + d)*d^2*x^3)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{acsc}\,(cx)}{x^4\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^4\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

# 3.147 $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

**Optimal**. Leaf size=252

$$\frac{bx\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(c$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^3+8/3*b*c*d^(3/2)*x*arctan((e*x^2+d
)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)-1/6*b*(9*c^2*d-e)*x*
arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(5/2)/(c^2*x^2)^
(1/2)-d^2*(a+b*arccsc(c*x))/e^3/(e*x^2+d)^(1/2)-2*d*(a+b*arccsc(c*x))*(e*x^
2+d)^(1/2)/e^3+1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/e^2/(c^2*x^2)^(1
/2)
```

**Rubi** [A]
time = 0.72, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {272, 45, 5347, 12, 1629, 163, 65, 223, 212, 95, 210}

$$-\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3}+\frac{8bcd^{3/2}x\,\mathrm{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{3e^3\sqrt{c^2x^2}}-\frac{bx(9c^2d-e)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{c^2x^2}}+\frac{bx\sqrt{c^2x^2-1}\,\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (b*x*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e^2*Sqrt[c^2*x^2]) - (d^2*(a
 + b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt[d + e*x^2]*(a + b*ArcCs
c[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) + (8*b*c*d^(
3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*e^3*Sqrt[c^
2*x^2]) - (b*(9*c^2*d - e)*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d
 + e*x^2])])/(6*c^2*e^(5/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}\,(a+b\csc}{3e^3}$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}\,(a+b\csc}{3e^3}$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{(d+ex^2)^{3/2}\,(a+b\csc}{3e^3}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\,(a+b\csc^{-1}(c}{e^3}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\,(a+b\csc^{-1}(c}{e^3}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\,(a+b\csc^{-1}(c}{e^3}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\,(a+b\csc^{-1}(c}{e^3}$$

$$= \frac{bx\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\,(a+b\csc^{-1}(c}{e^3}$$

**Mathematica** [A]
time = 0.23, size = 231, normalized size = 0.92

$$\frac{be\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex^2)-2ac(8d^2+4dex^2-e^2x^4)-2bc(8d^2+4dex^2-e^2x^4)\csc^{-1}(cx)}{6ce^3\sqrt{d+ex^2}} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(-16c^3d^{3/2}\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(-9c^2d+e)\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{6c^2e^3\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) - 2*a*c*(8*d^2 + 4*d*e*x^2 - e^2*x
^4) - 2*b*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*ArcCsc[c*x])/(6*c*e^3*Sqrt[d + e*
x^2]) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(-16*c^3*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[-1
 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(-9*c^2*d + e)*ArcTanh[(Sqrt[e]*Sqr
t[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(6*c^2*e^3*Sqrt[-1 + c^2*x^2])
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a + b\operatorname{arccsc}(cx))}{(e\,x^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 - 4*d*x^2*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1))*e - 8*d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
)) + 3*sqrt(x^2*e + d)*e^3*integrate(1/3*(c^2*x^5*e^2 - 4*c^2*d*x^3*e - 8*c
^2*d^2*x)*e^(-1/2*log(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^
2*x^2*e^3 + (c^2*x^2*e^3 - e^3)*e^(log(c*x + 1) + log(c*x - 1)) - e^3), x))
*b*e^(-3)/sqrt(x^2*e + d) + 1/3*(x^4*e^(-1)/sqrt(x^2*e + d) - 4*d*x^2*e^(-2
)/sqrt(x^2*e + d) - 8*d^2*e^(-3)/sqrt(x^2*e + d))*a
```

**Fricas [A]**
time = 0.60, size = 774, normalized size = 3.07

$$\left[-\frac{(9bc^2d^2 - bx^2e^2 + (9bc^2dx^2 - bd)e)e^{\frac{1}{2}}\log\left(c^4d^2 + 4(c^3d + (2c^3x^2 - c)e)\sqrt{c^2x^2-1}\sqrt{x^2e+d}\,e^{\frac{1}{2}} + (8c^4x^4 - 8c^2x^2 + 1)e^2 + 2(4c^4dx^2 - 3c^2d)e\right) - 16(bc^3dx^2e + bc^3d^2)\sqrt{-d}\log\left(\frac{c^4d^2x^4 - 8c^2d^2x^2 + x^4e^2 - 4(c^2dx^2 - x^2e - 2d)\sqrt{c^2x^2-1}\sqrt{x^2e+d}\sqrt{-d} + 8d^2 - 2(3c^2dx^4 - 4dx^2)e}{x^4}\right) - 4\left(2ac^3x^4e^2 - 8ac^3dx^2e - 16ac^3d^2 + 2(bc^3x^4e^2 - 4bc^3dx^2e - 8bc^3d^2)\operatorname{arccsc}(cx) + (bcx^2e^2 + bcde)\sqrt{c^2x^2-1}\right)\sqrt{x^2e+d}}{24(c^3x^2e^4 + c^3de^3)}, -\frac{(9bc^2d^2 - bx^2e^2 + (9bc^2dx^2 - bd)e)e^{\frac{1}{2}}\log\left(c^4d^2 + 4(c^3d + (2c^3x^2 - c)e)\sqrt{c^2x^2-1}\sqrt{x^2e+d}\,e^{\frac{1}{2}} + (8c^4x^4 - 8c^2x^2 + 1)e^2 + 2(4c^4dx^2 - 3c^2d)e\right) - 32(bc^3dx^2e + bc^3d^2)\sqrt{d}\arctan\left(-\frac{(c^2dx^2 - x^2e - 2d)\sqrt{c^2x^2-1}\sqrt{x^2e+d}\sqrt{d}}{2(c^2d^2x^2 - d^2 + (c^2dx^4 - dx^2)e)}\right) - 4\left(2ac^3x^4e^2 - 8ac^3dx^2e - 16ac^3d^2 + 2(bc^3x^4e^2 - 4bc^3dx^2e - 8bc^3d^2)\operatorname{arccsc}(cx) + (bcx^2e^2 + bcde)\sqrt{c^2x^2-1}\right)\sqrt{x^2e+d}}{24(c^3x^2e^4 + c^3de^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*((9*b*c^2*d^2 - b*x^2*e^2 + (9*b*c^2*d*x^2 - b*d)*e)*e^(1/2)*log(c^4
*d^2 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1
/2) + (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) - 16*(
b*c^3*d*x^2*e + b*c^3*d^2)*sqrt(-d)*log((c^4*d^2*x^4 - 8*c^2*d^2*x^2 + x^4*
e^2 - 4*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(-d
) + 8*d^2 - 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) - 4*(2*a*c^3*x^4*e^2 - 8*a*c^
3*d*x^2*e - 16*a*c^3*d^2 + 2*(b*c^3*x^4*e^2 - 4*b*c^3*d*x^2*e - 8*b*c^3*d^2
)*arccsc(c*x) + (b*c*x^2*e^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d))
/(c^3*x^2*e^4 + c^3*d*e^3), -1/24*((9*b*c^2*d^2 - b*x^2*e^2 + (9*b*c^2*d*x^
2 - b*d)*e)*e^(1/2)*log(c^4*d^2 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^
```

```
2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4
*d*x^2 - 3*c^2*d)*e) - 32*(b*c^3*d*x^2*e + b*c^3*d^2)*sqrt(d)*arctan(-1/2*(
c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(d)/(c^2*d^2
*x^2 - d^2 + (c^2*d*x^4 - d*x^2)*e)) - 4*(2*a*c^3*x^4*e^2 - 8*a*c^3*d*x^2*e
 - 16*a*c^3*d^2 + 2*(b*c^3*x^4*e^2 - 4*b*c^3*d*x^2*e - 8*b*c^3*d^2)*arccsc(
c*x) + (b*c*x^2*e^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d))/(c^3*x^2
*e^4 + c^3*d*e^3)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

# 3.148 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

**Optimal**. Leaf size=156

$$\frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2}-\frac{2bc\sqrt{d}\,x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}}+\frac{bx\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{\cdots}}{c\sqrt{\cdots}}\right)}{e^{3/2}\sqrt{c^2\cdots}}$$

```
[Out] b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(3/2)/(c^2*x^2)^
(1/2)-2*b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e^2
/(c^2*x^2)^(1/2)+d*(a+b*arccsc(c*x))/e^2/(e*x^2+d)^(1/2)+(a+b*arccsc(c*x))*
(e*x^2+d)^(1/2)/e^2
```

**Rubi** [A]
time = 0.17, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {272, 45, 5347, 12, 587, 163, 65, 223, 212, 95, 210}

$$\frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2}+\frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}}-\frac{2bc\sqrt{d}\,x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{e^2\sqrt{c^2x^2}}+\frac{bx\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (d*(a + b*ArcCsc[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Arc
Csc[c*x]))/e^2 - (2*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 +
 c^2*x^2])])/(e^2*Sqrt[c^2*x^2]) + (b*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2]
)/(c*Sqrt[d + e*x^2])])/(e^(3/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
```

```
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(bcx)\int \frac{2d+ex^2}{e^2x\sqrt{-1+c^2x^2}\sqrt{d\cdots}}}{\sqrt{c^2x^2}}$$

$$= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(bcx)\int \frac{2d+ex^2}{x\sqrt{-1+c^2x^2}\sqrt{d+\cdots}}}{e^2\sqrt{c^2x^2}}$$

$$= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(bcx)\text{Subst}\left(\int \frac{2d+e\cdots}{x\sqrt{-1+c^2x\cdots}}\right.}{2e^2\sqrt{c^2x\cdots}}$$

$$= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(bcdx)\text{Subst}\left(\int \frac{1\cdots}{x\sqrt{-1+c^2\cdots}}\right.}{e^2\sqrt{c^2x\cdots}}$$

$$= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} + \frac{(2bcdx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x\cdots\right.}{e^2\sqrt{c^2x^2}}$$

$$= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} - \frac{2bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+e\cdots}}{\sqrt{d}\sqrt{-1\cdots}}\right.}{e^2\sqrt{c^2x^2}}$$

$$= \frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^2} - \frac{2bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+e\cdots}}{\sqrt{d}\sqrt{-1\cdots}}\right.}{e^2\sqrt{c^2x^2}}$$

## **Mathematica [A]**

time = 0.14, size = 146, normalized size = 0.94

$$\frac{(2d+ex^2)\,(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(2c\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{d}\,\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{e^2\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] ((2*d + e*x^2)*(a + b*ArcCsc[c*x]))/(e^2*Sqrt[d + e*x^2]) + (b*Sqrt[1 - 1/(
c^2*x^2)]*x*(2*c*Sqrt[d]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2
]] + Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(e
^2*Sqrt[-1 + c^2*x^2])
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\,\mathrm{arccsc}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] (x^2*e^(-1)/sqrt(x^2*e + d) + 2*d*e^(-2)/sqrt(x^2*e + d))*a + ((x^2*e^3 + d
*e^2)*integrate((c^2*x^3*e + 2*c^2*d*x)*e^(-1/2*log(x^2*e + d) + 1/2*log(c*
x + 1) + 1/2*log(c*x - 1))/(c^2*x^2*e^2 + (c^2*x^2*e^2 - e^2)*e^(log(c*x +
1) + log(c*x - 1)) - e^2), x) + (x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)
)*e + 2*d*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(x^2*e + d))*b/(x^2*
e^3 + d*e^2)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(131) = 262.
time = 0.45, size = 572, normalized size = 3.67

$$\left[\frac{(bx^2e+bd)e^{\frac{1}{2}}\log\left(c^4d^2+4\,(c^3d+(2\,c^3x^2-c)e)\sqrt{c^2x^2-1}\,\sqrt{x^2e+d}\,e^{\frac{1}{2}}+(8\,c^4x^4-8\,c^2x^2+1)e^2+2\,(4\,c^4dx^2-3\,c^2d)e\right)+2\,(bcx^2e+bcd)\sqrt{-d}\,\log\left(\frac{c^4d^2x^4-8\,c^2d^2x^2+x^4e^2+4\,(c^2dx^2-x^2e-2\,d)\sqrt{c^2x^2-1}\,\sqrt{x^2e+d}\,\sqrt{-d}+8\,d^2-2\,(3\,c^2dx^4-4\,dx^2)e}{x^4}\right)+4\,(acx^2e+2\,acd+(bcx^2e+2\,bcd)\,\mathrm{arccsc}\,(cx))\sqrt{x^2e+d}}{4\,(cx^2e^3+cde^2)},\ \frac{(bx^2e+bd)e^{\frac{1}{2}}\log\left(c^4d^2+4\,(c^3d+(2\,c^3x^2-c)e)\sqrt{c^2x^2-1}\,\sqrt{x^2e+d}\,e^{\frac{1}{2}}+(8\,c^4x^4-8\,c^2x^2+1)e^2+2\,(4\,c^4dx^2-3\,c^2d)e\right)-4\,(bcx^2e+bcd)\sqrt{d}\,\arctan\left(-\frac{(c^2dx^2-x^2e-2\,d)\sqrt{c^2x^2-1}\,\sqrt{x^2e+d}\,\sqrt{d}}{2\,(c^2d^2x^2-d^2+(c^2dx^4-dx^2)e)}\right)+4\,(acx^2e+2\,acd+(bcx^2e+2\,bcd)\,\mathrm{arccsc}\,(cx))\sqrt{x^2e+d}}{4\,(cx^2e^3+cde^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*x^2*e + b*d)*e^(1/2)*log(c^4*d^2 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*s
qrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2
+ 2*(4*c^4*d*x^2 - 3*c^2*d)*e) + 2*(b*c*x^2*e + b*c*d)*sqrt(-d)*log((c^4*d^
2*x^4 - 8*c^2*d^2*x^2 + x^4*e^2 + 4*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2
- 1)*sqrt(x^2*e + d)*sqrt(-d) + 8*d^2 - 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) +
 4*(a*c*x^2*e + 2*a*c*d + (b*c*x^2*e + 2*b*c*d)*arccsc(c*x))*sqrt(x^2*e + d
))/(c*x^2*e^3 + c*d*e^2), 1/4*((b*x^2*e + b*d)*e^(1/2)*log(c^4*d^2 + 4*(c^3
*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*
x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4*d*x^2 - 3*c^2*d)*e) - 4*(b*c*x^2*e + b*
c*d)*sqrt(d)*arctan(-1/2*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x
^2*e + d)*sqrt(d)/(c^2*d^2*x^2 - d^2 + (c^2*d*x^4 - d*x^2)*e)) + 4*(a*c*x^2
*e + 2*a*c*d + (b*c*x^2*e + 2*b*c*d)*arccsc(c*x))*sqrt(x^2*e + d))/(c*x^2*e
^3 + c*d*e^2)]
```

**Sympy** [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\operatorname{acsc}(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

# 3.149 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=79

$$-\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{bcx\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}\,e\sqrt{c^2x^2}}$$

```
[Out] b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/d^(1/2)/(c^2*x^2)
^(1/2)+(-a-b*arccsc(c*x))/e/(e*x^2+d)^(1/2)
```

**Rubi** [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5345, 457, 95, 210}

$$\frac{bcx\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{\sqrt{d}\,e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcCsc[c*x])/(e*Sqrt[d + e*x^2])) + (b*c*x*ArcTan[Sqrt[d + e*x^2]/
(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(Sqrt[d]*e*Sqrt[c^2*x^2])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx &= -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{(bcx)\int \frac{1}{x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{e\sqrt{c^2x^2}}\\
&= -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e\sqrt{c^2x^2}}\\
&= -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{(bcx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}}\\
&= -\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}\,e\sqrt{c^2x^2}}
\end{aligned}$$

**Mathematica [A]**
time = 0.09, size = 96, normalized size = 1.22

$$-\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d}\,e\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcCsc[c*x])/(e*Sqrt[d + e*x^2])) - (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*A
rcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]])/(Sqrt[d]*e*Sqrt[-1 + c
^2*x^2])
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\mathrm{arccsc}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(x^2*e + d)*c^2*e*integrate(x*e^(-1/2*log(x^2*e + d) + 1/2*log(c*x +
1) + 1/2*log(c*x - 1))/(c^2*x^2*e + (c^2*x^2*e - e)*e^(log(c*x + 1) + log(c
*x - 1)) - e), x) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b*e^(-1)/sqrt(
x^2*e + d) - a*e^(-1)/sqrt(x^2*e + d)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(67) = 134$.
time = 0.40, size = 304, normalized size = 3.85

$$\left[-\frac{(bx^2e + bd)\sqrt{-d}\,\log\left(\frac{c^4d^2x^4 - 8\,c^2d^2x^2 + x^4e^2 + 4\,(c^2dx^2 - x^2e - 2\,d)\sqrt{c^2x^2 - 1}\,\sqrt{x^2e + d}\,\sqrt{-d} + 8\,d^2 - 2\,(3\,c^2dx^4 - 4\,dx^2)e}{x^4}\right) + 4\,(bd\,\mathrm{arccsc}\,(cx) + ad)\sqrt{x^2e + d}}{4\,(dx^2e^2 + d^2e)}, \frac{(bx^2e + bd)\sqrt{d}\,\arctan\left(-\frac{(c^2dx^2 - x^2e - 2\,d)\sqrt{c^2x^2 - 1}\,\sqrt{x^2e + d}\,\sqrt{d}}{2\,(c^2d^2x^2 - d^2 + (c^2dx^4 - dx^2)e)}\right) - 2\,(bd\,\mathrm{arccsc}\,(cx) + ad)\sqrt{x^2e + d}}{2\,(dx^2e^2 + d^2e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*x^2*e + b*d)*sqrt(-d)*log((c^4*d^2*x^4 - 8*c^2*d^2*x^2 + x^4*e^2
+ 4*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(-d) +
8*d^2 - 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) + 4*(b*d*arccsc(c*x) + a*d)*sqrt(
x^2*e + d))/(d*x^2*e^2 + d^2*e), 1/2*((b*x^2*e + b*d)*sqrt(d)*arctan(-1/2*(
c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(d)/(c^2*d^2
*x^2 - d^2 + (c^2*d*x^4 - d*x^2)*e)) - 2*(b*d*arccsc(c*x) + a*d)*sqrt(x^2*e
 + d))/(d*x^2*e^2 + d^2*e)]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\,\mathrm{acsc}\,(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

# 3.150 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)

**Rubi** [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)),x]

[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)), x]

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

**Mathematica** [A]
time = 25.42, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)),x]

[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)), x]

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(arcsinh(sqrt(d)*e^(-1/2)/abs(x))/d^(3/2) - 1/(sqrt(x^2*e + d)*d)) + b*i
ntegrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((x^3*e + d*x)*sqrt(x^2*e
+ d)), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsc(c*x) + a)/(x^5*e^2 + 2*d*x^3*e + d^2*x),
 x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{x (d + ex^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

# 3.151 $\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

**Rubi** [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

**Mathematica** [A]
time = 40.81, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x^3\left(e\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(3*arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e/d^(5/2) - 3*e/(sqrt(x^2*e + d)*
d^2) - 1/(sqrt(x^2*e + d)*d*x^2)) + b*integrate(arctan2(1, sqrt(c*x + 1)*sq
rt(c*x - 1))/((x^5*e + d*x^3)*sqrt(x^2*e + d)), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsc(c*x) + a)/(x^7*e^2 + 2*d*x^5*e + d^2*x^3
), x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^3\,\left(e\,x^2 + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

# 3.152 $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

**Mathematica** [A]
time = 7.50, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+b\,\mathrm{arccsc}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

**Maxima** [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^3*e^(-1)/sqrt(x^2*e + d) - 3*d*arcsinh(x*e^(1/2)/sqrt(d))*e^(-5/2) +
 3*d*x*e^(-2)/sqrt(x^2*e + d))*a + b*integrate(x^4*arctan2(1, sqrt(c*x + 1)
*sqrt(c*x - 1))/(x^2*e + d)^(3/2), x)
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsc(c*x) + a*x^4)*sqrt(x^2*e + d)/(x^4*e^2 + 2*d*x^2*e +
 d^2), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4 \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

# 3.153 $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

**Mathematica** [A]
time = 2.97, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\,\mathrm{arccsc}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] (arcsinh(x*e^(1/2)/sqrt(d))*e^(-3/2) - x*e^(-1)/sqrt(x^2*e + d))*a + b*inte
grate(x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(x^2*e + d)^(3/2), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(x^2*e + d)/(x^4*e^2 + 2*d*x^2*e +
 d^2), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{acsc}(cx))}{(d + ex^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

# 3.154 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

**Optimal**. Leaf size=108

$$\frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\,\sqrt{1+\frac{ex^2}{d}}\,F\left(\text{ArcSin}(cx)\,\middle|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\,\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}$$

[Out] x*(a+b*arccsc(c*x))/d/(e*x^2+d)^(1/2)+b*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)

**Rubi** [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {197, 5337, 12, 432, 430}

$$\frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\,\sqrt{\frac{ex^2}{d}+1}\,F\left(\text{ArcSin}(cx)\,\middle|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\,\sqrt{c^2x^2-1}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCsc[c*x]))/(d*Sqrt[d + e*x^2]) + (b*x*Sqrt[1 - c^2*x^2]*Sqrt[1
 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt[-
1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 5337

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx &= \frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{(bcx)\int \frac{1}{d\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}}\\
&= \frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{(bcx)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{c^2x^2}}\\
&= \frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{\left(bcx\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{c^2x^2}\sqrt{d+ex^2}}\\
&= \frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{\left(bcx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\right)\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\,dx}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\\
&= \frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,F\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

**Mathematica [A]**
time = 0.13, size = 112, normalized size = 1.04

$$\frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,F\left(\text{ArcSin}(cx)\,\middle|-\frac{e}{c^2d}\right)}{d\left(-c+c^3x^2\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCsc[c*x]))/(d*Sqrt[d + e*x^2]) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*
Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])
/(d*(-c + c^3*x^2)*Sqrt[d + e*x^2])
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(x^2*e + d)^(3/2), x) +
 a*x/(sqrt(x^2*e + d)*d)
```

**Fricas** [A]
time = 0.12, size = 79, normalized size = 0.73

$$-\frac{(bx^2e+bd)\sqrt{-d}\,\mathrm{ellipticF}\left(cx,-\frac{e}{c^2d}\right)-(bcdx\,\mathrm{arccsc}\,(cx)+acdx)\sqrt{x^2e+d}}{cd^2x^2e+cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] -((b*x^2*e + b*d)*sqrt(-d)*ellipticF(c*x, -e/(c^2*d)) - (b*c*d*x*arccsc(c*x
) + a*c*d*x)*sqrt(x^2*e + d))/(c*d^2*x^2*e + c*d^3)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{acsc}\,(cx)}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

# 3.155 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

**Optimal**. Leaf size=275

$$-\frac{bc\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}}+\frac{bc^2x\sqrt{1-c^2x^2}\,\sqrt{d+ex^2}\,E\big(\mathrm{ArcSin}(}{d^2\sqrt{c^2x^2}\,\sqrt{-1+c^2x^2}\,\sqrt{1+}}$$

```
[Out] (-a-b*arccsc(c*x))/d/x/(e*x^2+d)^(1/2)-2*e*x*(a+b*arccsc(c*x))/d^2/(e*x^2+d
)^(1/2)-b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)+b*c^2*x*E
llipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*
x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-b*(c^2*d+2*e)*x*EllipticF(c*
x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2
)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

**Rubi** [A]

time = 0.22, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {277, 197, 5347, 12, 597, 538, 438, 437, 435, 432, 430}

$$-\frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{bx\sqrt{1-c^2x^2}\,(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,F\left(\mathrm{ArcSin}(cx)\middle|-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\,\sqrt{c^2x^2-1}\,\sqrt{d+ex^2}}+\frac{bc^2x\sqrt{1-c^2x^2}\,\sqrt{d+ex^2}\,E\left(\mathrm{ArcSin}(cx)\middle|-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\,\sqrt{c^2x^2-1}\,\sqrt{\frac{ex^2}{d}+1}}-\frac{bc\sqrt{c^2x^2-1}\,\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] -((b*c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/(d^2*Sqrt[c^2*x^2])) - (a + b*Ar
cCsc[c*x])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b*ArcCsc[c*x]))/(d^2*Sqrt[d
+ e*x^2]) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x
], -(e/(c^2*d))])/(d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]
) - (b*(c^2*d + 2*e)*x*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcS
in[c*x], -(e/(c^2*d))])/(d^2*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^
2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-b/a, -d/c]))))))
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = -\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{-d-2ex^2}{d^2x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{c^2x^2}}$$

$$= -\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{-d-2ex^2}{x^2\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\sqrt{c^2x^2}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \cdots}{\cdots}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} + \frac{(bc^3x)\int \cdots}{d^2\cdots}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} + \frac{\left(bc^3x\sqrt{1-\cdots}\right.}{d^2\sqrt{\cdots}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} + \frac{\left(bc^3x\sqrt{1-\cdots}\right.}{d^2\sqrt{\cdots}}$$

$$= -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{1-\cdots}}{d^2\sqrt{\cdots}}$$

**Mathematica** [C] Result contains complex when optimal does not.
time = 3.96, size = 213, normalized size = 0.77

$$\frac{-bc\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex^2)-a(d+2ex^2)-b(d+2ex^2)\csc^{-1}(cx)}{d^2x\sqrt{d+ex^2}}+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)|-\frac{e}{c^2d}\right)-(c^2d+2e)\,F\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)|-\frac{e}{c^2d}\right)\right)}{\sqrt{-c^2}\,d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (-(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)) - a*(d + 2*e*x^2) - b*(d + 2*e*
x^2)*ArcCsc[c*x])/(d^2*x*Sqrt[d + e*x^2]) + (I*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*
Sqrt[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]
 - (c^2*d + 2*e)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-
c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

**Maple** [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsc}(cx)}{x^2\,(e\,x^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

**Maxima** [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(2*x*e/(sqrt(x^2*e + d)*d^2) + 1/(sqrt(x^2*e + d)*d*x)) - ((d^2*x^3*e +
d^3*x)*integrate((2*c^2*x^2*e + c^2*d)*e^(-1/2*log(x^2*e + d) + 1/2*log(c*x
 + 1) + 1/2*log(c*x - 1))/(c^2*d^2*x^2 - d^2 + (c^2*d^2*x^2 - d^2)*e^(log(c
*x + 1) + log(c*x - 1))), x) + (2*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
))*e + d*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(x^2*e + d))*b/(d^2*x
^3*e + d^3*x)
```

**Fricas** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

# 3.156 $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=243

$$\frac{bcdx\sqrt{-1+c^2x^2}}{3e^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}-\frac{8}{}$$

```
[Out] -1/3*d^2*(a+b*arccsc(c*x))/e^3/(e*x^2+d)^(3/2)+b*x*arctanh(e^(1/2)*(c^2*x^2
-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(5/2)/(c^2*x^2)^(1/2)-8/3*b*c*x*arctan((e*x^
2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e^3/(c^2*x^2)^(1/2)+2*d*(a+b*
arccsc(c*x))/e^3/(e*x^2+d)^(1/2)+1/3*b*c*d*x*(c^2*x^2-1)^(1/2)/e^2/(c^2*d+e
)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/e^3
```

**Rubi** [A]
time = 0.74, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {272, 45, 5347, 12, 1628, 163, 65, 223, 212, 95, 210}

$$-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}-\frac{8bc\sqrt{d}\,x\,\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e^3\sqrt{c^2x^2}}+\frac{bx\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{c^2x^2}}+\frac{bcdx\sqrt{c^2x^2-1}}{3e^2\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*d*x*Sqrt[-1 + c^2*x^2])/(3*e^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^
2]) - (d^2*(a + b*ArcCsc[c*x]))/(3*e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b*Arc
Csc[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^
3 - (8*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/
(3*e^3*Sqrt[c^2*x^2]) + (b*x*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d
 + e*x^2])])/(e^(5/2)*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{(b\cdots}{\cdots}$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{(b\cdots}{\cdots}$$

$$= -\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{e^3} + \frac{(b\cdots}{\cdots}$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \cdots$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \cdots$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \cdots$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \cdots$$

$$= \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \cdots$$

**Mathematica [A]**
time = 1.23, size = 237, normalized size = 0.98

$$\frac{bcde\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex^2)+a(c^2d+e)(8d^2+12dex^2+3e^2x^4)+b(c^2d+e)(8d^2+12dex^2+3e^2x^4)\csc^{-1}(cx)}{3e^3(c^2d+e)(d+ex^2)^{3/2}} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(8c\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)+3\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{3e^3\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) + a*(c^2*d + e)*(8*d^2 + 12*d*
e*x^2 + 3*e^2*x^4) + b*(c^2*d + e)*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4)*ArcCsc[
c*x])/(3*e^3*(c^2*d + e)*(d + e*x^2)^(3/2)) + (b*Sqrt[1 - 1/(c^2*x^2)]*x*(8
```

```
*c*Sqrt[d]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]] + 3*Sqrt[e]
*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(3*e^3*Sqrt[-1
 + c^2*x^2])
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+b\,\mathrm{arccsc}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*x^4*e^(-1)/(x^2*e + d)^(3/2) + 12*d*x^2*e^(-2)/(x^2*e + d)^(3/2) + 8
*d^2*e^(-3)/(x^2*e + d)^(3/2))*a + 1/3*(3*x^4*arctan2(1, sqrt(c*x + 1)*sqrt
(c*x - 1))*e^2 + 12*d*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e + 8*d^2
*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*(x^2*e^4 + d*e^3)*sqrt(x^2*e +
 d)*integrate(1/3*(3*c^2*x^5*e^2 + 12*c^2*d*x^3*e + 8*c^2*d^2*x)*e^(-1/2*lo
g(x^2*e + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*x^4*e^4 + (c^2*d*e
^3 - e^4)*x^2 - d*e^3 + (c^2*x^4*e^4 + (c^2*d*e^3 - e^4)*x^2 - d*e^3)*e^(lo
g(c*x + 1) + log(c*x - 1))), x))*b/((x^2*e^4 + d*e^3)*sqrt(x^2*e + d))
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(205) = 410.
time = 0.59, size = 1110, normalized size = 4.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*c^2*d^3 + b*x^4*e^3 + (b*c^2*d*x^4 + 2*b*d*x^2)*e^2 + (2*b*c^2*
d^2*x^2 + b*d^2)*e)*e^(1/2)*log(c^4*d^2 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*sqr
t(c^2*x^2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2 +
2*(4*c^4*d*x^2 - 3*c^2*d)*e) + 8*(b*c^3*d^3 + b*c*x^4*e^3 + (b*c^3*d*x^4 +
2*b*c*d*x^2)*e^2 + (2*b*c^3*d^2*x^2 + b*c*d^2)*e)*sqrt(-d)*log((c^4*d^2*x^4
```

```
 - 8*c^2*d^2*x^2 + x^4*e^2 + 4*(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*
sqrt(x^2*e + d)*sqrt(-d) + 8*d^2 - 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) + 4*(8
*a*c^3*d^3 + 3*a*c*x^4*e^3 + (8*b*c^3*d^3 + 3*b*c*x^4*e^3 + 3*(b*c^3*d*x^4
+ 4*b*c*d*x^2)*e^2 + 4*(3*b*c^3*d^2*x^2 + 2*b*c*d^2)*e)*arccsc(c*x) + 3*(a*
c^3*d*x^4 + 4*a*c*d*x^2)*e^2 + 4*(3*a*c^3*d^2*x^2 + 2*a*c*d^2)*e + (b*c*d*x
^2*e^2 + b*c*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d))/(c^3*d^3*e^3 + c*x^
4*e^6 + (c^3*d*x^4 + 2*c*d*x^2)*e^5 + (2*c^3*d^2*x^2 + c*d^2)*e^4), 1/12*(3
*(b*c^2*d^3 + b*x^4*e^3 + (b*c^2*d*x^4 + 2*b*d*x^2)*e^2 + (2*b*c^2*d^2*x^2
+ b*d^2)*e)*e^(1/2)*log(c^4*d^2 + 4*(c^3*d + (2*c^3*x^2 - c)*e)*sqrt(c^2*x^
2 - 1)*sqrt(x^2*e + d)*e^(1/2) + (8*c^4*x^4 - 8*c^2*x^2 + 1)*e^2 + 2*(4*c^4
*d*x^2 - 3*c^2*d)*e) - 16*(b*c^3*d^3 + b*c*x^4*e^3 + (b*c^3*d*x^4 + 2*b*c*d
*x^2)*e^2 + (2*b*c^3*d^2*x^2 + b*c*d^2)*e)*sqrt(d)*arctan(-1/2*(c^2*d*x^2 -
 x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(d)/(c^2*d^2*x^2 - d^2
+ (c^2*d*x^4 - d*x^2)*e)) + 4*(8*a*c^3*d^3 + 3*a*c*x^4*e^3 + (8*b*c^3*d^3 +
 3*b*c*x^4*e^3 + 3*(b*c^3*d*x^4 + 4*b*c*d*x^2)*e^2 + 4*(3*b*c^3*d^2*x^2 + 2
*b*c*d^2)*e)*arccsc(c*x) + 3*(a*c^3*d*x^4 + 4*a*c*d*x^2)*e^2 + 4*(3*a*c^3*d
^2*x^2 + 2*a*c*d^2)*e + (b*c*d*x^2*e^2 + b*c*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt
(x^2*e + d))/(c^3*d^3*e^3 + c*x^4*e^6 + (c^3*d*x^4 + 2*c*d*x^2)*e^5 + (2*c^
3*d^2*x^2 + c*d^2)*e^4)]
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

# 3.157 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

**Optimal**. Leaf size=163

$$-\frac{bcx\sqrt{-1+c^2x^2}}{3e\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{d(a+b\csc^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{2bcx\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3\sqrt{d}\,e^2\sqrt{c^2x^2}}$$

```
[Out] 1/3*d*(a+b*arccsc(c*x))/e^2/(e*x^2+d)^(3/2)+2/3*b*c*x*arctan((e*x^2+d)^(1/2
)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/d^(1/2)/(c^2*x^2)^(1/2)+(-a-b*arccsc(c*x))
/e^2/(e*x^2+d)^(1/2)-1/3*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(c^2*x^2)^(1/2
)/(e*x^2+d)^(1/2)
```

**Rubi** [A]
time = 0.16, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {272, 45, 5347, 12, 587, 157, 95, 210}

$$-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\csc^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}}+\frac{2bcx\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3\sqrt{d}\,e^2\sqrt{c^2x^2}}-\frac{bcx\sqrt{c^2x^2-1}}{3e\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c*x*Sqrt[-1 + c^2*x^2])/(e*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2
]) + (d*(a + b*ArcCsc[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcCsc[c*x]
)/(e^2*Sqrt[d + e*x^2]) + (2*b*c*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1
+ c^2*x^2])])/(3*Sqrt[d]*e^2*Sqrt[c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{d(a+b\csc^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{-2d-3ex^2}{3e^2x\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{d(a+b\csc^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{-2d-3ex^2}{x\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{3e^2\sqrt{c^2x^2}}$$

$$= \frac{d(a+b\csc^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{(bcx)\text{Subst}\left(\int \frac{-2d-3ex}{x\sqrt{-1+c^2x}\,(d+ex)^{3/2}}\,dx \cdots\right.}{6e^2\sqrt{c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{3e(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\csc^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bc\cdots}{\cdots}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{3e(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\csc^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bc\cdots}{\cdots}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{3e(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\csc^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(2b\cdots}{\cdots}$$

$$= -\frac{bcx\sqrt{-1+c^2x^2}}{3e(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\csc^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{2bc\cdots}{\cdots}$$

**Mathematica [A]**
time = 0.16, size = 173, normalized size = 1.06

$$\frac{-bce\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex^2) - a(c^2d+e)(2d+3ex^2) - b(c^2d+e)(2d+3ex^2)\csc^{-1}(cx)}{3e^2(c^2d+e)(d+ex^2)^{3/2}} - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\,x\text{ArcTan}\left(\frac{\sqrt{d}\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{3\sqrt{d}\,e^2\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (-(b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)) - a*(c^2*d + e)*(2*d + 3*e*x^
2) - b*(c^2*d + e)*(2*d + 3*e*x^2)*ArcCsc[c*x])/(3*e^2*(c^2*d + e)*(d + e*x
^2)^(3/2)) - (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x
^2])/Sqrt[d + e*x^2]])/(3*Sqrt[d]*e^2*Sqrt[-1 + c^2*x^2])
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\,\mathrm{arccsc}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
[Out] int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] -1/3*(3*x^2*e^(-1)/(x^2*e + d)^(3/2) + 2*d*e^(-2)/(x^2*e + d)^(3/2))*a + b*
integrate(x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((x^4*e^2 + 2*d*x^2*e
 + d^2)*sqrt(x^2*e + d)), x)
```

**Fricas [B]** Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(138) = 276$.
time = 0.48, size = 690, normalized size = 4.23

$$\left[-\frac{(bc^2d^3+bx^4e^3+(bc^2dx^4+2bdx^2)e^2+(2bc^2d^2x^2+bd^2)e)\sqrt{-d}\log\left(\frac{c^4d^2x^4-8c^2d^2x^2+x^4e^2+4(c^2dx^2-x^2e-2d)\sqrt{c^2x^2-1}\sqrt{x^2e+d}\sqrt{-d}+8d^2-2(3c^2dx^4-4dx^2)e}{x^4}\right)+2\left(2ac^2d^3+3adx^2e^2+(2bc^2d^3+3bdx^2e^2+(3bc^2d^2x^2+2bd^2)e)\operatorname{arccsc}(cx)+(3ac^2d^2x^2+2ad^2)e+(bdx^2e^2+bd^2e)\sqrt{c^2x^2-1}\right)\sqrt{x^2e+d}}{6(c^2d^4e^2+dx^4e^5+(c^2d^2x^4+2d^2x^2)e^4+(2c^2d^3x^2+d^3)e^3)},\ \frac{(bc^2d^3+bx^4e^3+(bc^2dx^4+2bdx^2)e^2+(2bc^2d^2x^2+bd^2)e)\sqrt{d}\arctan\left(-\frac{(c^2dx^2-x^2e-2d)\sqrt{c^2x^2-1}\sqrt{x^2e+d}\sqrt{d}}{2(c^2d^2x^2-d^2+(c^2dx^4-dx^2)e)}\right)-\left(2ac^2d^3+3adx^2e^2+(2bc^2d^3+3bdx^2e^2+(3bc^2d^2x^2+2bd^2)e)\operatorname{arccsc}(cx)+(3ac^2d^2x^2+2ad^2)e+(bdx^2e^2+bd^2e)\sqrt{c^2x^2-1}\right)\sqrt{x^2e+d}}{3(c^2d^4e^2+dx^4e^5+(c^2d^2x^4+2d^2x^2)e^4+(2c^2d^3x^2+d^3)e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] [-1/6*((b*c^2*d^3 + b*x^4*e^3 + (b*c^2*d*x^4 + 2*b*d*x^2)*e^2 + (2*b*c^2*d^
2*x^2 + b*d^2)*e)*sqrt(-d)*log((c^4*d^2*x^4 - 8*c^2*d^2*x^2 + x^4*e^2 + 4*(
c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(-d) + 8*d^2
 - 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) + 2*(2*a*c^2*d^3 + 3*a*d*x^2*e^2 + (2*
b*c^2*d^3 + 3*b*d*x^2*e^2 + (3*b*c^2*d^2*x^2 + 2*b*d^2)*e)*arccsc(c*x) + (3
*a*c^2*d^2*x^2 + 2*a*d^2)*e + (b*d*x^2*e^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sq
rt(x^2*e + d))/(c^2*d^4*e^2 + d*x^4*e^5 + (c^2*d^2*x^4 + 2*d^2*x^2)*e^4 + (
2*c^2*d^3*x^2 + d^3)*e^3), 1/3*((b*c^2*d^3 + b*x^4*e^3 + (b*c^2*d*x^4 + 2*b
*d*x^2)*e^2 + (2*b*c^2*d^2*x^2 + b*d^2)*e)*sqrt(d)*arctan(-1/2*(c^2*d*x^2 -
 x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(d)/(c^2*d^2*x^2 - d^2
+ (c^2*d*x^4 - d*x^2)*e)) - (2*a*c^2*d^3 + 3*a*d*x^2*e^2 + (2*b*c^2*d^3 + 3
*b*d*x^2*e^2 + (3*b*c^2*d^2*x^2 + 2*b*d^2)*e)*arccsc(c*x) + (3*a*c^2*d^2*x^
2 + 2*a*d^2)*e + (b*d*x^2*e^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d)
)/(c^2*d^4*e^2 + d*x^4*e^5 + (c^2*d^2*x^4 + 2*d^2*x^2)*e^4 + (2*c^2*d^3*x^2
 + d^3)*e^3)]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\operatorname{acsc}(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))/(d + e*x**2)**(5/2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

# 3.158 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=138

$$\frac{bcx\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{bcx\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}$$

```
[Out] 1/3*(-a-b*arccsc(c*x))/e/(e*x^2+d)^(3/2)+1/3*b*c*x*arctan((e*x^2+d)^(1/2)/d
^(1/2)/(c^2*x^2-1)^(1/2))/d^(3/2)/e/(c^2*x^2)^(1/2)+1/3*b*c*x*(c^2*x^2-1)^(
1/2)/d/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)
```

**Rubi** [A]
time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5345, 457, 98, 95, 210}

$$-\frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{bcx\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}+\frac{bcx\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x*Sqrt[-1 + c^2*x^2])/(3*d*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2])
- (a + b*ArcCsc[c*x])/(3*e*(d + e*x^2)^(3/2)) + (b*c*x*ArcTan[Sqrt[d + e*x^
2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(3*d^(3/2)*e*Sqrt[c^2*x^2])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5345

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Dist[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2])), Int[(d + e*x^2)^(p + 1)/(x*Sq
rt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx &= -\frac{a+b\csc^{-1}(cx)}{3e\,(d+ex^2)^{3/2}} - \frac{(bcx)\int \frac{1}{x\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{3e\sqrt{c^2x^2}} \\
&= -\frac{a+b\csc^{-1}(cx)}{3e\,(d+ex^2)^{3/2}} - \frac{(bcx)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\,(d+ex)^{3/2}}\,dx, x, x^2\right)}{6e\sqrt{c^2x^2}} \\
&= \frac{bcx\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{3e\,(d+ex^2)^{3/2}} - \frac{(bcx)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+c^2x}\cdots}\right.}{6de\sqrt{c^2x\cdots}} \\
&= \frac{bcx\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{3e\,(d+ex^2)^{3/2}} - \frac{(bcx)\mathrm{Subst}\left(\int \frac{1}{-d-x^2}\,dx, x, \cdots\right.}{3de\sqrt{c^2x^2}} \\
&= \frac{bcx\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{3e\,(d+ex^2)^{3/2}} + \frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x\cdots}}\right.}{3d^{3/2}e\sqrt{c^2x^2}}
\end{aligned}
$$

## **Mathematica** [A]

time = 0.13, size = 157, normalized size = 1.14

$$\frac{-ad(c^2d+e)+bce\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex^2)-bd(c^2d+e)\csc^{-1}(cx)}{3de\,(c^2d+e)\,(d+ex^2)^{3/2}}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\text{ArcTan}\left(\frac{\sqrt{d}\,\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{3d^{3/2}e\sqrt{-1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (-(a*d*(c^2*d + e)) + b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) - b*d*(c^2*
d + e)*ArcCsc[c*x])/(3*d*e*(c^2*d + e)*(d + e*x^2)^(3/2)) - (b*c*Sqrt[1 - 1
/(c^2*x^2)]*x*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]])/(3*d^(3
/2)*e*Sqrt[-1 + c^2*x^2])
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\mathrm{arccsc}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((x^4*e^2 + 2*d*x^2*e
 + d^2)*sqrt(x^2*e + d)), x) - 1/3*a*e^(-1)/(x^2*e + d)^(3/2)
```

**Fricas** [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(116) = 232.
time = 0.47, size = 596, normalized size = 4.32

$$\left[-\frac{(bc^2d^3+bx^4e^3+(bc^2dx^4+2\,bdx^2)e^2+(2\,bc^2d^2x^2+bd^2)e)\sqrt{-d}\,\log\left(\frac{c^4d^2x^4-8\,c^2d^2x^2+x^4e^2+4\,(c^2dx^2-x^2e-2\,d)\sqrt{c^2x^2-1}\,\sqrt{x^2e+d}\,\sqrt{-d}+8\,d^2-2\,(3\,c^2dx^4-4\,dx^2)e}{x^4}\right)+4\left(ac^2d^3+ad^2e+(bc^2d^3+bd^2e)\,\mathrm{arccsc}\,(cx)-(bdx^2e^2+bd^2e)\sqrt{c^2x^2-1}\right)\sqrt{x^2e+d}}{12\,(c^2d^5e+d^2x^4e^4+(c^2d^3x^4+2\,d^3x^2)e^3+(2\,c^2d^4x^2+d^4)e^2)},\ \frac{(bc^2d^3+bx^4e^3+(bc^2dx^4+2\,bdx^2)e^2+(2\,bc^2d^2x^2+bd^2)e)\sqrt{d}\,\arctan\left(-\frac{(c^2dx^2-x^2e-2\,d)\sqrt{c^2x^2-1}\,\sqrt{x^2e+d}\,\sqrt{d}}{2\,(c^2d^2x^2-d^2+(c^2dx^4-dx^2)e)}\right)-2\left(ac^2d^3+ad^2e+(bc^2d^3+bd^2e)\,\mathrm{arccsc}\,(cx)-(bdx^2e^2+bd^2e)\sqrt{c^2x^2-1}\right)\sqrt{x^2e+d}}{6\,(c^2d^5e+d^2x^4e^4+(c^2d^3x^4+2\,d^3x^2)e^3+(2\,c^2d^4x^2+d^4)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((b*c^2*d^3 + b*x^4*e^3 + (b*c^2*d*x^4 + 2*b*d*x^2)*e^2 + (2*b*c^2*d
^2*x^2 + b*d^2)*e)*sqrt(-d)*log((c^4*d^2*x^4 - 8*c^2*d^2*x^2 + x^4*e^2 + 4*
```

```
(c^2*d*x^2 - x^2*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(-d) + 8*d^
2 - 2*(3*c^2*d*x^4 - 4*d*x^2)*e)/x^4) + 4*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3
 + b*d^2*e)*arccsc(c*x) - (b*d*x^2*e^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(x
^2*e + d))/(c^2*d^5*e + d^2*x^4*e^4 + (c^2*d^3*x^4 + 2*d^3*x^2)*e^3 + (2*c^
2*d^4*x^2 + d^4)*e^2), 1/6*((b*c^2*d^3 + b*x^4*e^3 + (b*c^2*d*x^4 + 2*b*d*x
^2)*e^2 + (2*b*c^2*d^2*x^2 + b*d^2)*e)*sqrt(d)*arctan(-1/2*(c^2*d*x^2 - x^2
*e - 2*d)*sqrt(c^2*x^2 - 1)*sqrt(x^2*e + d)*sqrt(d)/(c^2*d^2*x^2 - d^2 + (c
^2*d*x^4 - d*x^2)*e)) - 2*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*arcc
sc(c*x) - (b*d*x^2*e^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(x^2*e + d))/(c^2*
d^5*e + d^2*x^4*e^4 + (c^2*d^3*x^4 + 2*d^3*x^2)*e^3 + (2*c^2*d^4*x^2 + d^4)
*e^2)]
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{acsc}(cx))}{(d + ex^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
[Out] Integral(x*(a + b*acsc(c*x))/(d + e*x**2)**(5/2), x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsc(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(a + b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2 + d\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

# 3.159 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)
```

**Rubi** [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

**Mathematica** [A]
time = 49.55, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x\,(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*arcsinh(sqrt(d)*e^(-1/2)/abs(x))/d^(5/2) - 3/(sqrt(x^2*e + d)*d^2
) - 1/((x^2*e + d)^(3/2)*d)) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*
x - 1))/((x^5*e^2 + 2*d*x^3*e + d^2*x)*sqrt(x^2*e + d)), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsc(c*x) + a)/(x^7*e^3 + 3*d*x^5*e^2 + 3*d^2
*x^3*e + d^3*x), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

# 3.160 $\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

**Rubi** [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

**Mathematica** [A]
time = 58.06, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x^3\left(e\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(15*arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e/d^(7/2) - 15*e/(sqrt(x^2*e + d
)*d^3) - 5*e/((x^2*e + d)^(3/2)*d^2) - 3/((x^2*e + d)^(3/2)*d*x^2)) + b*int
egrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((x^7*e^2 + 2*d*x^5*e + d^2*
x^3)*sqrt(x^2*e + d)), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsc(c*x) + a)/(x^9*e^3 + 3*d*x^7*e^2 + 3*d^2
*x^5*e + d^3*x^3), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^3\,{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

# 3.161 $\int \frac{x^6\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=26

$$\text{Int}\left(\frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

**Rubi** [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Defer[Int][(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

**Mathematica** [A]
time = 10.59, size = 0, normalized size = 0.00

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Integrate[(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^6(a+b\,\mathrm{arccsc}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*x^5*e^(-1)/(x^2*e + d)^(3/2) + 5*(3*x^2*e^(-1)/(x^2*e + d)^(3/2) + 2
*d*e^(-2)/(x^2*e + d)^(3/2))*d*x*e^(-1) - 15*d*arcsinh(x*e^(1/2)/sqrt(d))*e
^(-7/2) + 5*d*x*e^(-3)/sqrt(x^2*e + d))*a + b*integrate(x^6*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1))/((x^4*e^2 + 2*d*x^2*e + d^2)*sqrt(x^2*e + d)), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^6*arccsc(c*x) + a*x^6)*sqrt(x^2*e + d)/(x^6*e^3 + 3*d*x^4*e^2
 + 3*d^2*x^2*e + d^3), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^6\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^6*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

# 3.162 $\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=26

$$\mathrm{Int}\left(\frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Defer[Int][(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

**Mathematica** [A]
time = 9.39, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+b\,\mathrm{arccsc}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((3*x^2*e^(-1)/(x^2*e + d)^(3/2) + 2*d*e^(-2)/(x^2*e + d)^(3/2))*x - 3
*arcsinh(x*e^(1/2)/sqrt(d))*e^(-5/2) + x*e^(-2)/sqrt(x^2*e + d))*a + b*inte
grate(x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((x^4*e^2 + 2*d*x^2*e + d
^2)*sqrt(x^2*e + d)), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsc(c*x) + a*x^4)*sqrt(x^2*e + d)/(x^6*e^3 + 3*d*x^4*e^2
 + 3*d^2*x^2*e + d^3), x)
```

**Sympy [F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

# 3.163 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=276

$$\frac{bcx^2\sqrt{-1+c^2x^2}}{3d\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\text{ArcSin}(cx)\,|-\frac{e}{c^2d}\right)}{3de\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}+\frac{bx\sqrt{}}{}$$

```
[Out] 1/3*x^3*(a+b*arccsc(c*x))/d/(e*x^2+d)^(3/2)+1/3*b*c*x^2*(c^2*x^2-1)^(1/2)/d
/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)-1/3*b*c^2*x*EllipticE(c*x,(-e/c^
2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/e/(c^2*d+e)/(c^2*x^2)^(1/2
)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+1/3*b*x*EllipticF(c*x,(-e/c^2/d)^(1/2
))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/e/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/
2)/(e*x^2+d)^(1/2)
```

**Rubi** [A]
time = 0.19, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {270, 5347, 12, 482, 434, 438, 437, 435, 432, 430}

$$\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,F\left(\text{ArcSin}(cx)\,|-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\text{ArcSin}(cx)\,|-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{c^2x^2-1}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}}+\frac{bcx^2\sqrt{c^2x^2-1}}{3d\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x^2*Sqrt[-1 + c^2*x^2])/(3*d*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d + e*x^2]
) + (x^3*(a + b*ArcCsc[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c^2*x*Sqrt[1 - c
^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d*e*(c^2*d
 + e)*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (b*x*Sqrt[1 -
 c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d*e*
Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{x^3(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{(bcx)\int \frac{x^2}{3d\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{x^3(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{(bcx)\int \frac{x^2}{\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{3d\sqrt{c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} - \frac{(bcx)\int \frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\,dx}{3d\,(c^2d+e)\sqrt{c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{(bcx)\int \frac{1}{\sqrt{-1+c^2x^2}\sqrt{d}}}{3de\sqrt{c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} - \frac{\left(bc^3x\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{}}{\sqrt{1}}}{3de\,(c^2d+e)\sqrt{c^2x^2}\sqrt{}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} - \frac{\left(bc^3x\sqrt{1-c^2x^2}\sqrt{d+e}\right.}{3de\,(c^2d+e)\sqrt{c^2x^2}\sqrt{-}}$$

$$= \frac{bcx^2\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex}}{3de\,(c^2d+e)\sqrt{c^2x^2}\sqrt{-}}$$

**Mathematica [A]**
time = 0.19, size = 185, normalized size = 0.67

$$\frac{x^2\left(a(c^2d+e)\,x+bc\sqrt{1-\frac{1}{c^2x^2}}\,(d+ex^2)+b(c^2d+e)\,x\csc^{-1}(cx)\right)}{3d\,(c^2d+e)\,(d+ex^2)^{3/2}}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\,E\left(\mathrm{ArcSin}\left(\sqrt{-\frac{e}{d}}\,x\right)|-\frac{c^2d}{e}\right)}{3d\sqrt{-\frac{e}{d}}\,(c^2d+e)\sqrt{1-c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (x^2*(a*(c^2*d + e)*x + b*c*Sqrt[1 - 1/(c^2*x^2)]*(d + e*x^2) + b*(c^2*d +
e)*x*ArcCsc[c*x]))/(3*d*(c^2*d + e)*(d + e*x^2)^(3/2)) - (b*c*Sqrt[1 - 1/(c
^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt[-(e/d)]*x], -((c^2*d)/
e)])/(3*d*Sqrt[-(e/d)]*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\,\mathrm{arccsc}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(x*e^(-1)/(x^2*e + d)^(3/2) - x*e^(-1)/(sqrt(x^2*e + d)*d)) + b*inte
grate(x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((x^4*e^2 + 2*d*x^2*e + d
^2)*sqrt(x^2*e + d)), x)
```

**Fricas [F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac** [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

# 3.164 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

**Optimal**. Leaf size=296

$$-\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{x(a+b\csc^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E(}{3d^2\left(c^2d+e\right)\sqrt{c^2x^2}\sqrt{-1+}}$$

```
[Out] 1/3*x*(a+b*arccsc(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arccsc(c*x))/d^2/(e*x^
2+d)^(1/2)-1/3*b*c*e*x^2*(c^2*x^2-1)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(e
*x^2+d)^(1/2)+1/3*b*c^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2
)*(e*x^2+d)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/
d)^(1/2)+2/3*b*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^
2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

**Rubi** [A]
time = 0.16, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {198, 197, 5337, 12, 541, 538, 438, 437, 435, 432, 430}

$$\frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\csc^{-1}(cx))}{3d\left(d+ex^2\right)^{3/2}}+\frac{2bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,F\left(\text{ArcSin}(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\text{ArcSin}(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^2\sqrt{c^2x^2}\sqrt{c^2x^2-1}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}}-\frac{bcex^2\sqrt{c^2x^2-1}}{3d^2\sqrt{c^2x^2}\left(c^2d+e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c*e*x^2*Sqrt[-1 + c^2*x^2])/(d^2*(c^2*d + e)*Sqrt[c^2*x^2]*Sqrt[d +
 e*x^2]) + (x*(a + b*ArcCsc[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*Ar
cCsc[c*x]))/(3*d^2*Sqrt[d + e*x^2]) + (b*c^2*x*Sqrt[1 - c^2*x^2]*Sqrt[d + e
*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*(c^2*d + e)*Sqrt[c^2*x^2
]*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (2*b*x*Sqrt[1 - c^2*x^2]*Sqrt[1
 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*d^2*Sqrt[c^2*x^2]*Sq
rt[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 5337

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsc[c*x], u, x]
+ Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{x(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{3d+2ex^2}{3d^2\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{\sqrt{c^2x^2}}$$

$$= \frac{x(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{(bcx)\int \frac{3d+2ex^2}{\sqrt{-1+c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{3d^2\sqrt{c^2x^2}}$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \cdots$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \cdots$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \cdots$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \cdots$$

$$= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x(a+b\csc^{-1}(cx))}{3d\,(d+ex^2)^{3/2}} + \frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \cdots$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.
time = 0.35, size = 249, normalized size = 0.84

$$\frac{x\left(-bce\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex^2)+a(c^2d+e)\,(3d+2ex^2)+b(c^2d+e)\,(3d+2ex^2)\csc^{-1}(cx)\right)}{3d^2\,(c^2d+e)\,(d+ex^2)^{3/2}} + \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)|-\frac{e}{c^2d}\right)+2(c^2d+e)F\left(i\sinh^{-1}\left(\sqrt{-c^2}\,x\right)|-\frac{e}{c^2d}\right)\right)}{3\sqrt{-c^2}\,d^2\,(c^2d+e)\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (x*(-(b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)) + a*(c^2*d + e)*(3*d + 2*e
*x^2) + b*(c^2*d + e)*(3*d + 2*e*x^2)*ArcCsc[c*x]))/(3*d^2*(c^2*d + e)*(d +
 e*x^2)^(3/2)) + ((I/3)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^
2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(c^2*d + e)*Ellipt
```

```
icF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*(c^2*d + e)*Sq
rt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsc}(cx)}{(e\,x^2 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(x^2*e + d)*d^2) + x/((x^2*e + d)^(3/2)*d)) + b*integrate(a
rctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((x^4*e^2 + 2*d*x^2*e + d^2)*sqrt(x^
2*e + d)), x)
```

**Fricas** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

**Mupad [F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((a + b*asin(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

## 3.165 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\, dx$

**Optimal**. Leaf size=585

$$\frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)x}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$

```
[Out] d^3*(f*x)^(1+m)*(a+b*arccsc(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arccsc(c
*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arccsc(c*x))/f^5/(5+m)+e^3*(f*x)^(7
+m)*(a+b*arccsc(c*x))/f^7/(7+m)+b*(c^6*d^3*(2+m)*(4+m)*(6+m)/(1+m)+e*(1+m)*
(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+1
79*m^2+638*m+840))/(m^3+15*m^2+71*m+105))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2
+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^5/f/(1+m)/(2+m)/(4+m)/(6+
m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)+b*e*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m
)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))*x*(f*x)^(1+m)*(
c^2*x^2-1)^(1/2)/c^5/f/(6+m)/(m^2+6*m+8)/(m^3+15*m^2+71*m+105)/(c^2*x^2)^(1
/2)+b*e^2*(e*(5+m)^2+3*c^2*d*(m^2+13*m+42))*x*(f*x)^(3+m)*(c^2*x^2-1)^(1/2)
/c^3/f^3/(4+m)/(5+m)/(6+m)/(7+m)/(c^2*x^2)^(1/2)+b*e^3*x*(f*x)^(5+m)*(c^2*x
^2-1)^(1/2)/c/f^5/(6+m)/(7+m)/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 1.57, antiderivative size = 566, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {276, 5347, 1823, 1281, 470, 372, 371}

$$\frac{d^3(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\csc^{-1}(cx)\right)}{f^7(m+7)}+\frac{be^3x\sqrt{c^2x^2-1}(fx)^{m+5}}{cf^5(m+6)(m+7)\sqrt{c^2x^2}}+\frac{be^2x\sqrt{c^2x^2-1}(fx)^{m+3}\left(3c^2d(m^2+13m+42)+e(m+5)^2\right)}{c^3f^3(m+4)(m+5)(m+6)(m+7)\sqrt{c^2x^2}}+\frac{bcx\sqrt{1-c^2x^2}(fx)^{m+1}\left(\frac{e\left(3c^4d^2(m^4+22m^3+179m^2+638m+840)+3c^2de(m+3)^2(m^2+13m+42)+e^2(m^2+8m+15)^2\right)}{c^6(m+2)(m+3)(m+4)(m+5)(m+6)(m+7)}+\frac{d^3}{(m+1)^2}\right){}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)}{f\sqrt{c^2x^2}\sqrt{c^2x^2-1}}+\frac{bex\sqrt{c^2x^2-1}(fx)^{m+1}\left(3c^4d^2(m^4+22m^3+179m^2+638m+840)+3c^2de(m+3)^2(m^2+13m+42)+e^2(m^2+8m+15)^2\right)}{c^5f(m+2)(m+3)(m+4)(m+5)(m+6)(m+7)\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^
4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*x*(f*x)^(1 + m)*Sqrt[-1 + c^2
*x^2])/(c^5*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[c^2*x^2]
) + (b*e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*x*(f*x)^(3 + m)*Sqrt[-
1 + c^2*x^2])/(c^3*f^3*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[c^2*x^2]) + (b*
e^3*x*(f*x)^(5 + m)*Sqrt[-1 + c^2*x^2])/(c*f^5*(6 + m)*(7 + m)*Sqrt[c^2*x^2
]) + (d^3*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(
3 + m)*(a + b*ArcCsc[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*A
rcCsc[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCsc[c*x]))/(f^7*(
7 + m)) + (b*c*(d^3/(1 + m)^2 + (e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 +
 m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))
)/(c^6*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)))*x*(f*x)^(1 + m)*Sq
rt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*S
qrt[c^2*x^2]*Sqrt[-1 + c^2*x^2])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{d^3(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} +$$

$$= \frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}} + \frac{d^3(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{3a}{}$$

$$= \frac{be^2\left(e(5+m)^2+3c^2d\left(42+13m+m^2\right)\right)x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}}$$

$$= \frac{be\Big(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c}{c^5f(2+m)(3+m)(4+m)(}$$

$$= \frac{be\Big(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c}{c^5f(2+m)(3+m)(4+m)(}$$

$$= \frac{be\Big(e^2\left(15+8m+m^2\right)^2+3c^2de(3+m)^2\left(42+13m+m^2\right)+3c}{c^5f(2+m)(3+m)(4+m)(}$$

**Mathematica [F]**
time = 0.09, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsc[c*x]),x]
```

```
[Out] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsc[c*x]), x]
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(e\,x^2+d\right)^3 (a+b\operatorname{arccsc}(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] a*f^m*x^7*e^(m*log(x) + 3)/(m + 7) + 3*a*d*f^m*x^5*e^(m*log(x) + 2)/(m + 5)
 + 3*a*d^2*f^m*x^3*e^(m*log(x) + 1)/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1
)) + (((b*f^m*m^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^3 + 9*b*f^m*m^2
*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^3 + 23*b*f^m*m*arctan2(1, sqrt(c
*x + 1)*sqrt(c*x - 1))*e^3 + 15*b*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
))*e^3)*x^7 + 3*(b*d*f^m*m^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 +
11*b*d*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 + 31*b*d*f^m*m*a
rctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2 + 21*b*d*f^m*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1))*e^2)*x^5 + 3*(b*d^2*f^m*m^3*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1))*e + 13*b*d^2*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*
e + 47*b*d^2*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e + 35*b*d^2*f^m
*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e)*x^3 + (b*d^3*f^m*m^3*arctan2(1,
 sqrt(c*x + 1)*sqrt(c*x - 1)) + 15*b*d^3*f^m*m^2*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)) + 71*b*d^3*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 10
5*b*d^3*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x)*x^m + (m^4 + 16*m^3
 + 86*m^2 + 176*m + 105)*integrate(-(b*d^3*f^m*m^3 + 15*b*d^3*f^m*m^2 + (b*
f^m*m^3*e^3 + 9*b*f^m*m^2*e^3 + 23*b*f^m*m*e^3 + 15*b*f^m*e^3)*x^6 + 71*b*d
^3*f^m*m + 105*b*d^3*f^m + 3*(b*d*f^m*m^3*e^2 + 11*b*d*f^m*m^2*e^2 + 31*b*d
*f^m*m*e^2 + 21*b*d*f^m*e^2)*x^4 + 3*(b*d^2*f^m*m^3*e + 13*b*d^2*f^m*m^2*e
+ 47*b*d^2*f^m*m*e + 35*b*d^2*f^m*e)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(
m^4 + 16*m^3 - (c^2*m^4 + 16*c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^
2 + 86*m^2 + 176*m + 105), x))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*x^6*e^3 + 3*a*d*x^4*e^2 + 3*a*d^2*x^2*e + a*d^3 + (b*x^6*e^3 +
3*b*d*x^4*e^2 + 3*b*d^2*x^2*e + b*d^3)*arccsc(c*x))*(f*x)^m, x)
```

**Sympy** [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*acsc(c*x)),x)
[Out] Timed out
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^3*(a + b*asin(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^3*(a + b*asin(1/(c*x))), x)
```

# 3.166 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right) \, dx$

**Optimal**. Leaf size=371

$$\frac{be(e(3+m)^2+2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(3+m)(4+m)(5+m)\sqrt{c^2x^2}}+\frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}}+\frac{d^2(fx)^{1+m}\,(a+b}{f(1+m}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*arccsc(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arccsc(c*x
))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arccsc(c*x))/f^5/(5+m)+b*(c^4*d^2*(2+m)*(
3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(3+m)^2+2*c^2*d*(m^2+9*m+20)))*x*(f*x)^(1+m)*
hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^3/f/(1
+m)^2/(2+m)/(3+m)/(4+m)/(5+m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)+b*e*(e*(3+m
)^2+2*c^2*d*(m^2+9*m+20))*x*(f*x)^(1+m)*(c^2*x^2-1)^(1/2)/c^3/f/(4+m)/(5+m)
/(m^2+5*m+6)/(c^2*x^2)^(1/2)+b*e^2*x*(f*x)^(3+m)*(c^2*x^2-1)^(1/2)/c/f^3/(4
+m)/(5+m)/(c^2*x^2)^(1/2)
```

**Rubi** [A]

time = 0.32, antiderivative size = 352, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {276, 5347, 12, 1281, 470, 372, 371}

$$\frac{d^2(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+\frac{be^2x\sqrt{c^2x^2-1}\,(fx)^{m+3}}{cf^3(m+4)(m+5)\sqrt{c^2x^2}}+\frac{bcx\sqrt{1-c^2x^2}\,(fx)^{m+1}\left(\frac{e(2c^2d(m^2+9m+20)+e(m+3)^2)}{c^4(m+2)(m+3)(m+4)(m+5)}+\frac{d^2}{(m+1)^2}\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)}{f\sqrt{c^2x^2}\sqrt{c^2x^2-1}}+\frac{bex\sqrt{c^2x^2-1}\,(fx)^{m+1}\left(2c^2d(m^2+9m+20)+e(m+3)^2\right)}{c^3f(m+2)(m+3)(m+4)(m+5)\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2))*x*(f*x)^(1 + m)*Sqrt[-1 + c^2
*x^2])/(c^3*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*Sqrt[c^2*x^2]) + (b*e^2*x*(f*
x)^(3 + m)*Sqrt[-1 + c^2*x^2])/(c*f^3*(4 + m)*(5 + m)*Sqrt[c^2*x^2]) + (d^2
*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a +
 b*ArcCsc[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCsc[c*x]))/(f
^5*(5 + m)) + (b*c*(d^2/(1 + m)^2 + (e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m
^2)))/(c^4*(2 + m)*(3 + m)*(4 + m)*(5 + m)))*x*(f*x)^(1 + m)*Sqrt[1 - c^2*x
^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*Sqrt[c^2*x^2]
*Sqrt[-1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{e}{}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{e}{}$$

$$= \frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{2de}{}$$

$$= \frac{be\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{c^2x^2}} +$$

$$= \frac{be\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{c^2x^2}} +$$

$$= \frac{be\left(e(3+m)^2+2c^2d(20+9m+m^2)\right)x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{c^2x^2}} +$$

**Mathematica [F]**
time = 0.07, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsc[c*x]), x]
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(e\,x^2+d\right)^2 \left(a+b\,\mathrm{arccsc}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] a*f^m*x^5*e^(m*log(x) + 2)/(m + 5) + 2*a*d*f^m*x^3*e^(m*log(x) + 1)/(m + 3)
 + (f*x)^(m + 1)*a*d^2/(f*(m + 1)) + (((b*f^m*m^2*arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1))*e^2 + 4*b*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2
+ 3*b*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e^2)*x^5 + 2*(b*d*f^m*m^2
*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e + 6*b*d*f^m*m*arctan2(1, sqrt(c*
x + 1)*sqrt(c*x - 1))*e + 5*b*d*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
*e)*x^3 + (b*d^2*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*b*d^2*
f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 15*b*d^2*f^m*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1)))*x)*x^m + (m^3 + 9*m^2 + 23*m + 15)*integrate(-(b
*d^2*f^m*m^2 + 8*b*d^2*f^m*m + (b*f^m*m^2*e^2 + 4*b*f^m*m*e^2 + 3*b*f^m*e^2
)*x^4 + 15*b*d^2*f^m + 2*(b*d*f^m*m^2*e + 6*b*d*f^m*m*e + 5*b*d*f^m*e)*x^2)
*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(m^3 - (c^2*m^3 + 9*c^2*m^2 + 23*c^2*m + 1
5*c^2)*x^2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

**Fricas [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arccsc(c*x))*(f*x)^m, x)
```

**Sympy [F]**
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b\operatorname{acsc}(cx)) \left(d + ex^2\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsc(c*x))*(d + e*x**2)**2, x)
```

**Giac [F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

# 3.167 $\int (fx)^m (d+ex^2) \left(a+b\csc^{-1}(cx)\right) \, dx$

**Optimal**. Leaf size=215

$$\frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf\,(6+5m+m^2)\,\sqrt{c^2x^2}}+\frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)}+\frac{b(e(1+m)^2+c^2d(2}{cf(1}$$

```
[Out] d*(f*x)^(1+m)*(a+b*arccsc(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arccsc(c*x))/f^3
/(3+m)+b*(e*(1+m)^2+c^2*d*(2+m)*(3+m))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2+1/
2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c/f/(1+m)^2/(2+m)/(3+m)/(c^2*x
^2)^(1/2)/(c^2*x^2-1)^(1/2)+b*e*x*(f*x)^(1+m)*(c^2*x^2-1)^(1/2)/c/f/(m^2+5*
m+6)/(c^2*x^2)^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 202, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 5347, 12, 470, 372, 371}

$$\frac{d(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{bcx\sqrt{1-c^2x^2}\,(fx)^{m+1}\left(\frac{e}{c^2(m+2)(m+3)}+\frac{d}{(m+1)^2}\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)}{f\sqrt{c^2x^2}\,\sqrt{c^2x^2-1}}+\frac{bex\sqrt{c^2x^2-1}\,(fx)^{m+1}}{cf\,(m^2+5m+6)\,\sqrt{c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] (b*e*x*(f*x)^(1 + m)*Sqrt[-1 + c^2*x^2])/(c*f*(6 + 5*m + m^2)*Sqrt[c^2*x^2]
) + (d*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a
 + b*ArcCsc[c*x]))/(f^3*(3 + m)) + (b*c*(d/(1 + m)^2 + e/(c^2*(2 + m)*(3 +
m)))*x*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3
 + m)/2, c^2*x^2])/(f*Sqrt[c^2*x^2]*Sqrt[-1 + c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5347

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[b*c*(x/Sqrt[c^2*x^2]), Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\csc^{-1}(cx)\right)\, dx = \frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{(bcx)}{}$$

$$= \frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{(bcx)}{(}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^3}{}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^3}{}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^3}{}$$

**Mathematica** [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\csc^{-1}(cx)\right) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

```
[Out] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsc[c*x]), x]
```

**Maple** [F]
time = 9.31, size = 0, normalized size = 0.00

$$\int (fx)^m \left(e\,x^2+d\right) \left(a+b\,\mathrm{arccsc}(cx)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

```
[Out] a*f^m*x^3*e^(m*log(x) + 1)/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) + (((b*f
^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*e + b*f^m*arctan2(1, sqrt(c*x
+ 1)*sqrt(c*x - 1))*e)*x^3 + (b*d*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
 1)) + 3*b*d*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x)*x^m + (m^2 + 4
*m + 3)*integrate((b*d*f^m*m + 3*b*d*f^m + (b*f^m*m*e + b*f^m*e)*x^2)*sqrt(
c*x + 1)*sqrt(c*x - 1)*x^m/((c^2*m^2 + 4*c^2*m + 3*c^2)*x^2 - m^2 - 4*m - 3
), x))/(m^2 + 4*m + 3)
```

**Fricas** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsc(c*x))*(f*x)^m, x)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(a + b\operatorname{acsc}(cx)\right)\left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsc(c*x))*(d + e*x**2), x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

```
[Out] int((f*x)^m*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

# 3.168 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$

**Optimal**. Leaf size=26

$$\mathrm{Int}\left(\frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{d+ex^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx$$

**Mathematica** [A]
time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccsc}(cx)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

```
[Out] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(x^2*e + d), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*(f*x)^m/(x^2*e + d), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a + b\operatorname{acsc}(cx))}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsc(c*x))/(d + e*x**2), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2),x)
[Out] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2), x)
```

# 3.169 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{(d+ex^2)^2} dx$

**Optimal**. Leaf size=26

$$\mathrm{Int}\left(\frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

**Rubi** [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$$

**Mathematica** [A]
time = 4.24, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2, x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+b\,\mathrm{arccsc}(cx))}{(e\,x^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

**Maxima** [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(x^2*e + d)^2, x)
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*(f*x)^m/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

# 3.170 $\int (fx)^m (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\,dx$

**Optimal**. Leaf size=28

$$\mathrm{Int}\left((fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\,dx = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\,dx$$

**Mathematica** [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsc}(cx)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
[Out] integrate((x^2*e + d)^(3/2)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsc(c*x))*sqrt(x^2*e + d)*(f*x
)^m, x)
```

**Sympy** [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (f\,x)^m\,{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

# 3.171 $\int (fx)^m \sqrt{d+ex^2} \left(a+b\csc^{-1}(cx)\right) \, dx$

**Optimal**. Leaf size=28

$$\text{Int}\left((fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

**Rubi** [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Defer[Int][(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

Rubi steps

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

**Mathematica** [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
[Out] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m\left(a+b\,\mathrm{arccsc}(cx)\right)\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
[Out] int((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2*e + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2*e + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b\,\mathrm{acsc}\,(cx))\sqrt{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral((f*x)**m*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (f\,x)^m\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

# 3.172 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

**Optimal**. Leaf size=28

$$\text{Int}\left(\frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

**Mathematica** [A]
time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccsc}(cx)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/sqrt(x^2*e + d), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsc(c*x) + a)*(f*x)^m/sqrt(x^2*e + d), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+b\,\mathrm{acsc}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

# 3.173 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

**Optimal**. Leaf size=28

$$\mathrm{Int}\left(\frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

[Out] Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]

[Out] Defer[Int][((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]

Rubi steps

$$\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

**Mathematica** [A]
time = 1.03, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]

[Out] Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+b\,\mathrm{arccsc}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

**Maxima** [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(x^2*e + d)^(3/2), x)
```

**Fricas** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsc(c*x) + a)*(f*x)^m/(x^4*e^2 + 2*d*x^2*e +
 d^2), x)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

**Giac** [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

# 3.174 $\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

**Optimal**. Leaf size=401

$$-\frac{4b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{7b\sqrt{1-c^2x^2}\,(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{13b\sqrt{1-c^2x^2}\,(1+c^2x^2)^{5/2}}{150c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{3b\sqrt{1-c^2x^2}\,(1+c}{70c^{13}\sqrt{1-\frac{1}{c^2x}}}$$

```
[Out] 1/3*(-c^4*x^4+1)^(3/2)*(a+b*arccsc(c*x))/c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+b*
arccsc(c*x))/c^12+7/90*b*(c^2*x^2+1)^(3/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c
^2/x^2)^(1/2)-13/150*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2
/x^2)^(1/2)+3/70*b*(c^2*x^2+1)^(7/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^2
)^(1/2)-1/90*b*(c^2*x^2+1)^(9/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^2)^(1
/2)+4/15*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^
2)^(1/2)-4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^2)^(
1/2)-1/2*(a+b*arccsc(c*x))*(-c^4*x^4+1)^(1/2)/c^12
```

**Rubi** [A]
time = 1.75, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {272, 45, 5355, 12, 6853, 6874, 862, 52, 65, 214, 797}

$$-\frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^{12}}-\frac{b\sqrt{1-c^2x^2}\,(c^2x^2+1)^{9/2}}{90c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{3b\sqrt{1-c^2x^2}\,(c^2x^2+1)^{7/2}}{70c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{13b\sqrt{1-c^2x^2}\,(c^2x^2+1)^{5/2}}{150c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{7b\sqrt{1-c^2x^2}\,(c^2x^2+1)^{3/2}}{90c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{4b\sqrt{1-c^2x^2}\,\sqrt{c^2x^2+1}}{15c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{4b\sqrt{1-c^2x^2}\,\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{15c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (-4*b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(15*c^13*Sqrt[1 - 1/(c^2*x^2)]*x
) + (7*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(90*c^13*Sqrt[1 - 1/(c^2*x^
2)]*x) - (13*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(150*c^13*Sqrt[1 - 1/
(c^2*x^2)]*x) + (3*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(7/2))/(70*c^13*Sqrt[1
 - 1/(c^2*x^2)]*x) - (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(9/2))/(90*c^13*Sqr
t[1 - 1/(c^2*x^2)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/(2*c^12) +
((1 - c^4*x^4)^(3/2)*(a + b*ArcCsc[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)*(
a + b*ArcCsc[c*x]))/(10*c^12) + (4*b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2
*x^2]])/(15*c^13*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{3c^{12}} - \frac{(1-c^4x^4)^{5/}}{}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^{12}} + \frac{(1-c^4x^4)^{3/2}\,(a+}{3c^{12}}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{7b\sqrt{1-c^2x^2}\,(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{13b\sqrt{1-c^2x^2}\,(1+}{150c^{13}\sqrt{1-\frac{}{c^2}}}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{7b\sqrt{1-c^2x^2}\,(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{13b\sqrt{1-c^2x^2}\,(1+}{150c^{13}\sqrt{1-\frac{}{c^2}}}$$

**Mathematica [A]**
time = 0.19, size = 194, normalized size = 0.48

$$-\frac{105a\sqrt{1-c^4x^4}\,(8+4c^4x^4+3c^8x^8)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1-c^4x^4}\,(768+36c^2x^2+78c^4x^4+5c^6x^6+35c^8x^8)}{-1+c^2x^2}+105b\sqrt{1-c^4x^4}\,(8+4c^4x^4+3c^8x^8)\csc^{-1}(cx)+840b\mathrm{ArcTan}\left(\frac{c\sqrt{1-\frac{1}{c^2x^2}}\,x}{\sqrt{1-c^4x^4}}\right)}{3150c^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/3150*(105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*c*Sqrt[1
- 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]*(768 + 36*c^2*x^2 + 78*c^4*x^4 + 5*c^6*x
^6 + 35*c^8*x^8))/(-1 + c^2*x^2) + 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 +
 3*c^8*x^8)*ArcCsc[c*x] + 840*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]*x)/Sqrt[1 -
 c^4*x^4]])/c^12
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}(a+b\,\mathrm{arccsc}(cx))}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*sq
rt(-c^4*x^4 + 1)/c^12) + 1/30*(30*c^12*integrate(1/30*(3*c^10*x^11 + 3*c^8*
x^9 + 4*c^6*x^7 + 4*c^4*x^5 + 8*c^2*x^3 + 8*x)*e^(-1/2*log(c^2*x^2 + 1) + 1
/2*log(c*x - 1))/(c^10*e^(log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1))
+ sqrt(-c*x + 1)*c^10), x) - (3*c^8*x^8*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
 1)) + 4*c^4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1)))*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1))*
b/c^12
```

**Fricas [A]**
time = 0.43, size = 238, normalized size = 0.59

$$-\frac{(35\,bc^8x^8+5\,bc^6x^6+78\,bc^4x^4+36\,bc^2x^2+768\,b)\sqrt{-c^4x^4+1}\,\sqrt{c^2x^2-1}-840\,(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4+1}}{\sqrt{c^2x^2-1}}\right)+105\,(3\,ac^{10}x^{10}-3\,ac^8x^8+4\,ac^6x^6-4\,ac^4x^4+8\,ac^2x^2+(3\,bc^{10}x^{10}-3\,bc^8x^8+4\,bc^6x^6-4\,bc^4x^4+8\,bc^2x^2-8\,b)\,\mathrm{arccsc}\,(cx)-8\,a)\sqrt{-c^4x^4+1}}{3150\,(c^{14}x^2-c^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3150*((35*b*c^8*x^8 + 5*b*c^6*x^6 + 78*b*c^4*x^4 + 36*b*c^2*x^2 + 768*b)
*sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1) - 840*(b*c^2*x^2 - b)*arctan(sqrt(-c^
4*x^4 + 1)/sqrt(c^2*x^2 - 1)) + 105*(3*a*c^10*x^10 - 3*a*c^8*x^8 + 4*a*c^6*
x^6 - 4*a*c^4*x^4 + 8*a*c^2*x^2 + (3*b*c^10*x^10 - 3*b*c^8*x^8 + 4*b*c^6*x^
6 - 4*b*c^4*x^4 + 8*b*c^2*x^2 - 8*b)*arccsc(c*x) - 8*a)*sqrt(-c^4*x^4 + 1))
/(c^14*x^2 - c^12)
```

**Sympy** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((x^11*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

# 3.175 $\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

**Optimal**. Leaf size=268

$$-\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}\,x}+\frac{b\sqrt{1-c^2x^2}\,(1+c^2x^2)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{b\sqrt{1-c^2x^2}\,(1+c^2x^2)^{5/2}}{30c^9\sqrt{1-\frac{1}{c^2x^2}}\,x}-\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(}{2c^8}$$

```
[Out] 1/6*(-c^4*x^4+1)^(3/2)*(a+b*arccsc(c*x))/c^8+1/18*b*(c^2*x^2+1)^(3/2)*(-c^2
*x^2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/30*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+
1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)+1/3*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x
^2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^
(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/2*(a+b*arccsc(c*x))*(-c^4*x^4+1)^(1/2)/c^
8
```

**Rubi** [A]

time = 1.59, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {272, 45, 5355, 12, 6853, 6874, 862, 52, 65, 214, 797}

$$\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^8}-\frac{b\sqrt{1-c^2x^2}\,(c^2x^2+1)^{5/2}}{30c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{b\sqrt{1-c^2x^2}\,(c^2x^2+1)^{3/2}}{18c^9x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{b\sqrt{1-c^2x^2}\,\sqrt{c^2x^2+1}}{3c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{b\sqrt{1-c^2x^2}\,\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{3c^9x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/3*(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
+ (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(18*c^9*Sqrt[1 - 1/(c^2*x^2)]*x
) - (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(30*c^9*Sqrt[1 - 1/(c^2*x^2)]
*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/(2*c^8) + ((1 - c^4*x^4)^(3/2
)*(a + b*ArcCsc[c*x]))/(6*c^8) + (b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*
x^2]])/(3*c^9*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
```

```
reeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{6c^8} + \frac{b\int \frac{(-2-c^4x^4)}{6c^8\sqrt{1-}}}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{6c^8} + \frac{b\int \frac{(-2-c^4x^4)}{\sqrt{1-}}}{6}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{6c^8} + \frac{\left(b\sqrt{1-c^2x^2}\right.}{6}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right.}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right.}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right.}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{6c^8} - \frac{\left(b\sqrt{1-c^2x^2}\right.}{}$$

$$= -\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^8} + \frac{(1-c^4x^4)^{3/2}\,(a+b}{6c^8}$$

$$= -\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{b\sqrt{1-c^2x^2}\,(1+c^2x^2)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{b\sqrt{1-c^2x^2}\,(1+c^2x^2)}{30c^9\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} + \frac{b\sqrt{1-c^2x^2}\,(1+c^2x^2)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{b\sqrt{1-c^2x^2}\,(1+c^2x^2)}{30c^9\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

**Mathematica [A]**
time = 0.26, size = 159, normalized size = 0.59

$$-\frac{15a\sqrt{1-c^4x^4}\,(2+c^4x^4)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1-c^4x^4}\,(28+c^2x^2+3c^4x^4)}{-1+c^2x^2}+15b\sqrt{1-c^4x^4}\,(2+c^4x^4)\csc^{-1}(cx)+30b\text{ArcTan}\left(\frac{c\sqrt{1-\frac{1}{c^2x^2}}\,x}{\sqrt{1-c^4x^4}}\right)}{90c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/90*(15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*
Sqrt[1 - c^4*x^4]*(28 + c^2*x^2 + 3*c^4*x^4))/(-1 + c^2*x^2) + 15*b*Sqrt[1
- c^4*x^4]*(2 + c^4*x^4)*ArcCsc[c*x] + 30*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]
*x)/Sqrt[1 - c^4*x^4]])/c^8
```

**Maple [F]**
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^7(a+b\,\mathrm{arccsc}(cx))}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

**Maxima [F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*(c^8*x^8*
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 6*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)
*sqrt(-c*x + 1)*c^8*integrate(1/6*(c^6*x^7 + c^4*x^5 + 2*c^2*x^3 + 2*x)*e^(
-1/2*log(c^2*x^2 + 1) + 1/2*log(c*x - 1))/(c^6*e^(log(c*x + 1) + log(c*x -
1) + 1/2*log(-c*x + 1)) + sqrt(-c*x + 1)*c^6), x) + c^4*x^4*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1)) - 2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(sq
rt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8)
```

**Fricas [A]**
time = 0.36, size = 181, normalized size = 0.68

$$-\frac{(3\,bc^4x^4+bc^2x^2+28\,b)\sqrt{-c^4x^4+1}\,\sqrt{c^2x^2-1}-30\,(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4+1}}{\sqrt{c^2x^2-1}}\right)+15\,(ac^6x^6-ac^4x^4+2\,ac^2x^2+(bc^6x^6-bc^4x^4+2\,bc^2x^2-2\,b)\,\mathrm{arccsc}\,(cx)-2\,a)\sqrt{-c^4x^4+1}}{90\,(c^{10}x^2-c^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/90*((3*b*c^4*x^4 + b*c^2*x^2 + 28*b)*sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1
) - 30*(b*c^2*x^2 - b)*arctan(sqrt(-c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) + 15*(a
*c^6*x^6 - a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^6*x^6 - b*c^4*x^4 + 2*b*c^2*x^2 -
 2*b)*arccsc(c*x) - 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 - c^8)
```

**Sympy** [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Timed out
```

**Giac** [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

**Mupad** [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((x^7*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

# 3.176 $\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

**Optimal**. Leaf size=126

$$-\frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}-\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^4}+\frac{bx\text{ArcTan}\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{-1+c^2x^2}}\right)}{2c^3\sqrt{c^2x^2}}$$

```
[Out] 1/2*b*x*arctan((-c^4*x^4+1)^(1/2)/(c^2*x^2-1)^(1/2))/c^3/(c^2*x^2)^(1/2)-1/
2*(a+b*arccsc(c*x))*(-c^4*x^4+1)^(1/2)/c^4-1/2*b*x*(-c^4*x^4+1)^(1/2)/c^3/(
c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)
```

**Rubi** [A]
time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {267, 5355, 12, 1586, 1266, 862, 52, 65, 214}

$$-\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^4}-\frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{b\sqrt{1-c^2x^2}\tanh^{-1}\left(\sqrt{c^2x^2+1}\right)}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/2*(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(c^5*Sqrt[1 - 1/(c^2*x^2)]*x)
- (Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/(2*c^4) + (b*Sqrt[1 - c^2*x^2]*Ar
cTanh[Sqrt[1 + c^2*x^2]])/(2*c^5*Sqrt[1 - 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1586

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Dist[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(
x^mn*e)))^FracPart[q]))/x^(mn*FracPart[q]), Int[x^(m + mn*q)*(1 + d*(1/(x^m
n*e)))^q*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] && E
qQ[n2, -2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rule 5355

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Dist[a + b*ArcCsc[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/
(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /; F
reeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^4} + \frac{b\int -\frac{\sqrt{1-c^4x^4}}{2c^4\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{c}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^4} - \frac{b\int \frac{\sqrt{1-c^4x^4}}{\sqrt{1-\frac{1}{c^2x^2}}\,x^2}\,dx}{2c^5}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^4} - \frac{\left(b\sqrt{1-c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}\,dx}{2c^5\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^4} - \frac{\left(b\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1-c^2x}}\,dx, x, x^{\cdots}\right.}{4c^5\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^4} - \frac{\left(b\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x}\,dx, x, x^2\right.}{4c^5\sqrt{1-\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{2c^5\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^4} - \frac{\left(b\sqrt{1-c^2x^2}\right)\mathrm{Su}\cdots}{4c\cdots}$$

$$= -\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{2c^5\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^4} - \frac{\left(b\sqrt{1-c^2x^2}\right)\mathrm{Su}\cdots}{\cdots}$$

$$= -\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{2c^5\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\,(a+b\csc^{-1}(cx))}{2c^4} + \frac{b\sqrt{1-c^2x^2}\,\mathrm{tanh}\cdots}{2c^5\sqrt{1-\cdots}}$$

**Mathematica [A]**
time = 0.11, size = 138, normalized size = 1.10

$$\frac{\left(a-bc\sqrt{1-\frac{1}{c^2x^2}}\,x-ac^2x^2\right)\sqrt{1-c^4x^4}-b(-1+c^2x^2)\sqrt{1-c^4x^4}\,\csc^{-1}(cx)+(b-bc^2x^2)\,\mathrm{ArcTan}\left(\frac{c\sqrt{1-\frac{1}{c^2x^2}}\,x}{\sqrt{1-c^4x^4}}\right)}{2c^4\,(-1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] ((a - b*c*Sqrt[1 - 1/(c^2*x^2)]*x - a*c^2*x^2)*Sqrt[1 - c^4*x^4] - b*(-1 +
c^2*x^2)*Sqrt[1 - c^4*x^4]*ArcCsc[c*x] + (b - b*c^2*x^2)*ArcTan[(c*Sqrt[1 -
 1/(c^2*x^2)]*x)/Sqrt[1 - c^4*x^4]])/(2*c^4*(-1 + c^2*x^2))
```

**Maple** [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\,\mathrm{arccsc}(cx))}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

**Maxima** [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^4*integrate(1/2*(c^2*x^3 + x)*e^(-1/2*log(c^2*x^2 + 1) + 1/2*log(c
*x - 1))/(c^2*e^(log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1)) + sqrt(-c
*x + 1)*c^2), x) - sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(1
, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/c^4 - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4
```

**Fricas** [A]
time = 0.39, size = 124, normalized size = 0.98

$$-\frac{\sqrt{-c^4x^4+1}\,\sqrt{c^2x^2-1}\,b-(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4+1}}{\sqrt{c^2x^2-1}}\right)+\sqrt{-c^4x^4+1}\,(ac^2x^2+(bc^2x^2-b)\,\mathrm{arccsc}\,(cx)-a)}{2\,(c^6x^2-c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1)*b - (b*c^2*x^2 - b)*arctan(sqrt(
-c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) + sqrt(-c^4*x^4 + 1)*(a*c^2*x^2 + (b*c^2*x
^2 - b)*arccsc(c*x) - a))/(c^6*x^2 - c^4)
```

**Sympy** [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\,\mathrm{acsc}\,(cx))}{\sqrt{-\,(cx-1)\,(cx+1)\,(c^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acsc(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)),
 x)
```

**Giac** [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

**Mupad** [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((x^3*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

# 3.177 $\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

**Optimal**. Leaf size=29

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

**Rubi** [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

**Mathematica** [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsc}(cx)}{x\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

**Maxima** [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*inte
grate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x +
 1)*sqrt(-c*x + 1)*x), x)
```

**Fricas** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arccsc(c*x) + a)/(c^4*x^5 - x), x)
```

**Sympy** [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}{\left(cx \right)}}{x \sqrt{- \left(cx - 1\right) \left(cx + 1\right) \left(c^{2} x^{2} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))),
x)
```

**Giac** [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

**Mupad** [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

# 3.178 $\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

**Optimal**. Leaf size=29

$$\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

**Rubi** [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Defer[Int][(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

**Mathematica** [A]
time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

**Maple** [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsc}(cx)}{x^5\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

**Maxima [A]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) + 2
*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^5), x)
```

**Fricas [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arccsc(c*x) + a)/(c^4*x^9 - x^5), x)
```

**Sympy [A]**
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsc}(cx)}{x^5 \sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsc(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral((a + b*acsc(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
), x)
```

**Giac [A]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsc(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

**Mupad [A]**
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^5\,\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asin(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
[Out] int((a + b*asin(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

# Chapter 4

# Appendix

## Local contents

# 4.1 Download section

The following zip files contain the raw integrals used in this test.

**Mathematica format** Mathematica_syntax.zip

**Maple and Mupad format** Maple_syntax.zip

**Sympy format** SYMPY_syntax.zip

**Sage math format** SAGE_syntax.zip

# 4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

## 4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

### 4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```